INTERVAL APPROACH FOR ROBUST FAULT DIAGNOSIS

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ABSTRACT

It is seen that methodology for diagnosis of uncertain systems using the Bond Graph (BG) model in Linear Fractional Transformation (LFT) form is an effective way to detect the fault by systematic generation of robust adaptive thresholds. There are limitations associated, many of which are imposed due to the manner uncertain parameters can be treated. In this work, a new method of generating robust and adaptive thresholds is developed, where uncertain parameters can be treated as intervals that vary between certain upper and lower bounds. The developed method proposes to generate the envelopes, using the interval extension form of Analytic Redundancy Relations (ARRs) in continuous time domain. The interval extensions of ARR functions are obtained by replacing the uncertain parameters with their interval equivalent. A very optimum range of such a function can be obtained by taking into account parameter variation in the interval. Superior and inferior values of the range determine the thresholds that form the envelope. The methodology developed is then implemented over an uncertain model of DC Motor wherein cases of parameter deviation with time, multiple uncertain parameters are considered.

Keywords: Uncertain parameters, Bond-Graph, BG-LFT, Interval, Fault diagnosis.

1. INTRODUCTION

Dynamic systems can be represented by continuous state space models as in (1),

$$\dot{x} = f(x, u, d, \theta_n) \tag{1}$$

$$y = g(x, u, d, \theta_n)$$

where $x \in \Re^n$, $u \in \Re^m, d \in \Re^I, y \in \Re^r, \theta \in \Re^p$, denote respectively the state vector, the control input vector, the disturbance vector, the output vector and the vector regrouping the model parameters.

The system parameters may vary around their nominal value in certain range. For example, a resistor element R, of nominal value 10Ω, may have manufacturing tolerance as 10Ω±1Ω. A mechanical friction coefficient will increase from its nominal value (which may be zero or otherwise), to a value permissible for normal functioning of the system because of thermal effects and ageing of the materials.

To represent uncertainty in a parameter, different forms can be used. Additive uncertainty as represented in (2) is equivalent to (3) in an interval form. As generalization, a non symmetrical interval of this form is represented as in (4). Multiplicative uncertainty can be represented as shown in (5).

$$\theta = \theta_n \pm \Delta\theta \, , \Delta\theta > 0 \tag{2}$$

$$\theta = [\theta_n - \Delta\theta, \theta_n + \Delta\theta] \tag{3}$$

$$\theta = [\theta_n - \Delta\theta_1, \theta_n + \Delta\theta_2] \tag{4}$$
with
$$\Delta\theta_1 \geq 0 \, , \Delta\theta_2 \geq 0 \; \Delta\theta_1 \neq \Delta\theta_2$$

$$\theta = \theta_n (1 \pm \delta\theta) \, , \; \delta\theta = \frac{\pm\Delta\theta}{\theta_n} \tag{5}$$

Robust diagnosis of uncertain systems has been an interesting, widely studied area of recent research works (Patton et al. 1989, Djeziri et al, 2007). Efficient and optimal methods for robust fault detection and isolation are required. Dauphin-Tanguy et al (1999), Djeziri et al (2006, 2007) highlight the method of using Bond Graph in Linear Fractional Transformation (LFT) approach as a powerful tool for this purpose. When using Analytical Redundancy Relationships (ARRs) for Fault Detection and Isolation (FDI), uncertainties in parameters generate envelopes around the nominal trajectories defining a domain corresponding to thresholds inside which the behaviour of the system can be considered as "normal" or "non-faulty". To avoid false alarms, the thresholds must be formed accurately and optimally.

The determination of ARRs on a bond graph model in preferred derivative causality is done by elimination of unknown variables contained in the structural constraints of junctions 0 and 1. The equations of power balance on the junctions constitute the ARRs candidate (Ould Bouamama et al. 2005). The bond graph model in

LFT form, obtained from multiplicative form of the uncertainties, allows the generation of the ARRs $r_{j,}$ with j=1,2…n_r. The residual $r_{j,}$ is composed of two completely separated parts:

- a nominal part r_{j_n}, of the residual r_j ,with j=1,2…n_r, as shown in (6). Here n_r is the number of residuals.

$$r_{j_n} = \Phi_j \left\{ \begin{matrix} Se, Sf, De, Df, De_m, Df_m, R_n, C_n, I_n, TF_n, \\ GY_n, RS_n \end{matrix} \right\} \qquad (6)$$

where TF_n and GY_n are respectively the nominal values of TF and GY moduli. R_n; C_n; I_n and RS_n are the nominal values of elements R; C; I and RS, De and Df being the measured variables and De_m and Df_m are the dualised signal sources. The method is well developed in Djeziri et al. (2006)).

- an uncertain part b_j, serves for the calculation of adaptive thresholds and sensitivity analysis, as shown in (7) and (8).

$$b_j = \sum w_i \qquad (7)$$

$$w_i = \Theta_i \left\{ \begin{matrix} Se, Sf, De, Df, De_m, Df_m, R_n, C_n, I_n, TF_n, \\ GY_n, RS_n, \delta_R, \delta_I, \delta_C, \delta_{TF}, \delta_{GY}, \delta_{RS} \end{matrix} \right\} \qquad (8)$$

where, δ_R ,δ_I, δ_C, δ_{RS}, δ_{TF}, δ_{GY} are values of multiplicative uncertainties as shown in (6).

The threshold being defined on residual r_j equation (9), it generates an envelope around nominal residual as in equation (10).

$$a_j = \sum |w_i| \qquad (9)$$

$$-a_j < r_j < a_j \qquad (10)$$

Because of the definition of the thresholds itself as in Eq. (9), using the absolute values, the envelope determines a domain around the nominal trajectory with symmetrical upper and lower frontiers which is efficient when the uncertain parameters are defined as $\theta_n - \Delta\theta \leq \theta \leq \theta_n + \Delta\theta$ (case of manufactured components with a defined tolerance). It may induce an overestimation of the envelope if the bounds are not symmetrical as $\theta_n - \Delta\theta_1 \leq \theta \leq \theta_n + \Delta\theta_2$ with $\Delta\theta_1 \neq \Delta\theta_2$, or when the uncertainty appears in one side only as in case of irreversible deviation with $\theta_n \leq \theta \leq \theta_n + \Delta\theta$ (case of time deviation).

It is seen, even though BG-LFT approach offers an effective method for fault detection by adaptive threshold generation that it is not successful in accounting all various types of uncertainty found commonly. As such, treatment of uncertain parameters as intervals offers much scope and better methods for FDI.

In this work, the objective is to treat the uncertain parameters as intervals and use them to generate adaptive thresholds. The new method presented here generates adaptive thresholds and forms envelopes, using the interval extension form of the ARRs in continuous time domain which helps in fault detection and identification. Besides this section of Introduction, paper is divided into four other sections. Section 2 introduces the method of treating the uncertain parameters, features of INTLAB as the software tool to carry out simulations is given, interval extensions of real functions are briefly discussed to introduce the interval extensions of ARRs which are subsequently used to generate the thresholds. Further, the methodology is proposed .In section 3, the methodology is implemented over a model of standard direct current (DC) motor example. Section 4 contains the simulations and results. Finally, conclusions are drawn in the final section 5.

2. INTERVAL APPROACH FOR ROBUST DIAGNOSIS

The interval analysis was initially developed to hold account on the inaccuracies of the numbers. These inaccuracies come from data resulting from a chain of instrumentation or from computing tool. Interval analysis in the past one decade has been exploited to a great extent in many disciplines of engineering. Because of interval arithmetic's power to bound ranges of functions, interval arithmetic has been most successful in solution of nonlinear systems and global optimization (Hansen E.R, 1992). Much information can be found in the literature available, concerned with many disciplines of science and maths where it is used to obtain rigorous proofs or results (Jaulin, Keiffer, Didrit and Walter, 2001).

Early work on treatment of uncertain parameters as intervals and subsequent usage for diagnosis is found in works of Adrot et al, (2000). The approach, called bounded approach, represented these uncertainties by a set of possible values for which only their bounds were known. Ragot et al., (1997) proposed an interval technique for the detection and the isolation of sensor faults in the case of a static linear model .The case is treated for dynamic systems by Armengol. et al, (2003). D. Maquin and J. Ragot (Ragot et al, 2003a) treated the problem of data validation in the case of certain systems with uncertain measurements through interval approach. A method of data validation is developed based on bounded approach, for dynamic uncertain linear systems, where all the state variables of the system are measured (Ragot et al, 2003b). Here the proposed method treats the case where the state variables of the system are partially measured. The state estimation method is described which uses interval approach. Finally, the method is used to detect, isolate and correct the data affected by faults. In the works of Fagarasan (2004), Rinner and Weiss (2004), interval calculation laws are used to generate the exact estimate of the output, bounds of the estimates are computed using traditional numerical integration techniques from the uncertain parameter interval vertices, assuming that monotonicity property holds. Thus, the envelopes

generated, are primarily by the estimation of state or parameter.

2.1. INTLAB – The Choice

Many software tools are successful in implementation of interval arithmetic, prominent of them being, INTLAB (Rump, 1998), Sun's Forte C++ compilers with interval support (Sun Microsystems, 2001), filib++ (Lerch et al, 2000) and C-XSC (Hofschuster, 2001).Our choice to implement the interval computations and carry out simulations is INTLAB.

INTLAB (Rump, 1998; 1999) is a well-designed interval toolbox for the interactive programming environment MATLAB (The Math Works, 2001). It is integrated with MATLAB, providing immediate access to an extensive range of tools that lets us develop algorithms, analyze and visualize simulations. It allows the more traditional infimum-supremum as well as the midpoint-radius representations of intervals. That is to say, parameter variation with different value of deviations may be treated by declaring it as an interval by infimum-supremum .Variations with same deviations may be declared using 'midrad' notation. As example $\theta = [\ \theta_n - \Delta\theta_1,\ \theta_n + \Delta\theta_2\ ]$ may be declared as *infsup* (θ_n-$\Delta\theta_1,\ \theta_n + \Delta\theta_2$). When $\theta = [\ \theta_n - \Delta\theta,\ \theta_n + \Delta\theta\ ]$, interval may be declared in midpoint-radius format as *midrad*(θ_n, $\Delta\theta$). Here nominal value θ_n is the centre and deviation $\Delta\theta$ is the radius in positive and negative direction.

Every computation using INTLAB is rigorously verified to be correct, including input and output. All the algorithms written in INTLAB, may be implemented in MATLAB. Since it is implemented in MATLAB, it can also be used by SIMULINK for simulations. Mathematical routines written in MATLAB code can be modified and run in INTLAB. INTLAB offers predefined problem solving routines for dense and sparse systems of linear and nonlinear equations and eigenvalue problems. A multi precision interval arithmetic, a slope arithmetic as well as routines automatic differentiation are also included (Hargreaves, 2002). The main features of INTLAB can be explored through demo files included as .m files in the package. To work with systems in real time, Real-Time Windows Target™(RTWT) provides a real-time engine for executing Simulink® models on a Microsoft® Windows® PC .Thus, using INTAB will facilitate easy interfacing of systems involving interval computations with Simulink and thus, in real time with RTWT. However as direct libraries for interval variables are not available in SIMULINK, its execution in RT-WT for real-time application may face some major difficulties.

2.2. Interval Value Extensions of Real Functions

Interval-valued functions are obtained by selecting a real-valued function $\mathbf{f}$ and computing the range of values $\mathbf{f}(x)$ takes as x varies through some interval X. By definition, the result is equal to the set image $\mathbf{f}(X)$. Now, *interval valued extensions* of functions are obtained by *extending* a given real-valued function $\mathbf{f}$

by applying its *formula* directly to interval arguments (Moore, 1966).

Consider the real-valued function $\mathbf{f}$given by (11). The entity is a formula, not a function. Now the formula (12) is taken that describes function (11) and applied to the interval arguments. The resulting interval-valued function is the *interval extension* of the function as in (13).

$$\mathbf{f}(x) = 1 - x, \qquad x \in \text{R.} \tag{11}$$
$$f(x) = 1 - x \tag{12}$$
$$\mathbf{F}(X) = 1 - X, \quad X = [\underline{X}, \overline{X}] \tag{13}$$

This way the domain is enlarged to include non-degenerate intervals and the degenerate intervals x=[x, x]. $\mathbf{F} = \mathbf{F}(X_1,..., X_n)$ is called *inclusion isotonic* (Jaulin et al,2001) if equation (14) is satisfied when

$$Y_i \subseteq X_i \text{ for } i = 1,...,n$$
$$\mathbf{F}(Y_1,..., Y_n) \subseteq \mathbf{F}(X_1,..., X_n). \tag{14}$$

If $\mathbf{F}$ is an inclusion isotonic interval extension of $\mathbf{f}$, then according to the *Fundamental Theorem of Interval Analysis* (Moore, 1979),

$$\mathbf{f}(X_1,..., X_n) \subseteq \mathbf{F}(X_1,..., X_n) \tag{15}$$

This theorem is never violated in cases of inclusion isotonic functions. Now, all rational interval functions are inclusion isotonic. Thus, an interval value of $\mathbf{F}$ contains the range of values of the corresponding real function $\mathbf{f}$ when the real arguments of $\mathbf{f}$ lie in the intervals.

This forms a mean for the finite evaluation of upper and lower bounds on the ranges of values of real rational functions.

2.3. Interval Extensions of the Analytic Redundancy Relations (ARRs)-The Approach

The finite evaluation of upper and lower bounds on the ranges of values of real rational functions becomes very useful to find the range of interval extension of ARRs and thus, generate the thresholds. The analytic redundancy relations (ARRs) are essentially functions of measurement of state variables with physical parameters as coefficients. They are treated as rational functions, then their interval extension obeys the above relation as in Eq. (15).

Our approach is based on the above principle. The ARRs are obtained from bond-graph model in a preferred derivative causality. These ARRs are functions of known variables (sensor measurements, control inputs) with system uncertain parameters as coefficients. Consider an ARR as,

$$r_j = f_j(y, u, \theta_n) \tag{16}$$

with j=1,2.....n_r, where n_r is the total number of ARRs.

The interval extension of the ARRs is obtained by treating the ARR as a rational polynomial and putting intervals of the coefficient uncertain parameters as,

$$R_j = F_j(y, u, [\theta]) \qquad (17)$$

The upper bound and lower bound of R_j are evaluated as the value of θ varies within the interval $[\theta]$ and from Eq.(15), give the envelope such that

$$r_j \subseteq R_j, \quad \text{whenever} \quad \theta \in [\theta_n - \Delta\theta_1, \theta_n + \Delta\theta_2] \qquad (18)$$

These upper and lower bound values form the thresholds and hence, the envelope is generated.

Remarks:

- The envelope generated by R_j guarantees to contain the residual r_j only when all the considered parameters are within their respective interval bounds.

- If the residual r_j goes out of the envelope then it surely means that either one or more of the uncertain parameter has gone beyond its considered interval range.

- When one or more, *but not all* considered parameters go beyond their interval range, the residual may remain inside the envelope. So, the envelop guarantees to contain the residual only when *all* the parameters considered in ARR as intervals are within their bound.

Thus, in case of multiple parameter uncertainties, set of combination of envelops should be considered on various ARRs to identify the fault occurred.

3. APPLICATION ON DC MOTOR

This section presents the application of the proposed approach on a DC Motor model with uncertain parameters. Firstly, the specification of DC motor is given after which, BG model of same in derivative causality is used to generate the ARRs. Intervals extensions of ARRs are considered to generate the robust envelop. Importantly, cases of parameter deviation, ageing, or a catastrophic fault are considered through simulations.

3.1. Description of the DC Motor System

The model parameters are as resistance of stator, $R = 2.4\ \Omega$; inductance of the stator, $L = 0.44$ H; coefficient of couple, $k_e = 0.14$; moment of inertia of rotor, $J = 0.08$ kg m^2; coefficient of friction of motor shaft, $f = 0.01$ NS/ m ;the inputs being the voltage of the inductor $U(t)=220$ V, the load torque $\tau(t)=5$ Nm; and the observed outputs being current of inductor $i_m(t)$ and angular velocity of the motor shaft $\omega_m(t)$.
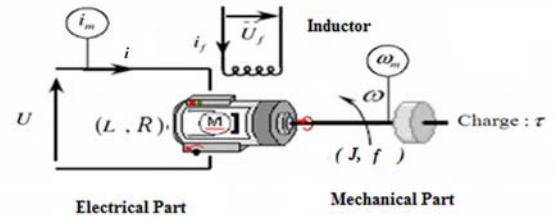

Figure 1: DC motor system representation.

The system can be modelled by bond graph in integral causality and can be represented in state space form as:

$$\begin{cases} \begin{bmatrix} \dot{i} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{F_l}{L} \\ \dfrac{F_l}{J} & -\dfrac{f}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} & 0 \\ 0 & -\dfrac{1}{J} \end{bmatrix} \begin{bmatrix} U \\ \tau \end{bmatrix} \\ \\ y = \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} \end{cases} \qquad (19)$$

3.2. Derivation of Analytic Redundancy Relations

The bond graph model of the system in derivative causality (BGD) is given in Fig.2. The electrical motor is characterized by the resistance R and inductance *L*. The mechanical part of the motor is characterized by friction *f*, inertia *J* and the load torque τ. The conversion of electrical energy into mechanical energy is represented by a transformer GY.
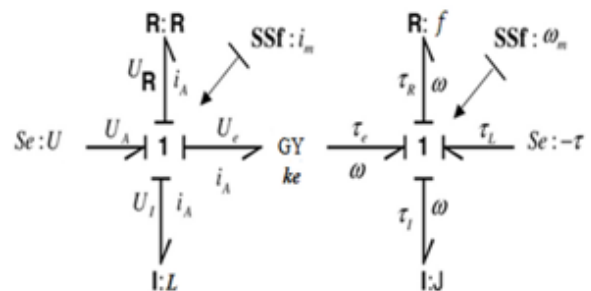

Figure 2: Bond graph model of DC motor in derivative causality with dualized sensors.

Here, the sensors have been dualized as corresponding sources of signal (SS). For simulation purpose, the measurements of sensors can be obtained from the BGI model, Eq.(19). But for unknown variable elimination to generate an ARR, the initial point is the source of signal. This covering of causal paths leads to well known oriented graphs. Any ARR candidate can be derived very easily from this model by considering the balance of powers on both junctions. The algorithm can be consulted in (Samantaray and Ould Bouamama, 2008).They are obtained as equations (20) and (21).

$$r_1 : U - R_n i_m - L_n \frac{di_m}{dt} - k_{en}\omega_m = 0 \qquad (20)$$

$$r_2 : k_{en} i_m - f_n \omega_m - J_n \frac{d\omega_m}{dt} - \tau = 0 \qquad (21)$$

Note that here all the parameters considered are with nominal values.

3.3. Interval Extensions of the ARRs

Next, the interval extension of r_1 and r_2 are considered by replacing the nominal parameters by their interval counterparts.

$$R_1 : U - [R]i_m - [L]\frac{d(i_m)}{dt} - \omega_m[ke] = 0 \qquad (22)$$

$$R_2 : i_m[k] - [f]\omega_m - [J]\frac{d(\omega_m)}{dt} - \tau = 0 \qquad (23)$$

Here, $[\theta]$ denotes $[\theta_n - \Delta\theta_1, \theta_n + \Delta\theta_2]$ and deviations in either direction may be equal (a special case). Interval bounds are considered on all the parameters. This can be done selectively too, depending upon which parameter we want to consider for envelope. Thus, R_1 as Eq. (22) and R_2 as Eq. (23) are the interval extensions of r_1 and r_2. They are evaluated by INTLAB in offline mode.

By definition, R_1 and R_2 are not rational polynomials as they have terms with differential operator such as $\frac{di_m}{dt}$ in R_1 and $\frac{d\omega_m}{dt}$ in R_2. However, if the values of such a term (say $\frac{d\omega_m}{dt}$) is obtained externally, then they can be directly substituted in the expression during evaluation by INTLAB. In our case, such expressions are evaluated "externally" in SIMULINK where the differential operation on i_m in (22) and on ω_m in (23) is done in continuous time domain. The obtained values are then "sent" to INTLAB where the interval expressions (22) and (23) are evaluated as a rational expression.

R_1 and R_2 can thus be treated as rational interval polynomials. This is essential, to have them inclusion – isotonic which is a necessary condition to satisfy the fundamental theorem of interval analysis.

4. SIMULATIONS AND RESULTS

Parameters are considered selectively, in different cases with each having interval bound as $\theta = [\theta_n - \Delta\theta_1, \theta_n + \Delta\theta_2]$ or $\theta \in [\theta_n - (0.1*\theta_n), \theta_n + (0.1*\theta_n)]$, for normal (non-faulty functioning).In following cases, $\Delta\theta_1 = \Delta\theta_2 = \Delta\theta$. But, in general such is not the case. Other cases when $\Delta\theta_1 \neq \Delta\theta_2$ can also be treated similarly and efficiently. It is not done here for sake of clarity.

Note: In the following cases parameter deviation such as $\Delta\theta_e$ denotes the parameter deviation considered to form the envelopes (subscript 'e' denotes deviation to form envelope) and is always considered as

$$\Delta\theta_e = 0.1*\theta_n .$$

4.1. Case 1: Only one parameter uncertain for envelope generation

The envelope is generated by considering interval on frictional parameter, f only. Thus, the interval extension of residual r_2 is obtained as R_2 in (24).

$$R_2 : i_m ke(t) - [f]\omega_m - J_n \frac{d(\omega_m)}{dt} - \tau = 0 \qquad (24)$$

The parameter deviation for envelope generation Δf_e, is taken as $\Delta f_e = 0.1* f_n$ so that, $[f] = [f_n - \Delta f_e, f_n + \Delta f_e]$. The envelope in Fig.3, Fig.4 and Fig.5 is deduced from the interval extension expression (24). Nominal parameters are considered.

In Fig 3, residual r_2 corresponds to $\Delta f = 0$, the ideal condition, residual is zero. In Fig 4, residual r_2 corresponds to $\Delta f = 0.08* f_n$, parameter deviates but below interval bound of envelope ($\Delta f_e = 0.1* f_n$), residual is below upper threshold and bound by the envelope. In Fig 5, residual r_2 corresponds to $\Delta f = 0.15* f_n$, the damage (faulty) condition, residual is well above the threshold, beyond the envelope.
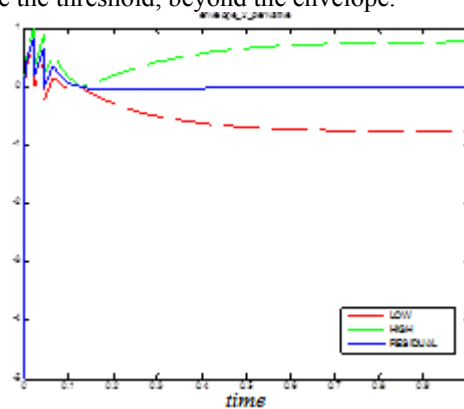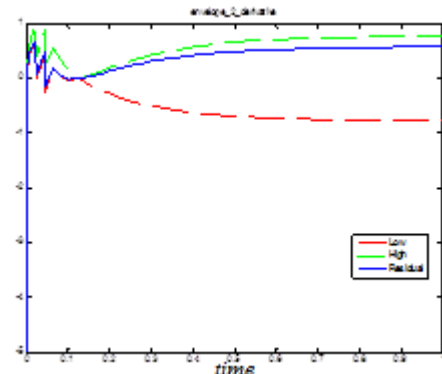

Figure 3: Residual r_2 with $\Delta f = 0$
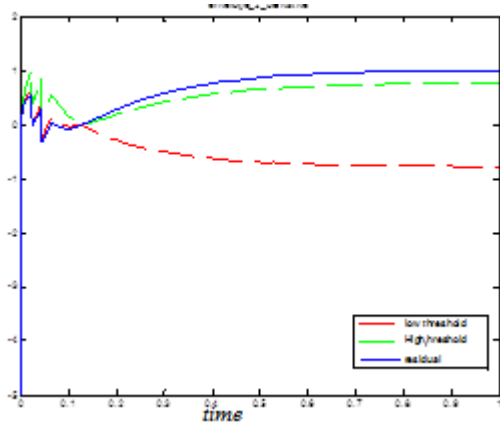

Figure 4: Residual r_2 with $\Delta f = 0.08* f_n$

Figure 5: Residual r_2 with $\Delta f = 0.15 * f_n$

4.2. Case 2: A Parameter (friction) variation with time

In this case, the parameter is considered to vary with time. So, after a finite time, it goes outside the permissible limit. It is shown in Fig.6. Envelope is built by considering interval on frictional parameter, $\Delta f_e = 0.1 * f_n$. The residual corresponds to situation when the parameter is time varying, and goes out of interval limit after a certain time ie. $f = \Theta(t)$. It can be seen, if and when a parameter deviates outside its interval limit. This way, a slow ageing (a case of slow parameter variation with time) of a parameter can be identified.
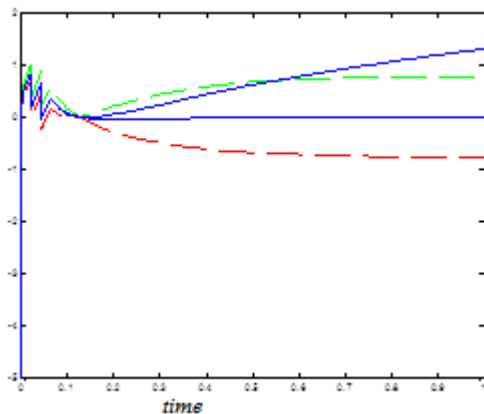

Figure 6: Parameter variation with time, Residual r_2 in blue.

4.3. Case 3: Abrupt-sudden change in a parameter

We consider the friction here to show a sudden, non-called for variation. A pulse is introduced as variation in friction (something similar associated with dry friction behaviour). The pulse variation can easily be detected by the residual. If the pulse amplitude is greater than that of interval limit, then it causes the residual to go outside the envelope. It is depicted in Fig.7. Here, the pulse is introduced at time 0.5 sec approx. in the frictional parameter, f. It is well reflected in the residual, causing it to go outside the envelope depending on the amplitude. Thus, fault is detected at

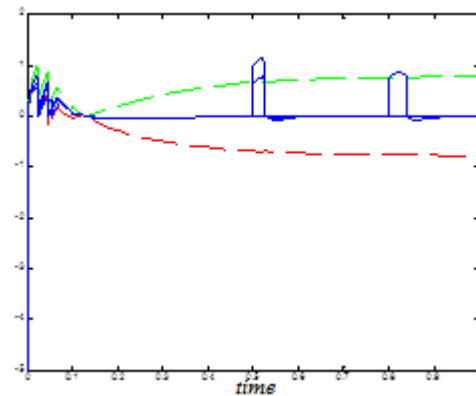instant t=0.5 s. At t=0.82 s, the pulse amplitude is just equal to the threshold limit

Figure7: Abrupt behaviour of a parameter

4.4. Case 4: Two parameters uncertain for envelope generation

In such cases, envelopes are built considering all the parameters that are likely to deviate. Envelope is built by considering interval on both friction f and ke. This generates the envelopes as seen in Fig.8 and Fig.9. In both figures envelope is generated considering,
$\Delta ke_e = 0.1 * ke_n$, and $\Delta f_e = 0.1 * f_n$.
In Fig 8, residual r_2 corresponds to $\Delta f = 0$, $\Delta ke = 0$ the ideal condition, residual is zero. In Fig.9, the residual corresponds to, $\Delta ke = 0.08 * ke_n$, and $\Delta f = 0.08 * f_n$ (both below the interval limit.) The residual remains successfully inside the envelope. In Fig.10, the residual r_2 is drawn for $\Delta ke = 0.15 * ke_n$, and $\Delta f = 0.15 * f_n$ (both above the interval limit.) The residual goes outside the envelope. So, it is seen that when both parameter go outside their limit, then residual goes out of the envelope too.
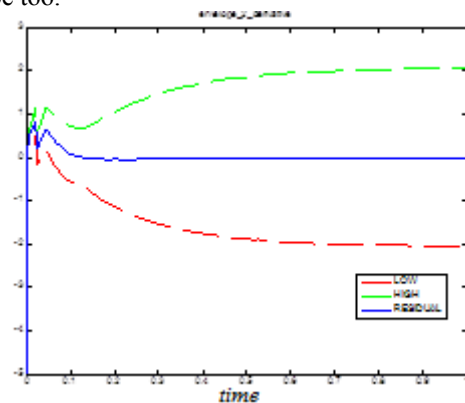

Figure 8: Residual r_2 with $\Delta f = 0$, $\Delta ke = 0$

The envelope generated, guarantees to contain the residual, only when <u>all</u> the considered parameters are within their interval bounds. Its converse is however, not true.

Conversely, (a) if one of them goes beyond its interval limit (say friction f. not ke) , then the residual might go outside. and equivalently,(b) one of the

parameter may go beyond its interval limit(say *ke*, not friction), yet the residual may be inside the envelope.

The former situation (a) is alright, as it however, results in fault detection as residual goes out. But, latter situation (b), can result in missed alarms.
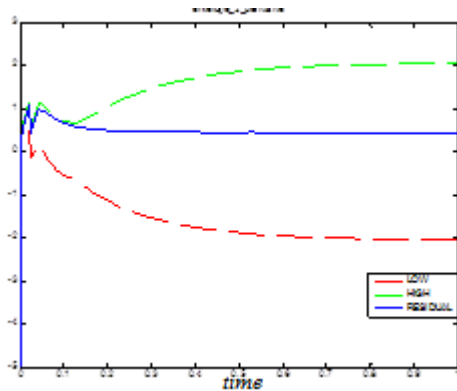

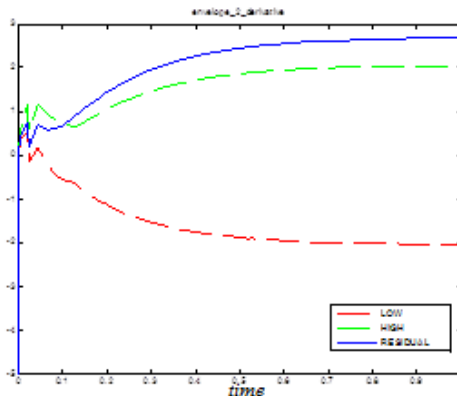Figure 9: Residual r_2 with $\Delta ke = 0.08 * ke_n$, $\Delta f = 0.08 *f_n$

Figure 10: Residual r_2 with $\Delta ke = 0.15 * ke_n$, $\Delta f = 0.15 *f_n$

In Fig.11, envelope is generated by considering, the two parameters friction *f* and *ke* (like in previous three figures.).One of the parameter, *ke* goes faulty, $\Delta ke = 0.12 * ke_n$ and friction *f* is nominal. $\Delta f = 0 *f_n$. The residual however, remains inside. This is due to the relative influence of the parameters and is linked to their order of magnitude.
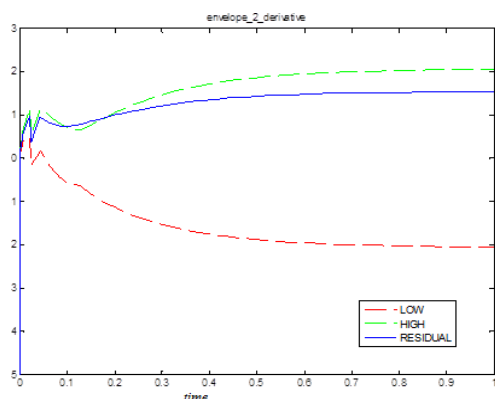

Figure11: Envelope considers both *f* and *ke* ($\Delta ke_e = 0.1 * ke_n$, and $\Delta f_e = 0.1 *f_n$). Residual r_2 corresponds to

situation when, $\Delta ke = 0.12 * ke_n$, $\Delta f = 0 *f_n$.

To diagnose such cases, set of envelopes are built as shown in Fig.12.Env 1- by considering only 'suspected' variable that will deviate *ke* and Env 2- by considering both *ke* and *f*. Then, as shown in Fig.12 the residual will not be included in Env 1- showing *ke* deviates and is included in Env -2, showing *f* doesn't deviate.
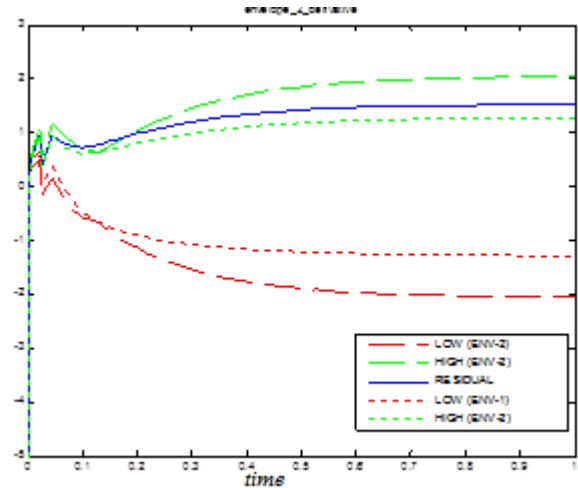

Figure 12: Env 1-considers only *ke* as $\Delta ke_e = 0.1 * ke_n$, Env-2 considers both *ke* and *f* as $\Delta ke_e = 0.1 * ke_n$, and $\Delta f_e = 0.1 *f_n$. Residual r_2 has $\Delta ke = 0.12 * ke_n$, $\Delta f = 0 *f_n$.

5. CONCLUSIONS

This method of fault diagnosis by generation of thresholds from the interval value extensions of analytical redundancy relations (ARRs) is robust and efficient. There are no limitations in terms of nonlinearities. The envelopes guarantee to contain the residual when all the parameters considered for the envelope, vary within their interval range. System with single or multiple uncertain parameters can be dealt effectively, by creating a series of envelopes. Parameters are treated as intervals for threshold generation, and thus a parameter with variations different in either side or with deviation that varies with time, can be treated easily, which is not possible with the BG-LFT approach. This renders a much more freedom in treating the uncertain parameters accurately. This results in efficiently optimized envelopes.

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