ON THE PARTIAL GUIDANCE OF AN AUTONOMOUS BLIMP BY VARIOUS FEEDBACK LAWS

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ABSTRACT
In this paper, the problem of partial asymptotic stabilization of the nonlinear autonomous under actuated airship (AUA) by various feedback laws is investigated. It has been shown that the AUA's is not stabilizable via continuous pure-state feedback. This is due to (Brockett 1983), necessary condition. In order to cope with this difficulty, we propose in asymptotically eleven components in finite-time or exponentially, while the remaining one converges.

Keywords: Airship, attitude control, discontinuous controllers, finite-time partial stabilizability.

1. INTRODUCTION

Control problems of aerospace engineering have recently drawn considerable attention in the control community. The rigid spacecraft, the rigid aircraft and the airship are examples of such systems. These systems are presented in cascade structure and have fewer actuators than the system degree of freedom.

For this reason, the tools from linear control theory are not sufficient, and stabilization techniques need to be reconsidered. Indeed, it has been proved by (Sontag and Sussmann 1980), that all nonlinear controlled systems in dimension one cannot be stabilized by continuous feedback laws. As a solution for this problem, the authors proposed piecewise continuous feedback laws. This obstruction to stabilizability in dimension one is generalized for nonlinear control systems by a number of authors. The first one was given by (Brockett 1983), for all controllable systems and (Ryan 1994), only for continuous control systems and (Coron and Rosier 1994), for the stabilizability of systems with drift. (Ryan 1994), proved that Brockett's condition is still necessary for stabilizability by discontinuous feedback laws in Ryan's sense. Also, (Coron and Rosier 1994), proved that Brockett's condition is still necessary for the stabilizability of systems with drift by means of discontinuous feedback laws and the solutions are defined in Filippov's sense. A strong homology necessary condition for stabilizability by dynamic feedback laws was given by (Coron 1990).

An article traced by (Samson 1991), has proved that continuous time-varying feedback laws can be interesting to stabilize many systems which cannot be stabilized by continuous pure-state feedbacks. This has been demonstrated by Coron's results in the famous paper (Coron 1995), which established that most STLC -small-time locally controllable- systems can be stabilized in finite time by continuous time-varying feedback. The obtained result leads to rich research in this area, namely:

- Time-varying periodic controllers: (Beji, Abichou, and Bestaoui 2004; Coron 1992; Coron 1995; Coron 2007; Coron and d'Andréa Novel 1992; Coron and Keraï 1996; M'Closkey and Murray 1997; Morin 2004; Morin and Samson 1997; Morin, Samson, Pomet, Ping, and Jiang 1995; Pettersen and Egeland 1996; Pettersen and Egeland 1999 ; Pettersen and Nijmeijer 2001; Samson 1991; Samson 1995), and the references therein,
- discontinuous controllers: (Astolfi 1996a; Astolfi 1996b; Coron and Rosier 1994; Sontag and Sussmann 1980; Sussmann 1979),
- the partial asymptotic or finite-time stabilization by continuous or discontinuous feedback laws: (Jammazi 2008a; Jammazi 2008b; Jammazi 2010; Jammazi 2011)

In this paper, we will focus our attention on the third approach. It consists of the concept of the Partial Asymptotic Stabilization (PAS). This concept means the asymptotic stabilization with respect to the maximum components of the system; while the remaining components are convergent and not necessarily toward an equilibrium point.

In (Jammazi 2008b), we have developed the backstepping techniques for the partial asymptotic stabilizability. This result was used to solve the partial asymptotic stabilization of many controllable cascaded systems that do not satisfy Brockett's necessary condition. Differentiable stabilizing feedback laws for the rigid spacecraft and for the ship are derived. For both systems, these stabilizing feedbacks make five components asymptotically stable and one component converges; in particular we have improved the (Zuyev 2001), feedbacks for the rigid spacecraft which states that the angular velocity of the third axes is only bounded, and for the ship system we have improved (Wichlund, Sørdalen, and Egeland 1995), feedbacks which states that the yaw angle is only bounded. Moreover, in (Jammazi 2010), we have provided a rigorous formulation of the theory of asymptotic partial stability, respectively, the finite-time partial stability of continuous autonomous systems. Sufficient conditions are derived with applications in control design. For example, we have proved in (Jammazi 2010) that the partial stabilization of the ship can be achieved in finite-time by continuous or discontinuous bounded state feedback laws.

In (Jammazi 2011), we have studied the finite-time partial stability of a prototype system of nonholonomic control systems which is the benchmark knife edge or the unicycle robot system called also the Brockett's integrator. We have proposed various feedback controllers that achieve the partial asymptotic stabilizability, or the finite-time partial stability of the mobile robot. These feedbacks are Hölderian for the rational partial stability, continuous and homogeneous of negative degree or discontinuous and quasi-homogeneous of negative degree for the finite-time partial stability.

The airship is the subject of numerous papers and thesis; (Hygounenc 2001), (Hygounenc 2003), (Zhang and Ostrowski 1999), (Beji and Abichou 2005; Beji, Abichou, and Bestaoui 2004), (Bestaoui 2006) and (Samaali, Abichou, and Beji 2007), and references therein.

As cited in (Samaali, Abichou, and Beji 2004), the problem of adding physical parameters of the blimp into the image plane for the performance of vision-guided control is discussed in (Zhang and Ostrowski 1999).

In (Beji and Abichou 2005), the problem of tracking control for ascent and descent flight with only three controllers is addressed. The authors supposed that roll is totally unactuated.

In (Bestaoui 2006), the problem of generation of characterization nominal trajectories (flight path) to be followed by an autonomous airship is addressed. In (Samaali, Abichou, and Beji 2007), the authors have studied the stabilization with respect to longitudinal and horizontal planes. By using iterative backstepping techniques combined with Lyapunov theory, homogeneity and averaging theorems, the authors have shown that the stabilization is possible via continuous time-varying feedback laws.

In this paper, our objective is to solve the stabilizing control problem of attitude and position for underactuated airship using only three available controls: the main and tail thrusters and the tilt angle of the propellers. The roll is totally unactuated. The same input controls both pitch and surge, while yaw and sway are related.

It was shown in (Beji, Abichou, and Bestaoui 2004), that the stabilization problem of autonomous airship by regular state feedback laws in the usual sense is not possible. As a solution of this problem, the authors have proposed time-varying feedback laws. The proposed method uses the averaging method and homogeneous exponential stability developed in (Morin and Samson 1997).

Note that all papers (Beji and Abichou 2005; Beji, Abichou, and Bestaoui 2004; Bennaceur 2009; Bestaoui 2006; Samaali, Abichou, and Beji 2007) cited here have treated the LSC'AS200 airship (Figure 1).

In fact, introducing the time in these feedback laws produces "undesirable" oscillations of the system around his equilibrium point, (Beji, Abichou, and Bestaoui 2004; Coron 1992 ; M'Closkey and Murray 1997; Morin and Samson 1997; Morin, Samson, Pomet, and Ping Jiang 1995; Pettersen and Egeland 1996; Samson 1991; Samson 1980), for more general systems. To get around the problem of impossibility to stabilize the autonomous airship by pure and regular feedback laws, and to overcome the drawback of the time dependence of these feedback laws, the stabilization of the airship should be solved via static feedbacks in partial asymptotic stabilizability sense.

The obtained results show that we can ensure the asymptotic stabilizability of eleven states of variables, and convergence of the remaining one. In the first approach, by using the backstepping techniques and partial asymptotic stabilizability developed in (Jammazi 2008a), we have shown that the LSC'AS-200 blimp can be stabilized partially exponentially by linear feedbacks. In the second approach, we have proved that the blimp can be stabilized partially in finite-time by means of continuous state feedback laws. However, the airship is an example of system with drift in which the (Coron and Rosier 1994), condition fails to be stabilized by discontinuous feedback laws. For this reason, to get around this obstruction, we have developed discontinuous state feedback laws that make the blimp stable in finite-time with respect to six components (which are the position $(x, y, z)'$ of the blimp in the inertial frame, and the linear velocities $(\mathrm{u}, \mathrm{v}, \mathrm{w})'$ in surge, sway and heave decomposed in the body-fixed frame), this leads by linearization to exponential stability of five components (which are $(p, q, r)'$: angular velocities in roll, pitch and yaw decomposed in the body-fixed frame, and (e_1, e_2) the orientation of principal axis (n_x, n_y)) and therefore the convergence of the orientation angle e_3 with respect to axis n_z. The stabilization by discontinuous feedback law appears significant, despite the presence of chattering phenomenon (Orlov 2009).

This paper is structured in this way: The section 2 contains some mathematical preliminaries. The stabilization strategies of the model of airship by various state feedback laws are the subject of Section 3. The theoretical results are confirmed by simulations in Section 4 and the conclusion is given in Section 5.

Throughout the paper, $|\,.\,|$ denotes the Euclidean norm in $\mathbb{R}^n$, $\|.\|$ denotes the Euclidean norm in $\mathbb{R}^{n\times m}$ defined by, $\|A\| = \sup_{i,j} |a_{ij}|$ for $A = a_{ij}, 1 \le i \le n, 1 \le j \le m$, $'$ is the symbol of transposition and sgn is the function "sign" and $A \cong B$ means A is diffeomorph to B.

2. PRELIMENAIRES

The double integrator is a key system that can appears in all underactuated dynamical systems. For this reason, the stabilization of such system is an interesting area of many works (Bhat and Bernstein 1998; Hong, Yang, Cheng, and Spurgeon 2004; Huang, Lin, and Yang 2005; Orlov 2005; Orlov 2009). In this section, we begin with review some results concerning the stabilization in finite-time of the double integrator

$$\begin{cases} \dot{x} = y, \\ \dot{y} = u. \end{cases} \qquad (1)$$

The system (1) can be stabilized by two classes of feedbacks: continuous and discontinuous state static feedback laws which are presented in the following lemmas.

Lemma 1: (Bhat and Bernstein 1997) *The system (1) is finite-time stabilizable under the continuous feedback*

$u(x,y) = -sgn(y)|y|^{\alpha} - sgn(x)|x|^{\frac{\alpha}{2-\alpha}}, \alpha \in (0,1).$

Lemma 2: *(Orlov 2009) the system (1) is finite-time stabilizable under the discontinuous feedback*

$$u(x,y) = -a\, sgn(x) - b\, sgn(y), a > b > 0.$$

3. PARTIAL ATTITUDE CONTROL OF AUTONOMOUSUNDERACTUATED AIRSHIP SYSTEM

This section is devoted to studying the complete system of underactuated airship which is the AS-200 by Airspeed Airships, see (Beji and Abichou 2005; Bestaoui 2006), for more details. It was shown in (Beji, Abichou, and Bestaoui 2004) that no continuous time-invarying feedback law which makes the origin of the airship asymptotically stable exists, because the latter system does not satisfy Brockett's condition (Brockett 1983). In order to overcome the Brockett's obstruction, the stabilization of the airship is treated in partial asymptotic stabilization sense.

Equation of motion: The autonomous underactuated airship is a complex nonlinear system described by
twelve variables of state and three controls. The model was found in (Beji and Abichou 2005; Beji, Abichou, and Bestaoui 2004):

$$\begin{cases}
\dot{u} = \frac{1}{m_{11}}(X_u u - 2(B-mg)(e_1 e_3 - e_2\sqrt{1-e_1^2-e_2^2-e_3^2}) - m_{33} wq + m_{22} v r + \tau_1), \\
\dot{v} = \frac{1}{m_{22}}(Y_v v - 2(B-mg)(e_1\sqrt{1-e_1^2-e_2^2-e_3^2} + e_2 e_3) - m_{11} ur + m_{33} pw + \tau_2), \\
\dot{w} = \frac{1}{m_{33}}(Z_w w - (B-mg)(1-2(e_1^2+e_2^2)) + m_{11} uq - m_{22} vp + \tau_3), \\
\dot{p} = \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2Bz_b I_{33}\left(e_2 e_3 + e_1\sqrt{1-e_1^2-e_2^2-e_3^2}\right) + (I_{33}^2 + I_{13}^2 - I_{33}I_{22}) qr + I_{13}(I_{11} - I_{22} + I_{33}) pq + I_{13}(Y_v - X_u) uv + I_{33}(Y_v - Z_w) vw - P_2^1 I_{13}\tau_2), \\
\dot{q} = \frac{1}{I_{22}}(M_q q - 2Bz_b(e_2\sqrt{1-e_1^2-e_2^2-e_3^2} - e_1 e_3) + (X_u - Z_w) uw + I_{13}(p^2 - r^2) + (I_{11} - I_{33}) pr + P_1^3\tau_1), \\
\dot{r} = \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2Bz_b I_{13}\left(e_1\sqrt{1-e_1^2-e_2^2-e_3^2} + e_2 e_3\right) + (-I_{13}^2 - I_{11}^2 + I_{11}I_{22}) pq + I_{13}(-I_{11} + I_{22} - I_{33}) qr + I_{11}(X_u - Y_v) uv + I_{13}(Z_w - Y_v) vw + P_2^1 I_{11}\tau_2), \\
\dot{x} = (1-2(e_2^2+e_3^2)) u + 2\left(e_1 e_2 - e_3\sqrt{1-e_1^2-e_2^2-e_3^2}\right) v + 2\left(e_1 e_3 + e_2\sqrt{1-e_1^2-e_2^2-e_3^2}\right) w, \\
\dot{y} = 2\left(e_1 e_2 + e_3\sqrt{1-e_1^2-e_2^2-e_3^2}\right) u + (1-2(e_1^2+e_3^2)) v + 2\left(e_2 e_3 - e_1\sqrt{1-e_1^2-e_2^2-e_3^2}\right) w, \\
\dot{z} = 2\left(e_1 e_3 - e_2\sqrt{1-e_1^2-e_2^2-e_3^2}\right) u + 2\left(e_2 e_3 + e_1\sqrt{1-e_1^2-e_2^2-e_3^2}\right) v + \left(1-2(e_1^2+e_2^2)\right) w, \\
\dot{e}_1 = \frac{1}{2}\left(p\sqrt{1-e_1^2-e_2^2-e_3^2} + e_2 r - e_3 q\right), \\
\dot{e}_2 = \frac{1}{2}\left(e_3 p + q\sqrt{1-e_1^2-e_2^2-e_3^2} - e_1 r\right), \\
\dot{e}_3 = \frac{1}{2}\left(-e_2 p + e_1 q + r\sqrt{1-e_1^2-e_2^2-e_3^2}\right).
\end{cases} \qquad (2)$$

The constants m_{ij} and I_{ij} are the coefficients of the inertia matrix M supposed to be symmetric and positive definite.

The constants X_u, Y_v, Z_w, L_p, N_r and M_q are the aerodynamic coefficients. The vector $\nu := (u, v, w, p, q, r)'$ denotes the linear velocities in surge, sway and heave, and the angular velocities in roll, pitch and yaw, decomposed in the body-fixed frame.

Define the vector $\eta := (\eta_1, \eta_2)$ where $\eta_1 := (x, y, z)'$ is the position of the airship in the inertial frame. The vector $\eta_2 := (e_0, e_1, e_2, e_3)'$ is defined as follows: the vector $(e_0, e_1, e_2, e_3)'$ defined the unit quaternion (i.e. $e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$), the component e_0 is supposed non negative and given by $e_0 := \sqrt{1 - e_1^2 - e_2^2 - e_3^2}$. The vector $\tau = (\tau_1, \tau_2, \tau_3)'$ denotes the control forces decomposed in the body-fixed frame.

As in (Beji and Abichou 2005), the lighter than air platform used in this paper is the AS-200 by Airspeed Airships (Figure 1), for more description of this type of blimp the reader is referred to (Bestaoui 2006). The blimp's parameters are as follows in the International System Units:

- blimp's total mass: $m = 9.07$, the nacelle mass m_n =1.58,
- added masses $X_x = 1.13, Y_y = 7.25, Z_z = 7.25, K_x = 0, M_y = 8.87, N_z = 8.87$,
- inertial parameters around the principal axes of inertia: $I_x x = 2.19, I_y y = 18.85, I_z z = 18.76$ and $I_x z = 0$,
- inertial terms $I_{11} = I_{xx} + K_x = 2.19, I_{22} = I_{yy} + M_y = 27.73$, $I_{33} = I_{zz} + N_z$ =27.63, $I_{13} = -I_{xz} + K_x = 0.22$,
- Δ term: $\Delta = I_{13}^2 - I_{11} I_{22} = -60.89$,
- positions of input forces F_1 and F_2: $P_1^3 = -1$ and P_2^1=-3,
- aerodynamic coefficients: $X_u = Y_v = Z_w = L_p = N_r = M_q$ =-10
- buoyancy and gravity magnitudes: mg =89 and $B = 72.2$ and $z_b = -0.1$.


Figure 1: The AS-200 Airship (Bestaoui 2006).

Consider the function $\sigma : B(0,1) \sqsubset \mathbb{R}^3 \to \mathbb{R}$ defined by

$$\sigma(\eta_2) = \sigma(e_1, e_2, e_3) = \sqrt{1 - e_1^2 - e_2^2 - e_3^2} \quad (3)$$

where $B(0,1)$ is the open ball of $\mathbb{R}^3$. A Taylor's expansion on the neighborhood B of $0_{\mathbb{R}^3}$ of σ gives

$$\sigma(\eta_2) = \sigma(0) + \sigma'(0)\eta_2 + g(\eta_2), \quad (4)$$

where g is a smooth function on the open ball $B(0,1)$ satisfying $g(0) = 0$.

Since $\sigma(0) = 1$, and

$$\sigma'(\eta_2) = ((-e_i)((1 - e_1^2 - e_2^2 - e_3^2)^{-1/2}))_{1 \le i \le 3},$$

then $\sigma'(0) = 0_{\mathbb{R}^3}$

and we get $\sigma(\eta_2) = 1 + g(\eta_2)$. (5)

Thus, the system (2) can be transformed as follows:

$$\dot{\xi} = f(\xi, \tau) + g(\xi),$$

where $\xi \in \mathbb{R}^{12}$ is the state and $\tau \in \mathbb{R}^3$ the control. The function $f(\xi, \tau)$ contains the linear terms of the system (2) where g contains the nonlinear part of the rest of the system.

We begin by studying the system $\dot{\xi} = f(\xi, \tau)$ which is given by

$$\begin{cases} \dot{u} = (\frac{1}{m_{11}} X_u u + 2(B - mg)e_2 + \tau_1), \\ \dot{v} = \frac{1}{m_{22}} (Y_v v - 2(B - mg)e_1 + \tau_2), \\ \dot{w} = \frac{1}{m_{33}} (Z_w w - (B - mg) + \tau_3), \\ \dot{p} = \frac{1}{\Delta} (- L_p I_{33} p + N_r I_{13} r - 2Bz_b I_{33} e_1 - P_2^1 I_{13} \tau_2), \\ \dot{q} = \frac{1}{I_{22}} (M_q q - 2Bz_b e_2 + P_1^3 \tau_1), \\ \dot{r} = \frac{1}{\Delta} (L_p I_{13} p - N_r I_{11} r + 2Bz_b I_{13} e_1 + P_2^1 I_{11} \tau_2), \\ \dot{x} = u, \\ \dot{y} = v, \\ \dot{z} = w, \\ \dot{e}_1 = \frac{1}{2} p, \\ \dot{e}_2 = \frac{1}{2} q, \\ \dot{e}_3 = \frac{1}{2} r. \end{cases} \quad (6)$$

3.1. Obstruction to stabilizability

We show that (6) cannot satisfy the Brockett's necessary condition for stabilizability.

Proposition 1: *There is no continuous state feedback that can stabilize asymptotically the system (6).*

Proof: Let τ′ be the feedback transformation defined by

$$\tau'_3 = \tau_3 - (B - mg),$$

this means that the term $-(B - mg)$ is crushed by a component of control τ_3, and the airship is at position h above the ground. With the new input τ'_3, the system (6) becomes $\dot{\xi} = f(\xi,\tau) :=$

$$\begin{pmatrix} \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2 + \tau_1) \\ \frac{1}{m_{22}}(Y_v v - 2(B-mg)e_1 + \tau_2) \\ \frac{1}{m_{33}}(Z_w w + \tau'_3) \\ \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2Bz_b I_{33} e_1 - I_{13} P_2^1 \tau_2) \\ \frac{1}{I_{22}}(M_q q - 2Bz_b e_2 + P_1^3 \tau_1) \\ \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2Bz_b I_{13} e_1 + P_2^1 I_{11} \tau_2) \\ u \\ v \\ w \\ \frac{1}{2}p \\ \frac{1}{2}q \\ \frac{1}{2}r \end{pmatrix}$$

All points M_ϵ in the form $(0,0,*,0,0,\varepsilon,0,0,0,0,0,0)'$ where $\varepsilon \neq 0$ are not in the image of f. Indeed, if it was the case, the equation $f(\xi,\tau) = M_\epsilon$ admits a solution. So, by combining the equation 2 and the equation 4 we get $2(B-mg)e_1 = \tau_2$ and $2Bz_b I_{33} e_1 = -P_2^1 I_{13} \tau_2$ then $2Bz_b I_{33} e_1 = -P_2^1 I_{13}(B-mg)e_1$ which implies $e_1 = \tau_2 = 0$; then $\varepsilon = 2Bz_b I_{13} e_1 + P_2^1 I_{11} \tau_2 = 0$.
We obtain a contradiction. □

3.2. First strategy: Partial exponential stabilizability

To get an adequate form of the system (6), we adopt the following transformation

$$u_1 := \frac{1}{m_{11}}(X_u u + 2(B-mg)e_2 + \tau_1),$$
$$u_2 := \frac{1}{m_{22}}(Y_v v - 2(B-mg)e_1 + \tau_2),$$
$$u_3 =: \frac{1}{m_{33}}(Z_w w - (B-mg) + \tau_3).$$

Then (6) is equivalent to

$$\begin{cases} \dot{u} = u_1, \\ \dot{v} = u_2, \\ \dot{w} = u_3, \\ \dot{p} = \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2(Bz_b I_{33} + P_2^1 I_{13}(B - mg))\, e_1 - P_2^1 I_{13}(m_{22}u_2 - X_v v)), \\ \dot{q} = \frac{1}{I_{22}}(M_q q - 2(Bz_b + P_1^3(B-mg))e_2 + P_1^3(m_{11}u_1 - X_u u)), \\ \dot{r} = \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2(Bz_b I_{13} + P_2^1 I_{11}(B - mg))e_1 + P_2^1 I_{11}(m_{22}u_2 - Y_v v)), \\ \dot{x} = u, \\ \dot{y} = v, \\ \dot{z} = w, \\ \dot{e}_1 = \frac{1}{2}p, \\ \dot{e}_2 = \frac{1}{2}q, \\ \dot{e}_3 = \frac{1}{2}r. \end{cases} \tag{7}$$

In the sequel, we will be interested in system (7) with respect to $\zeta = (u,v,w,p,q,r,x,y,z,e_1,e_2)'$. In order to apply the backstepping techniques in partial asymptotic stabilizability developed in (Jammazi 2008b), we start by studying the reduced system which is given by

$$\begin{cases} \dot{p} = \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2Bz_b I_{33} e_1 + P_2^1 I_{13}((Y_v - m_{22})u_2 - 2(B-mg)e_1)), \\ \dot{q} = \frac{1}{I_{22}}(M_q q - 2Bz_b e_2 + P_1^3((m_{11} - X_u)u_1 + -2(B-mg)e_2)), \\ \dot{r} = \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2Bz_b I_{13} e_1 + P_2^1 I_{11}((m_{22} - Y_v)u_2 + 2(B-mg)e_1)), \\ \dot{x} = u_1, \\ \dot{y} = u_2, \\ \dot{z} = u_3, \\ \dot{e}_1 = \frac{1}{2}p. \\ \dot{e}_2 = \frac{1}{2}q. \end{cases} \tag{8}$$

Stabilization of the system (2)

Proposition 2: *Let $k_i (i = 1,2,3)$ three nonnegative reel numbers. Then, with the action of the following feedbacks*

$$v_1 := -k(u + k_1 x),\ v_2 := -k(v + k_2 y),$$
$$v_3 := -k(w + k_3 z), \tag{9}$$

where k is large enough, the system (2) is eleven locally partially exponentially stable. More precisely, the partial state $\zeta = (u,v,w,p,q,r,x,y,z,e_1,e_2)' \in \mathbb{R}^{11}$ is locally exponentially stable and e_3 converges.

Proof: The proof of the proposition comes from ((Jammazi 2010), Corollary 7) which states that if the linearized system is p-partially exponentially stable then the initial system is p-locally exponentially partially stable. In closed loop, the linearization of (2) with respect to ζ around the equilibrium point is given by the system

$$\begin{cases} \dot{u} = -k(u + k_1 x), \\ \dot{v} = -k(v + k_2 y), \\ \dot{w} = -k(w + k_3 z), \\ \dot{p} = \frac{1}{\Delta}(-L_p I_{33} p + N_r I_{13} r - 2(Bz_b I_{33} + P_2^1 I_{13}(B - mg))e_1 - P_2^1 I_{13}(-km_{22}(v + k_2 y) - Y_v v)), \\ \dot{q} = \frac{1}{I_{22}}(M_q q - 2(Bz_b + P_1^3(B - mg))e_2, + P_1^3(-km_{11}(u + k_1 x) - X_u u)), \\ \dot{r} = \frac{1}{\Delta}(L_p I_{13} p - N_r I_{11} r + 2(Bz_b I_{13} + P_2^1 I_{11}(B - mg)e_1 + P_2^1 I_{11}(-km_{22}(v + k_2 y) - Y_v v)), \\ \dot{x} = u, \\ \dot{y} = v, \\ \dot{z} = w, \\ \dot{e}_1 = \frac{1}{2} p, \\ \dot{e}_2 = \frac{1}{2} q. \end{cases} \qquad (10)$$

Clearly the system (10) is exponentially stable with respect to $(u, v, w, x, y, z)'$. The linear system (10) with respect to $\delta = (p, q, r, e_1, e_2)' \in \mathbb{R}^5$ admits the following set of eigenvalues which are with negative real parts

$\lambda_1 = -0.7597, \lambda_2 = -3.6869 + 0.4501i,$
$\lambda_3 = -3.6869 - 0.4501i, \lambda_4 = -0.0970$ *and* $\lambda_5 = -3.5560.$

Straightforward computations show that (10) is exponentially stable with respect to. Clearly nonlinear part of (2) with respect to ζ vanish when the "uncontrolled part" e_3 is zero. Then, by using, ((Jammazi 2010), Corollary 7), the system (2) is locally exponentially stable with respect to ζ.

Consequently, there exists $\alpha > 0$ and $\lambda > 0$ such that

$$|\zeta(t)| \le \alpha|\zeta(0)|e^{-\lambda t}, t > 0. \qquad (11)$$

In particular, we get

$$|p(t)|, |q(t)|, |r(t)|, |e_1(t)|, |e_2(t)| \le \alpha|\zeta(0)|e^{-\lambda t}. \qquad (12)$$

Since $\dot{e}_3 = \frac{1}{2}\left(-e_2 p - e_1 q + r\sqrt{(1 - e_1^2 - e_2^2 - e_3^2}\right)$, and $\sqrt{1 - e_1^2 - e_2^2 - e_3^2} \le 1$, then we get

$$|\dot{e}_3| \le \alpha^2|\zeta(0)|^2 e^{-2\lambda t} + \frac{1}{2}\alpha|\zeta(0)|e^{-\lambda t}. \qquad (13)$$

From (13) we easily deduce that $\dot{e}_3(t)$ is Lebesgue integrable and therefore e_3 converges. This completes the proof. □

3.3. Second alternative: Finite-time partial stabilizability

In this section, we give other strategies to stabilize the airship. This alternative is based on the theory of partial stabilization and on continuous feedback laws given in Lemma 1 (respectively, discontinuous feedback laws given in Lemma (2). We begin with the continuous finite time stabilizing feedback laws.

Proposition 3: *Let be* $\alpha \in (0,1)$, *then under the following feedback laws*

$$v_1 := -sgn(\dot{x})|\dot{x}|^{\alpha} - sgn(x)|x|^{\frac{\alpha}{2-\alpha}},$$

$$v_2 := -sgn(\dot{y})|\dot{y}|^{\alpha} - sgn(y)|y|^{\frac{\alpha}{2-\alpha}},$$

$$v_3 := -sgn(\dot{z})|\dot{z}|^{\alpha} - sgn(z)|z|^{\frac{\alpha}{2-\alpha}}, \qquad (14)$$

the underactuated system (2) is finite-time stable with respect to $(x, u, y, v, z, w)'$ *and locally exponentially stable with respect to* $(p, q, r, e_1, e_2)'$ *which implies that* e_3 *converges.*

Proof: We consider the system (2) and taking the feedback transformation

$$\bar{\tau}_1 := \frac{1}{m_{11}}(X_u u - 2(B - mg)(e_1 e_3 - e_2\sqrt{1 - e_1^2 - e_2^2 - e_3^2}\,) - m_{33} wq + m_{22} v r + \tau_1),$$

$$\bar{\tau}_2 := \frac{1}{m_{22}}(Y_v v - 2(B - mg)(e_1\sqrt{1 - e_1^2 - e_2^2 - e_3^2} + e_2 e_3) - m_{11} ur + m_{33} pw + \tau_2),$$

$$\bar{\tau}_3 := \frac{1}{m_{33}}(Z_w w - (B - mg)(1 - 2(e_1^2 + e_2^2)) + m_{11} uq - m_{22} vp + \tau_3), \qquad (15)$$

then the dynamic of the states u, v and w become $\dot{u} = \bar{\tau}_1$, $\dot{v} = \bar{\tau}_2$, and $\dot{w} = \bar{\tau}_3$. Then by taking the time derivative respectively of $\dot{x}$, $\dot{y}$ and $\dot{z}$ we get

$$\ddot{x} = \left(1 - 2(e_2^2 + e_3^2)\right)\dot{u} + \{\text{nonlinear terms}\}$$

$$= (1 - 2(e_2^2 + e_3^2))\, \bar{\tau}_1 + \{\text{nonlinear terms}\}. \quad (16)$$

Let g_1 be the smooth function defined *by* $g_1(\eta_2) = 1 - 2(e_2^2 + e_3^2)$. Since $g_1(0) = 1 \neq 0$, then there exists a neighborhood V_0 of zero such that $g_1(\eta_2) \neq 0$ for all $\eta_2 \in V_0$. In this case, the system (16) is locally feedback equivalent to

$\ddot{x} = v_1$, where $v_1 = (1 - 2(e_2^2 + e_3^2))\, \bar{\tau}_1 +$ {nonlinear terms}. (17)

Since the functions $g_2(\eta_2) = 1 - 2(e_1^2 + e_3^2)$ and $g_3(\eta_2) = 1 - 2(e_1^2 + e_2^2)$ satisfies $g_2(0) = g_3(0) = 1 \neq 0$, by the same above argument, the dynamic of y and z are locally equivalent to

$$\ddot{y} = v_2 := (1 - 2(e_1^2 + e_3^2))\, \bar{\tau}_2 + \{\text{nonlinear terms}\}.$$

$\ddot{z} = v_3 := (1 - 2(e_1^2 + e_2^2))\,\bar{\tau}_3 \quad + \quad \{\text{nonlinear terms}\}.$ (18)

Here, these nonlinear terms vanish in the equilibrium point. To summarize, the dynamic equation of (x, y, z) is now in the following double integrator form:

$$\ddot{x} = v_1,\quad \ddot{y} = v_2,\quad \ddot{z} = v_3. \tag{19}$$

Then, according to Lemma 1, by choosing feedbacks given in (14), we get easily x, y and z are stable in finite time and $\dot{x}, \dot{y}$ and $\dot{z}$ are too, which give also u, v and w are stable in finite-time. Then there exists a settling time T such that $\forall\, t \geq T$,

$$x(t) = y(t) = z(t) = u(t) = v(t) = w(t) = 0.$$

In this case, the system (2) becomes

$$\begin{cases} \dot{p} = \frac{1}{\Delta}\big(-L_p I_{33} p + N_r I_{13} r + (I_{33}^2 + I_{13}^2 - I_{33} I_{22})\, qr + I_{13}(I_{11} - I_{22} + I_{33}) pq \\ \quad -2\left(e_1\sqrt{1 - e_1^2 - e_2^2 - e_3^2} + e_2\, e_3\right)(B z_b I_{33} + (B - mg) P_2^1 I_{13})\big), \\ \dot{q} = \frac{1}{I_{22}}\big(M_q q + I_{13}(p^2 - r^2) + (I_{11} - I_{33})\, pr \\ \quad -2\left(e_2\sqrt{1 - e_1^2 - e_2^2 - e_3^2} - e_1\, e_3\right)(B z_b + P_1^3(B - mg))\big), \\ \dot{r} = \frac{1}{\Delta}\big(L_p I_{13} p - N_r I_{11} r + (-I_{13}^2 - I_{11}^2 + I_{11} I_{22})\, pq + I_{13}(-I_{11} + I_{22} - I_{33}) qr \\ \quad +2(e_1\sqrt{1 - e_1^2 - e_2^2 - e_3^2} + e_2\, e_3)(P_2^1 I_{11}\,(B - mg) + B z_b I_{13})\big), \\ \dot{e}_1 = \frac{1}{2}\left(p\sqrt{1 - e_1^2 - e_2^2 - e_3^2} + e_2 r - e_3 q\right), \\ \dot{e}_2 = \frac{1}{2}\left(e_3 p + q\sqrt{1 - e_1^2 - e_2^2 - e_3^2} - e_1\, r\right), \\ \dot{e}_3 = \frac{1}{2}\left(-e_2 p + e_1\, q + r\sqrt{1 - e_1^2 - e_2^2 - e_3^2}\right). \end{cases} \tag{20}$$

The system (20) can be expressed as:

$$\begin{cases} \dot{\delta} = A\delta + S(\delta, e_3), \\ \dot{e}_3 = R(\delta, e_3), \end{cases} \tag{21}$$

where $\delta = (p, q, r, e_1, e_2)'$, S and R represent higher order nonlinear terms and vanish when $\delta = 0$.

The linearized system of (20) with respect to δ is given as follows:

$$\begin{cases} \dot{p} = \frac{1}{\Delta}\big(-L_p I_{33} p + N_r I_{13} r - 2(B z_b I_{33} + P_2^1 I_{13}(B - mg) e_1)\big), \\ \dot{q} = \frac{1}{I_{22}}\big(M_q q - 2(B z_b + P_1^3(B - mg)) e_2\big), \\ \dot{r} = \frac{1}{\Delta}\big(L_p I_{13} p - N_r I_{11} r + 2(B z_b I_{13} + P_2^1 I_{11}(B - mg)) e_1\big), \\ \dot{e}_1 = \frac{1}{2} p, \\ \dot{e}_2 = \frac{1}{2} q. \end{cases} \tag{22}$$

By using MATLAB Toolbox, the linear system (22) around the partial equilibrium point $(p, q, r, e_1, e_2)' = (0,0,0,0,0)'$ admits the following eigenvalues $-0.7597, -3.6869 + 0.4501i$, $-3.6869 + 0.4501i, -0.0970, -3.5560$. Clearly the linearized system is asymptotically stable, and therefore by using partial exponential stability and linearization theorem (Jammazi 2010), the initial system (21) is locally exponentially stable with respect to δ.

Since the function R satisfies the property $R(0, e_3) = 0$, then the state e_3 converges.

Moreover, we have $e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$, then the state e_1, e_2 and e_3 are Lyapunov stable. Therefore, the system (20) is locally exponentially stable with respect to the partial state $(p, q, r, e_1, e_2)'$, stable with respect to $(p, q, r, e_1, e_2, e_3)'$ and the "uncontrolled" state e_3 converges. This achieves the proof. □

The airship is an example of system with drift in which the Coron and Rosier's condition fails to be stabilized by discontinuous feedback laws (Coron and Rosier 1994). In order to overcome this obstruction, the next proposition introduces the finite-time partial stabilizability by discontinuous feedback laws. By the same argument as in the proof of Proposition 3 we show the following proposition.

Proposition 4: *Let be $\alpha \in (0,1)$, then under the following feedback laws*

$$\begin{aligned} v_1 &= -a_1 sgn(x) - b_1\ sgn(\dot{x}), \\ v_2 &= -a_2\, sgn(y) - b_2\ sgn(\dot{y}), \\ v_3 &= -a_3\, sgn(z) - b_3\ sgn(\dot{z}), \end{aligned} \tag{23}$$

where $a_i > b_i > 0$, the underactuated system (2) is finite time stable with respect to $(x, u, y, v, z, w)'$ and locally exponentially stable with respect to $(p, q, r, e_1, e_2)'$ and therefore e_3 converges.

Now, we are ready to give the open question.

Open Question: Is the feedbacks proposed in the section 4 are robust with respect to measurement noise on the state variables and with respect to unmodeled dynamics ?

4. SIMULATION RESULTS

The performances of our feedback laws are tested by numerical simulations on the nonlinear model of airship. The advantage our method resides in obtaining a static stabilization. Moreover, the state variable, which is not "controllable" converges, which makes it possible to avoid the oscillation of the system in the neighborhood of the equilibrium point. For space reason, the simulations of the exponential stabilizability are omitted. Only the simulations of the finite-time partial stabilizability by continuous feedbacks are considered.

4.1. Second approach: Finite-time partial stabilizability by continuous feedback laws.

In this strategy we have used the initial condition:
$(u^0, v^0, w^0, p^0, q^0, r^0, x^0, y^0, z^0, e_1^0, e_2^0, e_3^0)'$
$= (0.3, 0.5, -1.5, 0.2, 0.1, 0.5, 0.5, -0.2, 0.5, -0.5, 0.5, 0.5)'$,
and the feedbacks

$$u_1 := -0.2\, sgn(u)|u|^{\frac{1}{3}} - 0.2\ sgn(x)|x|^{\frac{1}{5}}$$
$$u_2 := -0.2\, sgn(v)|v|^{\frac{1}{3}} - 0.2\ sgn(y)|y|^{\frac{1}{5}}$$
$$u_3 := -0.2\, sgn(w)|w|^{\frac{1}{3}} - 0.2\ sgn(z)|z|^{\frac{1}{5}}.$$

This simulation shows the finite time stability of $(u, v, w, w, y, z)'$. The asymptotic stability of $(p, q, r, e_1, e_2)'$ and convergence of $e_3\ to \simeq 0.21$.

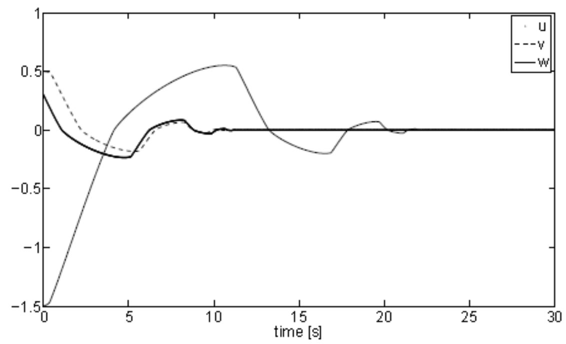

Figure 2: Velocities u and v and w

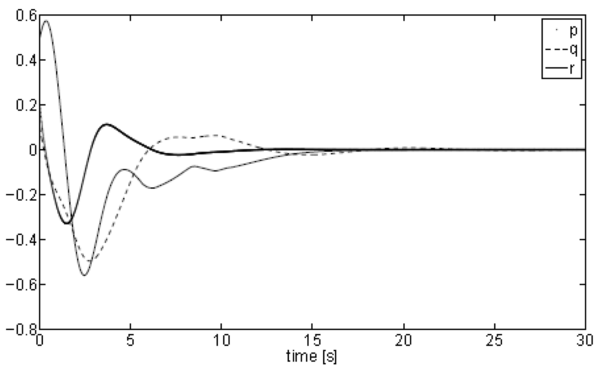

Figure 3: Velocities p and q and r.

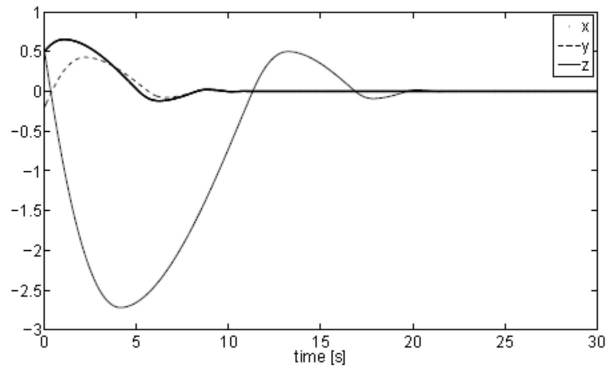

Figure 4: Positions x and y and z.

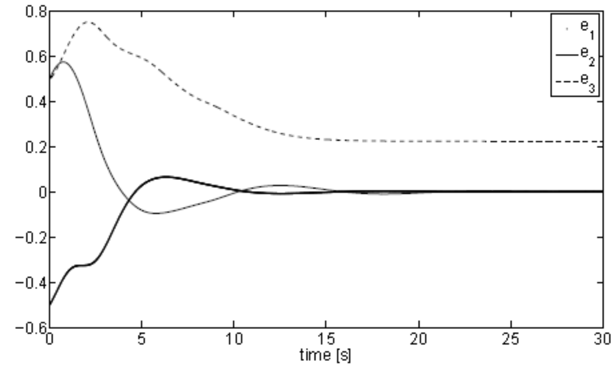

Figure 5: Positions e_1, e_2 and convergence of e_3.

5. CONCLUSION

The model of airship cannot be stabilized by continuous pure state feedback laws, this due to Brockett necessary condition. To overcome this problem, various controllers are proposed to study the position and the partial attitude of the airship; summarizing, these feedbacks makes eleven states of variables asymptotically stable once only one variable remains convergent. In the airship model, the "uncontrolled part "e_3" is the yaw angle which leads the system to revolve around the axis n_z attached to the frame airship. We have shown with the action of our feedback that the airship is asymptotically stable without taking into consideration its orientation with respect to axis n_z, since the latter angle converges. Clearly this stabilization seems sufficient.

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