REDUCING VIBRATIONS ON FLEXIBLE ROTATING ARMS THROUGH THE MOVEMENT OF SLIDING MASSES: MODELING, OPTIMAL CONTROL AND SIMULATION

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ABSTRACT

This paper brings contributions on the proposal of use of translational motions of sliding masses to minimize vibrations induced by the rotational motion of a light flexible manipulator (rotating arm). This system is inspired by rotating cranes used to transport loads. Optimal control methods have been used to generate the slider trajectories while the flexible manipulator performs a rotational maneuver from a fixed to other fixed configuration. This approach has led to good solutions even in case of quite quick maneuvers, as, for example, a 90^o beam rotation in just 1 second, using 1 or 2 sliders (Terceiro, 2002). In the present paper, the complete motion equations for any number of masses are firstly presented, in order to emphasize the complexity of the coupled elastic-rotational-translational motions. Simplifying assumptions are pointed out and the corresponding optimal control problems (OCP) are obtained. Optimal trajectories, generated according to different Indexes of Performance and different problem parameters, are analysed and compared in order to get feasible movements for the set.

Keywords: Optimal Control, Vibration Control, Flexible Robotics, Lightweight Structures, Composed Motion

1 INTRODUCTION

For many applications, structural flexibilities must be considered in early design stages in order to assure good vibration attenuation in modern machine design. The problem of flexible structures has worried many authors, today's literature on the subject is extensive. We can cite as authors interested: (Junkins and Kim, 1993) dealing with the problem of dynamic and flexible control structure and (Meirotvitch, 1980), (Meirotvitch, 1990) that contributed to the disclosure of the issue of flexible structures.

Based on the examples of rotating cranes and rotational/prismatic joint robots, this work explores simultaneous rotating/translational motions to minimize vibrations on a light one-link manipulator that performs large rotational maneuvers. The basic question investigated is how the motion of independent parts may contribute to reduce the vibration levels of the whole system. As the results achieved are encouraging, it seems feasible to extend the research to other more complex applications. Then, the objectives here are the achievement of suitable system models for a very light flexible manipulator and the synthesis of optimal controllers using the torque applied to the hub where the flexible arm is fixed and the forces applied to the sliding masses as control variables. The case of a single sliding mass has brought surprising results, as shown in (Fleury and Oliveira, 2004). The investigation has been extended to model a mechanism which include any number of sliders and structural modes (Terceiro, 2002). The full approach is firstly introduced in this paper. In all cases, the dynamical models of the structural system have been derived through the Extended Hamilton Principle resulting in a set of coupled integro-differential non linear equations where system parameters are time and space dependent due to changes in the inertia terms. Using substructuring techniques, arm and sliders motions have been separated and systems responses have been expanded in products of spatial and time functions. Many Control techniques (LQR, for example) have been used in order to minimize vibrations induced by the rotational movements, but this reduction resulted dependent on the prescribed motions of the sliders. A bad choice for the slider movements can lead to larger vibrations amplitudes when compared to a situation where the masses remain fixed on the rotating arm (Fleury and Oliveira, 2004). An Optimal Control line of investigation became mandatory to understand the very influence of the composed torque-sliders position controls on the elastic vibrations. Slider trajectories become control variables, among hub torque leading to Optimal Control Problems (OCP). The resulting models are non linear and time variant and analytical solutions are not feasible. Then, optimal arm and slider trajectories are investigated through the use of RIOTS'95 (Schwartz et al, 1997), a computational package based on the Consistent Approximation Theory (Schwartz, 1966). Among many already simulated cases, the results presented in this paper demonstrate the importance and influence of the choice of Objective Functions in System performance. Early results appear in the work of (Oliveira, 2000) when the problem was treated with only one mass sliding. Then, in (Terceiro, 2002) problem has been generalized to different masses has been established, and then using the RIOTS'95 was simulated problem with two sliding masses. In a research work hard, many results have appeared as shown in (Terceiro and Fleury, 2008). Sometimes, the differences seem subtle, and are basically set the simulation time, the performance criterion and the initial guess necessary for the simulation. Even these few changes have produced a wealth of results that are now published and others that need to be studied further. The numerical difficulties inherent problems of this size are increased by the large number of parameters available for analysis. Just a few results are presented and refer to the most interesting cases selected from a broader set of results that were obtained by the choice of all parameters involved and discussed in the previous paragraph.

2 SYSTEM FULL MODEL

As shown in Figure 1, system is composed by a long, slender, flexible beam (the arm) that can rotate in an horizontal plan driven by the torque delivered in a rigid hub. Angular acceleration and disacceleration of the flexible arm should cause large amplitude vibrations. Here, we propose to move some masses (sliders) simultaneously to the arm motion, thus changing rotational inertia properties to minimize arm vibrations, represented by the arm tip excursions. In order to use Hamiltons Extended Principle, kinetic and potential energies of each component, hub, arm and masses must be calculated. Then, the elastic potential energy of the flexible arm, $U(t)$, is given by:

$$U(t) = \frac{1}{2}\int_0^L EI_v \left(\frac{\partial^2 e}{\partial x^2}\right)^2 dx \tag{1}$$

with **e** the deformation of the arm at a generic point x, I_v is the moment of inertia, b is the width h the height of a tipical section. L is the length of the flexible arm. The total kinetic energy of the arm, T_B,is:

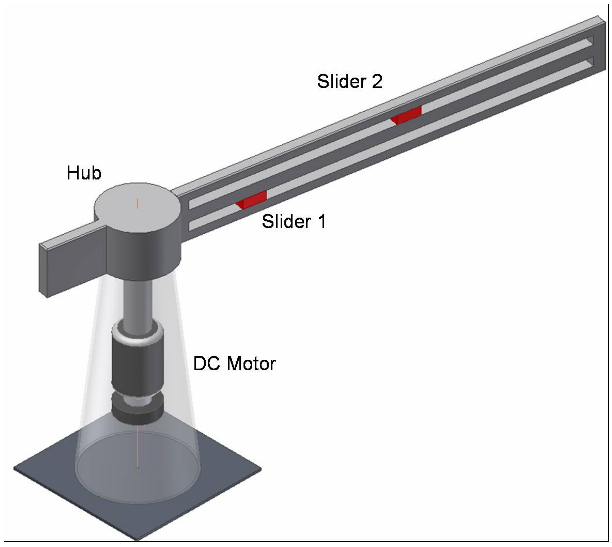

Figure 1: Flexible Arm Model

$$T_B = \int_0^L \frac{1}{2}\rho(e\dot{\theta})^2 dx + \int_0^L \frac{1}{2}\rho(\dot{e} + \dot{x}\theta)^2 dx \tag{2}$$

with ρ arm linear mass density, θ the angular displacement and $\dot{\theta}$ the angular velocity.

The kinetic energy of the two masses, T_M, is given by:

$$T_M = \frac{1}{2}\sum_{i=1}^{2} M_i\left((\dot{l}_i - e_i\dot{\theta})^2 + (\dot{e}_i + l_i\dot{\theta})^2\right) \tag{3}$$

l_i is the location on the mass of each mass over the arm.

The kinetic energy of the hub, T_c is:

$$T_c = \frac{1}{2}\frac{M_c L_c^2}{2}\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{2}J_c\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{2}J_c\dot{\theta}^2 \tag{4}$$

L_C is the radius of the hub, M_C its mass and J_C is its moment of inertia of the cube.

The work of the nonconservative forces, W, is given by:

$$\delta W = \tau\delta\theta + F_1\delta l_1 + F_2\delta l_2 \tag{5}$$

The first term refers to the virtual work of the applied torque, the other two terms refer to the virtual work of the tangent forces applied to the masses m_1 and m_2 respectively.

Explicitly, τ is the torque applied by the motor to produce rotational movement of the flexible arm, and F_1,

F_2 is the force applied by the sliders m_1, m_2 to reduce the vibration of the flexible arm

The Extended Hamilton's Principle states that between two instants t_1 and $t_2$the system energy follows:

$$\int_{t_1}^{t_2} (\delta L_g + \delta W)dt = 0 \tag{6}$$

with L_g, the Lagrangian of the system, is given by:

$$L_g = T - U = T_B + T_M + T_C - U \tag{7}$$

That is true for the system:

$$\int_{t_1}^{t_2} (\delta T_B + \delta T_M + \delta T_C - \delta U + \delta W)dt = 0 \tag{8}$$

The application of the principle leads to the following model

$$\begin{array}{rcl}
\int_0^L \rho(-x\ddot{e} - (x^2 + e^2)\ddot{\theta})dx & & \\
+M_1 \int_0^L (e_1\ddot{l}_1 - l_1\ddot{e}_1 - e_1^2\ddot{\theta})\Delta l_1 dx & & \\
+M_2 \int_0^L (e_2\ddot{l}_2 - l_2\ddot{e}_2 - e_2^2\ddot{\theta})\Delta l_2 dx & & \\
-(M_1 l_1^2 + M_2 l_2^2 + J_c)\ddot{\theta} + \tau & = & 0 \\
\int_0^L \rho(\ddot{e} - x\ddot{\theta} + e\theta^2)dx - \int_0^L EI_v \frac{\partial^4 e}{\partial x^4} dx & = & 0 \\
M_1 \int_0^L (-\ddot{e}_1 + e_1\dot{\theta}^2 + -\dot{l}_1\dot{\theta})\Delta l_1 dx & = & 0 \\
M_2 \int_0^L (-\ddot{e}_2 + e_2\dot{\theta}^2 + -\dot{l}_2\dot{\theta})\Delta l_2 dx & = & 0 \\
M_1 \int_0^L [e_1\ddot{\theta} + e_1\dot{\theta}^2]\Delta l_1 dx + M_1(l_1\dot{\theta}^2\ddot{l}_1) + F_1 & = & 0 \\
M_2 \int_0^L [e_2\ddot{\theta} + e_2\dot{\theta}^2]\Delta l_2 dx + M_2(l_2\dot{\theta}^2\ddot{l}_2) + F_2 & = & 0
\end{array} \tag{9}$$

with boundary conditions:

$$\begin{array}{ll}
e\Big|_{x=0} = 0 & \frac{\partial^2 e}{\partial x^2}\Big|_{x=l} = 0 \\
\frac{\partial e}{\partial x}\Big|_{x=0} = 0 & \frac{\partial^3 e}{\partial x^3}\Big|_{x=l} = 0
\end{array} \tag{10}$$

3 SUBSTRUCTURE SYNTHESIS

The difficulties of the mathematical analysis of the above problem, which involves all the terms of the interaction energy of each of the parties established because requires solving the equations 9 with boundary condition 10 where all parts of the structure appear mixed when to implementing Extended Hamiltons Principle which involves all the terms of the interaction energy of each of the parties established then the strategy was to consider a system consisting of several substructures, determine the motion equations that govern these substructures and then consider the interaction of each structure with the others and their effects on the structure as a whole. This approach is known as Substructure Synthesis and its roots can be found in papers like (Meirovitch and Kwak, 1991). In our case, the arm-hub has been considered as one substructure and each sliding mass as another ones. Hamiltons Extended Principle has been rewritten for each substructure. After many algebraic manipulations, which include disregarding quadratic terms (Terceiro, 2002), the substructured model is given by:

$$\begin{array}{l}
\int_0^L \rho(-x\ddot{e} - (x^2 + e^2)\ddot{\theta})dx - J_c\ddot{\theta} + \tau = 0 \\
\int_0^L \rho(\ddot{e} - x\ddot{\theta} + e\theta^2)dx - \int_0^L EI_v \frac{\partial^4 e}{\partial x^4} dx = 0 \\
\ddot{e}_1 + l_1\ddot{\theta} + 2\dot{l}_1 - e_1\dot{\theta}^2 = 0 \\
\ddot{e}_2 + l_2\ddot{\theta} + 2\dot{l}_2 - e_2\dot{\theta}^2 = 0
\end{array} \tag{11}$$

with the same boundary conditions as in equation 10.

The free vibration of the arm, after disregarding some second order terms is given by:

$$\begin{array}{l}
-\int_0^L \rho x\ddot{e} dx - (J_B + J_C)\ddot{\theta} = 0 \\
\rho(\ddot{e} - x\ddot{\theta}) + EI_v \frac{\partial^4 e}{\partial x^4} = 0
\end{array} \tag{12}$$

A change of variables is then introduced:

$$e(x,t) = z(x,t) - x\theta(t) \tag{13}$$

leading the model to a form like:

$$\begin{array}{l}
\int_0^L \rho x\ddot{z} dx + J_c\ddot{\theta} = 0 \\
\rho\ddot{z} + EI_v \frac{\partial^4 e}{\partial x^4} dx = 0
\end{array} \tag{14}$$

with boundary conditions:

$$z\Big|_{x=0} = 0 \frac{\partial^2 z}{\partial x^2}\Big|_{x=l} = 0 \tag{15}$$

$$\frac{\partial z}{\partial x}\Big|_{x=0} = 0 \frac{\partial^3 z}{\partial x^3}\Big|_{x=l} = 0$$

All these transformations have been necessary to express the system in coordinates where an expansion on independent orthogonal functions of time and space can be performed:

$$z(x,t) = \sum_{r=1}^{\infty} \phi_r(x)\eta_r(t) \tag{16}$$

This allows separated spatial and time descriptions through the equations:

$$\ddot{\eta}_r(t) + \omega_r^2 \eta_r(t) = 0 \tag{17}$$

$$EI_v \frac{\partial^4 \phi_r(x)}{\partial x^4} - \omega_r^2 \rho \phi_r(x) = 0$$

For the eigenvector equation, 17, admissible solutions are proposed as:

$$\begin{aligned} \phi_r(x) &= a_r sin(\beta_r x) + b_r cos(\beta_r x) \\ &+ c_r sinh(\beta_r x) + d_r cosh(\beta_r x) \end{aligned} \tag{18}$$

The determination of the coefficients is made by solving a linear system. After many algebraic manipulations one can arrive at a standard form:

$$\delta_{rs} = \int_0^L \frac{d^2\phi_s}{dx^2}\frac{d^2\phi_r}{dx^2} dx \tag{19}$$

$$= \frac{1}{EI_v}\left\{ \int_0^L \omega_r^2 \rho \phi_s \phi_r dx - J_c \omega_r [\phi_s' \phi_r']_{x=0} \right\}$$

Considering the forced system, one can find:

$$\sum_{r=1}^{\infty} \left[\int_0^L \rho \phi_s \phi_r dx - J_c [\phi_s' \phi_r']_{x=0} \right] \ddot{\eta}_r + \tag{20}$$

$$+EI_v \sum_{r=1}^{\infty} \left[\omega_r^2 [\int_0^L \rho \phi_s \phi_r dx - J_c [\phi_s' \phi_r']_{x=0} \right] \eta_r = \tau \phi_s' \Big|_{x=0}$$

Observing the adopted norm, we finally have:

$$\ddot{\eta}_r + \omega_r^2 \eta_r = \tau \phi_r'|_{x=0} \tag{21}$$

Then, the interaction between substructures becomes:

$$\ddot{\eta}_r + \omega_r^2 \eta_r = \int_0^L F_E(l_i) \phi_r dx + \tau \phi_s'|_{x=0} \tag{22}$$

where $F_E(l_i)$ is the force due to the presence of a sliding mass at this point.

After some manipulations:

$$\begin{aligned} \ddot{\eta}_r + \omega_r^2 \eta_r = &- \int_0^L M_i \sum_{s=1}^{\infty} \left[\phi_s \big|_{x=l_i} \ddot{\eta}_s \right] \phi_r dx \\ &- \int_0^L M_i 2\dot{l}_i \dot{\theta} \phi_r dx + \tau \phi_s' \Big|_{x=0} \end{aligned} \tag{23}$$

This equation is very important because it shows the relationship between the vibration modes in time and the influence of motions in space, that is, the interaction between the sliding masses, applied at the point l_i, and the structure and their effects transferred as functions of spatial forms. It also shows the effect of the torque applied to the hub on the flexible robotic arm.

4 STATE SPACE FOR THE DYNAMICAL SYSTEM

In order to design any control strategy, the system model should be written in state space variables. Then, substructure motions are synchronized and normalized to get a set of matrix equations in the form:

$$\ddot{\eta}_r = -[T]^{-1} \left(W\eta_r - 2M_i \dot{l}_i \dot{\theta} \left[\int_0^L \phi_r dx \right] + \tau \dot{\phi}_r|_{x=0} \right) \tag{24}$$

where $[W] = \omega_i^2 \times I_{nxn}$ and $[S] = [T]^{-1}$ is given by:

$$S_{rs} = \begin{cases} \dfrac{M_i + M_i^2 \sum\limits_{k=1,k\neq r}^{p} \phi_k \Big|_{x=l_i} \int\limits_0^L \phi_k dx}{1 + \sum\limits_{k=1}^{p} M_i \phi_k \Big|_{x=l_i} \int\limits_0^L \phi_k dx} & \text{for} \quad r = s \\ \dfrac{-M_i^2 \phi_r \Big|_{x=l_i} \int\limits_0^L \phi_s dx}{1 + \sum\limits_{k=1}^{p} M_i \phi_k \Big|_{x=l_i} \int\limits_0^L \phi_k dx} & \text{for} \quad r \neq s \end{cases} \tag{25}$$

Using Newton's Law on the rotational motion, one can get:

$$\left(J_B + J_C + M_L l_l^2 \ddot{\theta} \right) = \tau + F_{Ej}(l_i) l_i \tag{26}$$

Equation 26 describes the rotational motion, includes the torque applied to the hub and the slider reactions F_{Ej}, at positions l_i and takes into account the beam moment of inertia, J_B. Finally, a state model can be written:

$$
\begin{aligned}
\dot{x}_{2r-1} &= x_{2r} \\
\dot{x}_{2r} &= -\frac{2\left(M_1 x_{2p+4}+M_2 x_{2p+6}\right) x_{2p+2} \sum_{s=1}^{p} S_{rs} \int_0^L \phi_s dx}{M_1+M_2} \\
&\quad -\frac{\sum_{s=1}^{p} w_s^2 S_{rs} x_{2s-1}}{M_1+M_2} + \frac{\sum_{s=1}^{p} S_{rs} \dot{\phi}_s \Big|_{x=0}}{M_1+M_2} u_i \\
\dot{x}_{2p+1} &= x_{2p+2} \\
\dot{x}_{2p+2} &= \frac{\sum_{r=1}^{p} \sum_{s=1}^{p} \omega_s^2 \phi_r(l_i) S_{rs} x_{2s-1}}{J_B+J_C+M_1 x_{2p+3}^2+M_2 x_{2p+5}^2} + \\
&\quad \frac{1-\sum_{r=1}^{p} \sum_{s=1}^{p} S_{rs} \dot{\phi}\Big|_{x=0} \phi_r(l_i)}{J_{BC+M_1 x_{2p+3}^2+M_2 x_{2p+5}^2}} \tau \\
&\quad +2 \frac{\left(M_1 x_{2p+4}+M_2 x_{2p+6}\right) x_{2p+2}}{J_B+J_C+M_2 x_{2p+5}^2+M_1 x_{2p+3}^2} \times \\
&\quad \frac{\left(\sum_{r=1}^{p} \sum_{s=1}^{p} \phi_r(l_i) S_{rs} \int_0^L \phi_s dx - 1\right)}{J_B+J_C+M_2 x_{2p+5}^2+M_1 x_{2p+3}^2} \\
\dot{x}_{2p+3} &= x_{2p+4} \\
\dot{x}_{2p+4} &= \frac{u_2}{M_1} + x_{2p+3} x_{2p+2}^2 \\
\dot{x}_{2p+5} &= x_{2p+6} \\
\dot{x}_{2p+6} &= \frac{u_3}{M_1} + x_{2p+5} x_{2p+2}^2
\end{aligned}
\tag{27}
$$

The elements of matrix S are written according to eq. 25 with the necessary adaptations, p indicates the number of vibration modes of the system.

In this model, the state variables represent $\dot{x}_{2r}$ are the normal modes of the system, x_{2r-1} are the velocities of normal modes, x_{2p+1} is the angular motion, x_{2p+2} is the velocity angular motion, and x_{2p+3} and x_{2p+5} are the position of sliders 1 and 2 respectively, and x_{2p+4} and x_{2p+5} are the velocities of sliders.

5 THE OPTIMAL CONTROL PROBLEM

A general Optimal Control Problem (OCP) can be stated as one where the control laws $u_j(t),\ j=1,\cdots m$ and the initial and final conditions $x_i(a)$and $x_i(b)$, $i=1,\cdots,n$ have to be chosen so as to minimize an Index of Performance

$$IP = \Phi(x(a),x(b)) + \int_a^b L(x(t),u(t),t)dt \tag{28}$$

subject to:

1. Dynamic Constraints:

$$\dot{x}_i = f_u(x(t),u(t),t), i=1,\cdots,n \tag{29}$$

2. Boundary Constraints:

$$\phi_k(x(a),x(b),a,b) = 0, k=1,\cdots,r \tag{30}$$

Although easily included, this system does not require the use of Control or State Inequality Constraints.

6 ILLUSTRATIVE CASES STUDY

The Indexes of Performance were always chosen as combinations of arm tip displacements and velocities since we are interested in minimizing tip vibrations. For robotic problems, minimum time or minimum control energy are not as relevant as tip vibrations because in the case the rotating arm is carrying a tool or other device, getting the final position with the tool ready to use is the most important figure. Manoeuver time may be incorporated to the Index of Performance (1 or 2 or 3 seconds, or more) and the control efforts to move the sliders are low. The structural problem under investigation corresponds to a fast angular manouever of the arm, from 0 to 45^o in 3s. A flexible arm will vibrate with large amplitudes if the system is not controlled and induced vibrations should be attenuated. Then, in our investigations, the objective is to reduce arm tip displacement and velocity to the smallest levels at the end of the angular manoeuver to guarantee a quick start. To do this, the Index of Performance is a combination of the weighted squares of tip displacement and velocity in the fundamental mode of vibration.

In the next figures the main dynamical arm parameters are addressed and two different IPs are simulated. In each case, the computational scheme performed successfully (assured numerical convergence) and 8 simulation results, considering different slider initial positions, are presented to facilitate comparisons, that is, one slider starts motion from a fixed arbitrary point, while the other has its initial position changed each time a new run is initiated.

Table 1shows the main parameters of the flexible arm.

Table 1: Physical parameters of the Optimal Control Problem

Physical Parameters	
Length of the arm	$L = 0,7m$
Arm Thickness	h = 0,001 m
Arm Width	b=0.0254
Arm Mass	0,0482 kg
Arm Linear mass density	$\rho_0 = m/L$
Arm Moment of inertia	$J_v = \rho_0 * \frac{L^3}{3}$
Hub Moment of inertia	$J_c = 1.3510^{-4}$
Aluminum Density	$\rho_B = 27 \cdot 10 kg/m^3$ (Al)
Aluminum Young Modulus	$E = 7.1 \cdot 10^{10}$ PA
Slider 1 mass	$M_1 = 0.05 * m$
Slider 2 mass	$M_2 = 0.05 * m$

In the first case, the sliding masses are arranged as follows: sliding mass 1 is initially put in position $l_1 = 0.3556$, and sliding body 2 has initial positions

$l_2 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7$ for the 8 simulations. The performance index is $IP = \int_0^3 (4x_1^2 + x_2^2)dt$ All computer simulations were run using the RIOTS (Recursive Integration Optimal Trajectory Solver) 95 package by (Schwartz, 1966). This software solves the OCP allowing the use of 1-st, 2-nd, 3-rd and 4-th order fixed step-size Runge-Kutta integrator and 1-st to 4-th order splines. The optimization problem is solved with a class of conjugate-gradient techniques or with an SQP (Sequential Quadratic Programming) solver. User-defined cost and constraint functions, as well as their symbolic derivatives, are written in C code and dynamically linked to RIOTS. For the second case study, a small modification in the IP is made $IP = \int_0^3 (2x_1^2 + x_2^2)dt$ Through a relatively heavier weight on the variable x_2, one intends to diminish the tip velocity during the flexible arm rotation.

The variables x_1 represent the vibration of the flexible arm and x_2 its derivative. Thus the criterion established intended to minimize the vibration during rotation. The performance criteria are presented with the intention of showing the difficulties in the numerical solution. Comparing the position of the sliding masses, figures 4 and 9, and the external action (forces and torque) applied, figures 6 and 11, to the flexible arm makes it clear the influence of the parameters in performance criterion to obtain the solution.

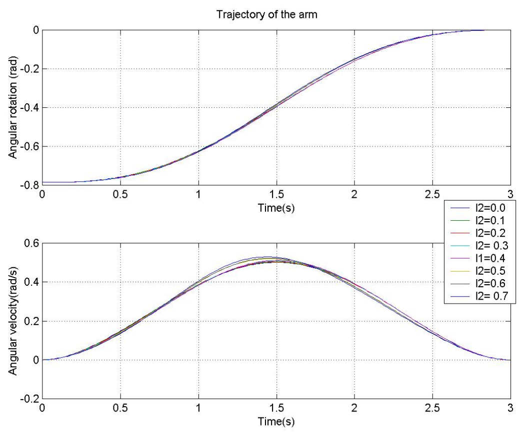

Figure 2: Flexible Arm Motion

Figures 2 and 7 show smooth movements of the rotating structure, as can be observed from the angular displacements, despite high angular velocities, since the structures start from null velocities, accelerate and ceases motion in just 3 seconds. Arm vibrations are quite small and movements are almost rigid ones.

In Figures 3 and 8, the tip vibrations on the end of the flexible arm are shown. Amplitudes are quite small when one considers the large flexibility of the structure.

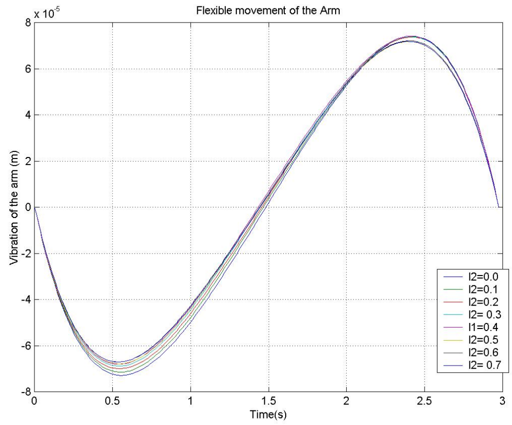

Figure 3: Vibration of the end of the flexible arm

In the cases shown, the smoothness of the curve is determined by the IP system.

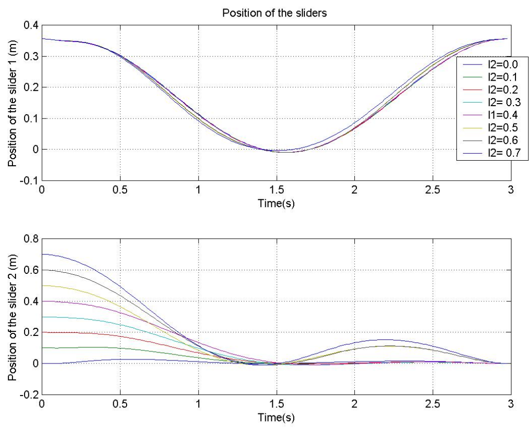

Figure 4: Trajectories of the sliders

Figures 4 and 9 exhibits the trajectories of the two sliders during the manoeuver of the flexible arm. One may observe in these simulations, a movement throughout the flexible arm on the first half of the time interval for the sliding mass m_2. For the mass m_1 , movements are virtually the same in all the simulations, noting that this slider moves toward the inner end of the flexible arm on the first half of the time and returns to its original position in the second half.

In Figures 5 and 10, the velocities of the two sliders are presented. Note that in both cases at the end of movement (T = 3s) their speeds are null, a condition that was deliberately imposed in the simulation scheme.

In Figures 6 and 11, the external forces acting on the flexible arm, due to the mass M_1 and M_2 and the external torque on the hub are shown. Forces to move the sliders are small and the torque is as required to perform the rigid motion.

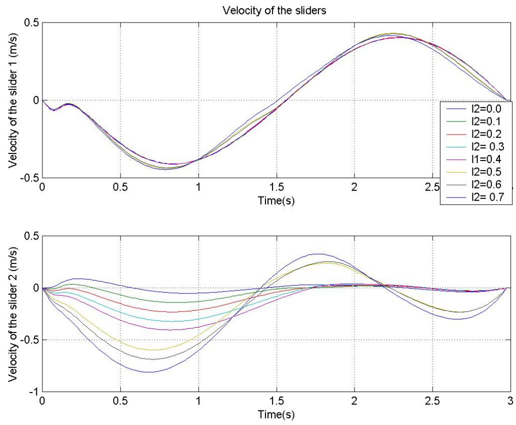

Figure 5: Velocity of the sliders

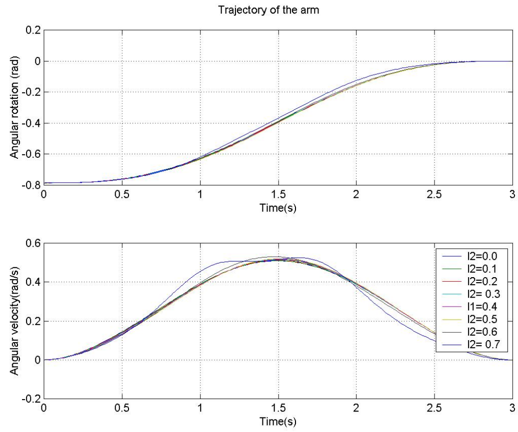

Figure 7: Flexible Arm Motion

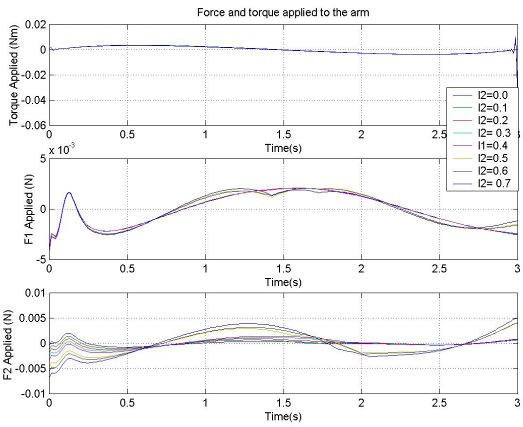

Figure 6: External Forces

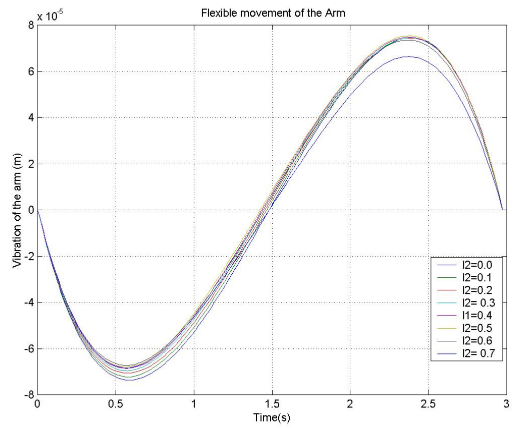

Figure 8: Vibration of the end of the flexible arm

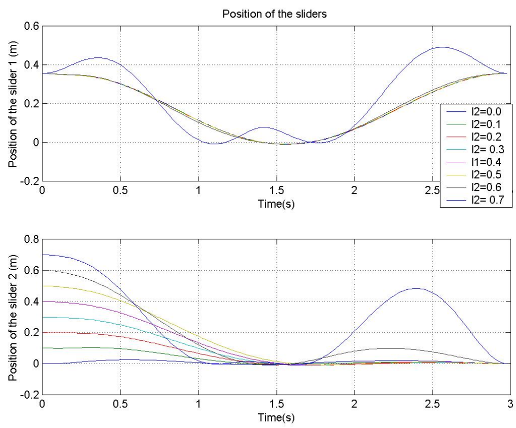

Figure 9: Trajectories of the sliders

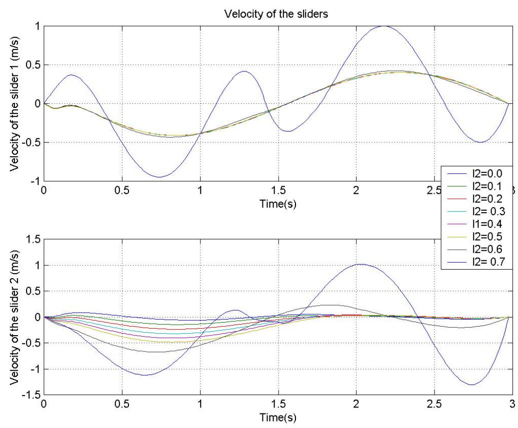

Figure 10: Velocity of the sliders

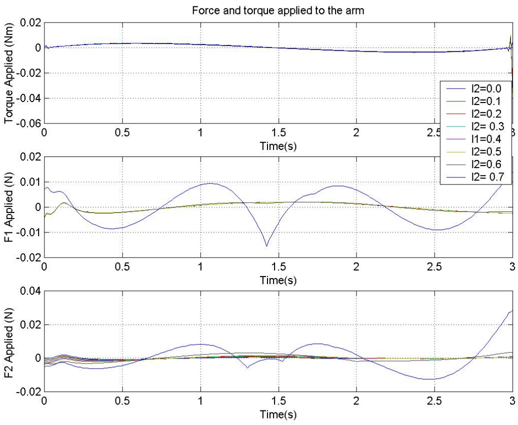

Figure 11: External Forces

7 FINAL COMMENTS

In this paper, we have presented a new way to reduce the vibrations induced on a flexible structure rotating around some axis. Vibration reduction is achieved through the translational motion of mass sliders. The full model of a narrow beam carrying n sliders is deduced and, in the sequence, the equations of motion are simplified by substruturing the system to make it feasible for control design. The non linear nature of the complex problem demands for Optimal Control approaches to find the trajectories the sliders shall perform in order to achieve the best vibration figures. Two slightly different cases considering two sliders have been proposed and simulated to demonstrate the feasibility of the proposed scheme. When comparing the two examples, a small change in the index of performance (other figures have been kept the same) leads to significant change in only one aspect, relative to the region where one of the sliding masses moves, the outer or the inner edge of the flexible arm, leading to the conclusion that the system behaves accordingly, despite the complexity of the motion of each part of the system. The authors intend to implement these and other already published results in an experimental device, already designed but not yet assembled, to confirm the performance of the proposed approaches in a near future.

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