Unknown input observer: a physical approach

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Abstract— **The object of this paper is the study of a new unknown input observer for linear models. This new observer has some classical restrictive conditions proposed for other ones: infinite structure condition and Hurwitz conditions for invariant zeros. The main contribution are twofold: this new observer has the same state space representation as the initial state model and it is shown to be very accurate. Simulation results are proposed for a DC motor example with analysis of two other classical methodologies.**

Keywords: Unknown input observers, Bond graph, linear models, invariant zeros

1. Introduction

The unknown input and state observability problem (UIO) is a well known problem because for control design with a state space approach, the state vector $x(t)$ cannot be entirely measured and the system is often subject to unknown inputs $d(t)$ (disturbance or failure...) which must be estimated, as proposed in the state space representation (1). In this state equation $x \in \Re^n$ is the state vector, $z \in \Re^p$ is the vector set of measured variables and $y \in \Re^p$ is the vector of output variables to be controlled. The input variables are divided into two sets $u \in \Re^m$ and $d(t) \in \Re^q$ which represent known and unknown input variables respectively.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Fd(t) \\ z(t) = Hx(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

Different approaches give solvability conditions and constructive solutions for the unknown input observer problem. For LTI models, constructive solutions with reduced order observers are first proposed with the geometric approach [13], [4], [2]. Constructive solutions based on generalized inverse matrices taking into account properties of invariant zeros are given in [19] and then in [20] and [15] with observability and detectability properties. Full order observers are then proposed in a similar way (based on generalized inverse matrices) in [7] and [6], but with some restriction on the infinite structure of the model. The algebraic approach is proposed in [25] and in [5] for continuous and discrete time systems, without restriction on the infinite structure of the model. The structural invariants which play a fundamental role in this problem have been extensively studied in many papers and books [2], [21], [23], [16], [12], [17]. The knowledge of zeros is often an important issue because zeros are directly related to some stability conditions of the controlled system and the infinite structure is often related to solvability conditions.

The objective of this paper is the development of a new observer for linear systems when there are two kinds of inputs: measured and unmeasured inputs. The second section deals with the problem statement in an usual way, with the recall of two classical approaches and some conditions of application. In the third section, the new approach is proposed and then an application on a DC motor is proposed. Simulations are proposed for this new observer and for two classical approaches.

2. Preliminaries

2.1. UIO existence conditions

In the literature, the different proposed approaches consider first the finite structure of $\Sigma(H,A,F)$ and then its infinite structure. The finite structure gives some stability conditions on the UIO and the infinite structure some conditions on the existence of the UIO.

The concepts of strong detectability, strong* detectability and strong observability have been proposed in [14]. System (1) (with only unknown input $d(t)$) is strongly detectable if $z(t) = 0$ for $t > 0$ implies $x(t) \to 0$ with $(t \to \infty)$ and system (1) is strong* detectable if $z(t) = 0$ for $t \to \infty$ implies $x(t) \to 0$ with $(t \to \infty)$.

The strong detectability corresponds to the minimum-phase condition, directly related to the zeros of system $\Sigma(H,A,F)$ (finite structure) defined as to be the values of $s \in \mathscr{C}$ (the complex plane) for which (2) is verified.

$$\texttt{rank}\begin{pmatrix} sI-A & -F \\ H & 0 \end{pmatrix} < n + \texttt{rank}\begin{pmatrix} -F \\ 0 \end{pmatrix} \quad (2)$$

Proposition 1: [14] The system $\Sigma(H,A,F)$ in (1) is strongly detectable if and only if all its zeros s satisfy $Re(s) < 0$ (minimum phase condition).

Proposition 2: [14] The system $\Sigma(H,A,F)$ in (1) is strong* detectable if and only if it is strongly detectable and in addition rank$[HF]$=rank$[F]$.

Proposition 3: [14] The system $\Sigma(H,A,F)$ in (1) is strongly observable if and only if it has no zeros.

The infinite structure of multivariable linear models is characterized by different integer sets. $\{n'_i\}$ is the set of infinite zero orders of the global model $\Sigma(C,A,B)$ and $\{n_i\}$ is the set of row infinite zero orders of the row sub-systems $\Sigma(c_i,A,B)$. The infinite structure is well defined in case of LTI models [8] with a transfer matrix representation or with a graphical representation (structured approach), [9].

The row infinite zero order n_i verifies condition $n_i = \min\left\{k|c_iA^{(k-1)}B \neq 0\right\}$. n_i is equal to the number of derivations of the output variable $y_i(t)$ necessary for at least one of the input variables to appear explicitly. The global infinite zero orders [10] are equal to the minimal number of derivations of each output variable necessary so that the input variables appear explicitly and independently in the equations. The infinite structure can also be defined for others models, such as $\Sigma(H,A,F)$.

In order to solve the UIO problem for systems in (1), a necessary condition called *observer matching condition* for the existence of observers is often required (see [19]; [7]): rank$[HF]$ = rank$[F]$. For a SISO model, the infinite zero order of model $\Sigma(H,A,F)$ is equal to 1. When this condition is not satisfied [11] proposed unknown input sliding mode observers after implementing a procedure to get a canonical observable form of systems. This method can also be extended in the nonlinear case. [5] and [1] gave an intrinsic solution with an algebraic approach. Necessary and sufficient conditions are that system $\Sigma(H,A,F)$ is left invertible and minimum phase. The LTI system $\Sigma(H,A,F)$ in (1), supposed to be asymptotically observable with unknown input, is rapidly observable if, and only if, it zero dynamics is trivial.

2.2. UIO Synthesis

Two approaches are recalled in this paper. The goal is to compare the performances of these observers with the new one proposed in this paper.

Approach with pseudo-inverse: An observer proposed by [6] has the form (3).

$$\begin{cases} \dot{\xi}(t) = N\xi(t) + Jz(t) + Gu(t) \\ \hat{x}(t) = \xi(t) - Ez(t) \end{cases} \tag{3}$$

where $\hat{x}(t) \in \Re^n$ is the estimate of $x(t)$. Matrices N, J, G and E with constant entries have appropriate dimensions. [6] studied the model with unknown inputs in the state and in the measurement equations. Here the model is simplified without considering unknown inputs in the measurement equation, i.e., $z = Hx(t) + Dd(t), D = 0$.

Let $P = I + EH$, the observer reconstruction error is $e = x - \hat{x} = Px - \xi$. The dynamic of the estimation error is given by $\dot{e} = Ne + (PA - NP - JH)x + (PB - G)u + PFd$. Hence, in the error variable equation some relations must be satisfied: $PA - NP - JH = PB - G = PF = 0$. In order to solve these equations, some generalized inverse matrices must be defined because in the previous equations some matrices are not square. Moreover, model $\Sigma(H,A,F)$ has a stable observer if the model is strong* detectable.

Approach with output derivation: The unknown input observer for a SISO model [5] with control input is written in (4).

$$\begin{cases} \dot{\hat{x}} = (PA - LH)\hat{x} + Q\left(z^{(r)} - U\right) + Lz + Bu \\ \hat{d} = \left(HA^{r-1}F\right)^{-1}\left(z^{(r)} - HA^r\hat{x} - U\right) \end{cases} \tag{4}$$

r is the infinite zero order of $\Sigma(H,A,F)$. $\hat{d}$ is the estimation of d and the matrices Q and P verify: $Q = F\left(HA^{r-1}F\right)^{-1}$, $P = I_n - QHA^{r-1}$, and $U = \sum_{i=0}^{r-1} HA^iBu^{(r-1-i)}$. The main idea of the method is to implement derivations on the output variable $z(t)$ to let the unknown input variable $d(t)$ appears explicitly. Note that the control input must be derivable ($r-1$ times). For MIMO models, the extension of the procedure was proposed by [11].

The dynamic of the estimation error of state variables is $\dot{e} = \dot{x} - \dot{\hat{x}} = (PA - LH)(x - \hat{x})$. One has $\lim_{t\to\infty} e(t) = 0$ for any $x(0)$, $\hat{x}(0)$, $d(t)$ and $u(t)$. The estimation of d can be written as $\hat{d} = \left(HA^{r-1}F\right)^{-1}HA^r(x - \hat{x}) + d$. As $\lim_{t\to\infty} e(t) = 0$, then $\lim_{t\to\infty} \hat{d}(t) = d(t)$.

This observer is stable if the finite structure of $\Sigma(H,A,F)$ is stable.

3. New approach

3.1. New UIO

If a somewhat physical approach is proposed, some assumptions are possible for the state space model deduced for example from a bond graph representation. The model $\Sigma(H,A,F)$ is also supposed to be a SISO model.

Asumption 1. It is supposed that the SISO system $\Sigma(H,A,F)$ defined in (1) is controllable/observable and that the state matrix A is invertible.

With Asumption 1, a derivative causality assignment is possible for bond graph models (physical model without null pole). The extension to models with non invertible state matrix is straight for bond graph models, because a graphical approach can be proposed in that case. It is not proposed in this paper.

The state equation (1) without output variable y is now rewritten as (5).

$$\begin{cases} x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t) \\ z(t) = HA^{-1}\dot{x}(t) - HA^{-1}Bu(t) - HA^{-1}Fd(t) \end{cases} \tag{5}$$

If matrix $HA^{-1}F$ is invertible (Model $\Sigma(H,A,F)$ has no null invariant zero), the disturbance variable can be written in equation (6) and then the estimation of the disturbance variable can be written in equation (7).

$$d(t) = -(HA^{-1}F)^{-1}[z(t) - HA^{-1}\dot{x}(t) + HA^{-1}Bu(t)] \tag{6}$$

$$\hat{d}(t) = -(HA^{-1}F)^{-1}[z(t) - HA^{-1}\dot{\hat{x}}(t) + HA^{-1}Bu(t)] \tag{7}$$

From the state equation (5), a new estimation is proposed for the state vector, defined in equation (8), which can also be written as (9), which is similar to a classical estimation, but with a difference in the last term. It needs the derivation of the measured variable. Matrix K is used for pole placement.

$$\hat{x}(t) = A^{-1}\dot{\hat{x}}(t) - A^{-1}Bu(t) - A^{-1}F\hat{d}(t) + K(\dot{z}(t) - \dot{\hat{z}}(t)) \tag{8}$$

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - AK(\dot{z}(t) - \dot{\hat{z}}(t)) \quad (9)$$

For these three observers, the estimate of the state vector is the solution of a first order differential state equation which is not the state equation of the model for the two first observers. In our approach, the state equation is the same (model and observer), with only an extra term for the observer. This new observer is thus much more simpler. Note that most of the works proposed in the literature do not take into account the control inputs.

3.2. Properties of the observer

In this section, some properties of the UIO are enounced and proved. It is proved that this new observer requires the matching condition defined in some well known approaches [14], [6] and that in that case, fixed poles of the estimation error are all the invariant zeros of system $\Sigma(H,A,F)$, which means that this system must be strong* detectable.

The convergence of the disturbance variable can be verified with equation (10), obtained from (6) and (7).

$$d(t) - \hat{d}(t) = (HA^{-1}F)^{-1}HA^{-1}(\dot{x}(t) - \dot{\hat{x}}(t)) \quad (10)$$

The estimation of the disturbance variable converges to the disturbance variable only if $(\dot{x}(t) - \dot{\hat{x}}(t))$ converges asymptotically. Convergence of the state estimation must be proved with the study of the observer fixed poles.

In order to simplify notations, new matrices N_{BO} and N_{BF} are introduced in (11).

$$\begin{cases} N_{BO} = A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} \\ N_{BF} = A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} - KH \end{cases} \quad (11)$$

From (5) and (8), with $e(t) = x(t) - \hat{x}(t)$ it comes (12).

$$e(t) = N_{BF}\dot{e}(t) \quad (12)$$

In equation (12), conditions for pole placement are studied. If matrix N_{BF} is invertible, a classical pole placement is studied, and the error variable $e(t) = x(t) - \hat{x}(t)$ does not depend on the disturbance variable. The conditions for (8) to be an asymptotic state observer of $x(t)$ is that N_{BF} must be an Hurwitz matrix, i.e., has all its eigenvalues in the left-hand side of the complex plane. Properties of the observer are studied in the next part.

A necessary condition for the existence of the state estimator is proposed in Proposition 4.

Proposition 4: A necessary condition for matrix N_{BF} defined in (11) to be invertible is that $HF \neq 0$.

Proof In Proposition 4, matrix $N_{BF}F$ is equal to $[A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} - KH]F$, thus it can be rewritten as $N_{BF}F = A^{-1}F - A^{-1}F(HA^{-1}F)^{-1}HA^{-1}F - KHF = KHF$. If condition $HF \neq 0$ is not satisfied, the Kernel of matrix N_{BF} is not empty, which means that matrix N_{BF} is not invertible and that this matrix contains at least one null mode, thus pole placement is not possible (all its eigenvalues are not in the left-hand side of the complex plane).□

Condition defined in proposition 4 is exactly the same condition defined for the well-known observers defined in [14] and [6]. It means that the infinite zero order between the disturbance variable $d(t)$ and the measured variable $z(t)$ is equal to 1.

It is now supposed that $HF \neq 0$ is satisfied. Two properties are proved. First, it is proved that matrix N_{BO} has one eigenvalue equal to 0 and that the other eigenvalues are the inverse of the invariant zeros of system $\Sigma(H,A,F)$. In that case in matrix N_{BF}, fixed modes are the inverse of the invariant zeros of model $\Sigma(H,A,F)$ and the only eigenvalue which can be chosen is related to the null eigenvalue of matrix N_{BO}.

Proposition 5: In matrix N_{BF} defined in (11), only 1 pole can be chosen with matrix K.

Proof Since $HN_{BO} = HA^{-1} - HA^{-1}F(HA^{-1}F)^{-1}HA^{-1}$, it comes $HN_{BO} = 0$. If a state model is considered with the state matrix N_{BO} and the output vector H, it is proved that this model is not observable since vector H^t is orthogonal to the state matrix. Moreover, the rank of the observability matrix of $\Sigma(H,N_{BO})$ is equal to 1. If a classical state estimation is proposed for this model, only 1 pole can be assigned, which is also true for matrix N_{BF}, because matrix N_{BF} can be considered as the state matrix of an equation error when estimating the state of system $\Sigma(H,N_{BO})$.□

Proposition 6: The eigenvalues of matrix N_{BO} defined in (11) are the inverse of the invariant zeros of system $\Sigma(H,A,F)$ ($n-1$ modes) plus 1 eigenvalue equal to 0.

Proof: appendix A

Proposition 7: The fixed poles of the estimation equation defined in (12) are the invariant zeros of system $\Sigma(H,A,F)$.

Proof From Proposition 6, the eigenvalues of matrix N_{BO} are the inverse of the invariant zeros of system $\Sigma(H,A,F)$ with an eigenvalue equal to 0, and since N_{BF} is invertible and only one pole can be chosen, all the fixed poles are the non null eigenvalues.□

4. EXAMPLE

The previous procedures are applied on a DC motor modeled by bond graph [18] and [22]. At the analysis step, proposed methods on bond graph models do not require the knowledge of the value of parameters, because intrinsic solvability conditions can be given and a formal calculus can be proposed at the synthesis level. First some properties of bond graph models are recalled and then the example is studied. Note that this system could be studied without the bond graph approach, and that other state variables could be chosen.

4.1. Bond graph approach

In a bond graph model, causality and causal paths are useful for the study of properties, such as controllability, observability and systems poles/zeros. Bond graph models with integral causality assignment (BGI) can be used to determine reachability conditions and the number of invariant zeros by studying the infinite structure. The rank of the controllability matrix is derived from bond graph models with derivative causality (BGD) [24].

An LTI bond graph model is controllable if and only if the two following conditions are satisfied: first there is a causal path between each dynamical element and one of the input sources and secondly each dynamical element can have a derivative causality assignment in the bond graph model with a preferential derivative causality assignment (with a possible duality of input sources). The observability property can be studied in a similar way, but with output detectors. Systems invariant zeros are poles of inverse systems. Inverse systems can be constructed by bond graph models with bicausality (BGB) which are thus useful for the determination of invariant zeros.

The concept of causal path is used for the study of the infinite structure of the model.The order of the infinite zero for the row sub-system $\Sigma(h_i, A, F)$ is equal to the length of the shortest causal path between the i^{th} output detector z_i and the set of disturbance input sources. The global infinite structure is defined with the concepts of different causal paths (not recalled here). The number of invariant zeros is determined by the infinite structure of the BGI model. The number of invariant zeros associated to a controllable, observable, invertible and square bond graph model is equal to $n - \sum n'_i$. For bond graph models, invariant zeros equal to zero can be directly deduced from the infinite structure of the BGD model [3].

An example of a DC motor is used to show the procedure for designing the UIO observer. The BGI model of the system with a disturbance signal is given in Fig. 1, and the state-space equations are presented in (13), with $x = (p_L, p_J)^t = (x_1, x_2)^t$ the state vector. Since the state equation is written from a bond graph model, the state vector contains energy variables, for example p_L is the magnetic flux in the inductance. z is the measured output variable, it is the rotational speed of the motor drive shaft, $z = \frac{p_J}{J}$. u is the control input variable and d the disturbance input variable (disturbance torque). The input $u(t)$ is the Heaviside unit step function, i.e. $u(t) = 100\Gamma(t)$. The disturbance d of the system is a pulse signal with start time $0.005s$, end time $0.006s$ and amplitude $10N$.

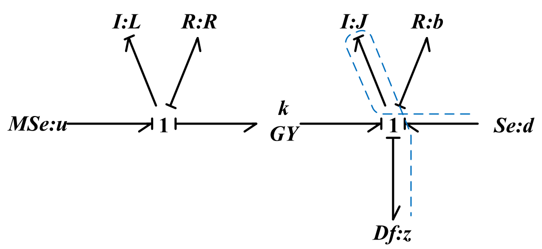

Fig. 1. BGI model of the DC moteur

$$\begin{cases} \dot{x}_1 = -\frac{R}{L}x_1 - \frac{k}{J}x_2 + u \\ \dot{x}_2 = \frac{k}{L}x_1 - \frac{b}{J}x_2 + d \\ z = \frac{1}{J}x_2 \end{cases} \tag{13}$$

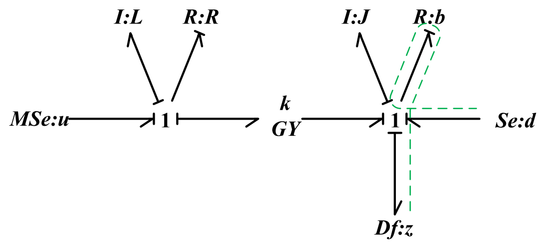

Fig. 2. BGD model of the DC moteur

The bond graph model is controllable and observable (a derivative causality can be assigned, fig. 2). The numerical values of system parameters are shown in Table I. In this part, some simulation results will be shown by the bond graph software 20-sim®.

TABLE I
NUMERICAL VALUES OF SYSTEM PARAMETERS

L	R	k	J	b
1.6×10^{-4} H	0.29 Ω	0.225	1×10^{-4} kgm^2	1×10^{-4} Nm/Wb

The rotor angular velocity ω and it's estimate $\hat{\omega}$ with an initial condition for the state variables $x_1 = 0$ and $x_2 = 0.01$ are studied in each case. Then the disturbance variable d and it's estimate $\hat{d}$ and the estimation errors for the state variables are drawn.

4.2. New observer

The design of the observer proposed in the previous section can thus be redesigned from a bond graph approach.

The causal path length between the output detector $Df : z$ and the disturbance input $Se : d$ is equal to 1, Fig. 1, path $Df : z \to I : J \to Se : d$, thus the matching condition is verified, and there is an invariant zero in the system $\Sigma(H, A, F)$. After calculations or analysis of the bond graph model with a bicausal assignment, the invariant zero is $s = -\frac{R}{L}$ which verifies the minimum phase condition. The bond graph representation of the observer is drawn in Fig. 3 in a general form without values for parameters.

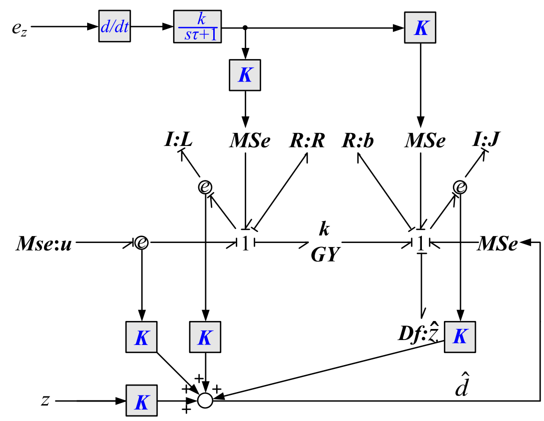

Fig. 3. Observer with the bond graph representation

In the state estimation equation defined in (8), matrix $K = [k_1, k_2]^t$ is used for pole placement. With some formal calculus, the two poles of matrix N_{BF}^{-1} defined in the state estimation error equation (12) are $s = -\frac{R}{L}$ and $s = -\frac{J}{k_2}$. The first fixed pole is equal to $-\frac{R}{L} = -1812.5$. The second one is chosen at $s = -2000$, thus $k_2 = 0.2$.

The two estimated variables $\hat{\omega}$ and $\hat{d}$ are very close to the real variables, Fig. 4, Fig. 5 and the estimation errors for the state variables converge rapidly to zero, Fig. 6. The different figures prove the accuracy of this new UIO.

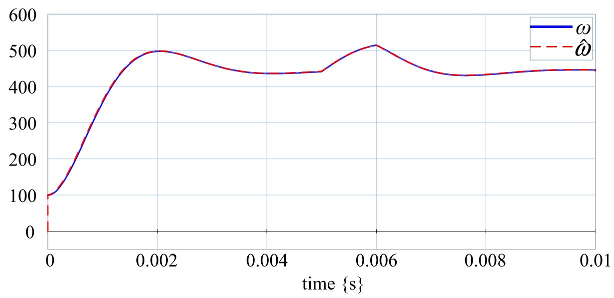

Fig. 4. The rotor angular velocity ω and it's estimate $\hat{\omega}$

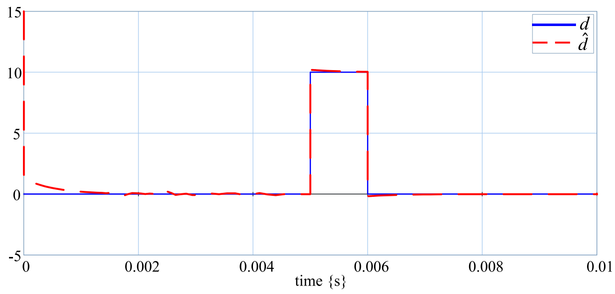

Fig. 5. The disturbance variable d and it's estimate $\hat{d}$

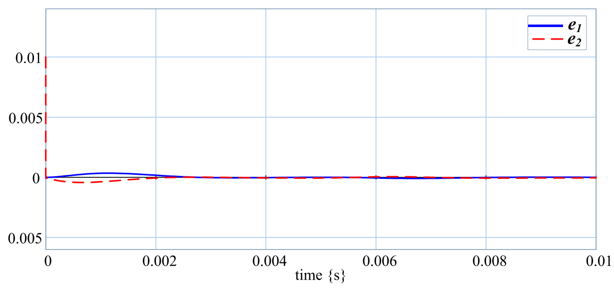

Fig. 6. Trajectories $e_i = x_i - \hat{x}_i, i = 1,2$ with UIO in (9)

4.3. Other observers

In order to compare the different observers, the same model is studied with the two other observers.

Matrix Z is used to place poles of the observer for the UIO defined with generalized inverse matrices. One pole is fixed (invariant zero of system $\Sigma(H,A,F)$), another is placed at $s = -2000$. The matrices of the observer are

$$N = \begin{bmatrix} -1812.5 & -2250 \\ 0 & -2000 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ -0.0001 \end{bmatrix}$$

$$J = \begin{bmatrix} -0.225 \\ 0 \end{bmatrix} \qquad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Variables ω and $\hat{\omega}$, Fig. 7, are very close. The estimation errors for the two state variables are displayed in Fig. 8. [6] did not proposed the estimate of the unknown input. In that case, results are similar, but the structure of the observer is much more complex.

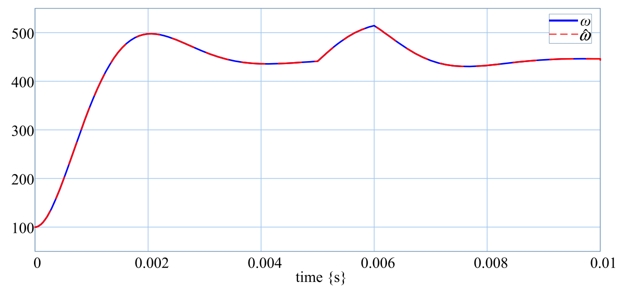

Fig. 7. The rotor angular velocity ω and it's estimate $\hat{\omega}$

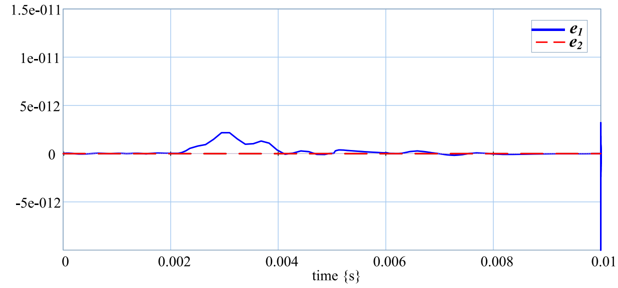

Fig. 8. Trajectories $e_i = x_i - \hat{x}_i, i = 1,2$ of the system

With the algebraic approach, Matrix L is used to place poles of the observer. One pole is fixed (invariant zero of system $\Sigma(H,A,F)$), another is placed at $s = -2000$. The matrices of the observer are $Q = \begin{bmatrix} 0 \\ 0.0001 \end{bmatrix}$ $PA - LH = \begin{bmatrix} -1812.5 & -2250 \\ 0 & -2000 \end{bmatrix}$

$$L = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \qquad U = 0$$

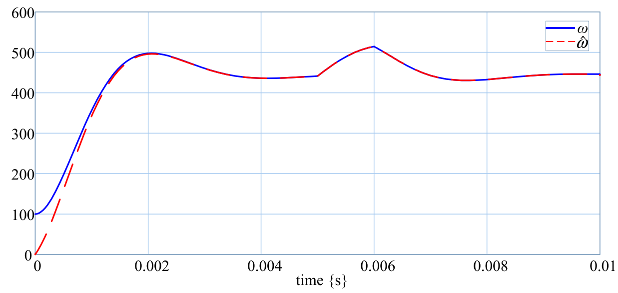

Fig. 9. The rotor angular velocity ω and it's estimate $\hat{\omega}$

Variables ω and $\hat{\omega}$ are drawn in Fig. 9. The estimation of d is $\hat{d} = 0.0001(\dot{y} - \begin{bmatrix} 14062500 & -10000 \end{bmatrix} \hat{x})$. The comparison of the estimation of the unknown input $\hat{d}(t)$ and $d(t)$ is shown in Fig. 10. The estimation errors for the two state variables are displayed in Fig. 11. In that case, our new observer is more accurate.

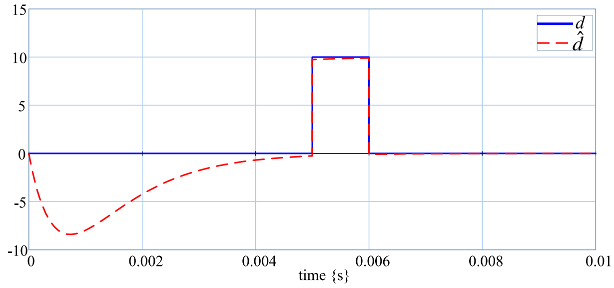

Fig. 10. Trajectories d and $\hat{d}$ of the system

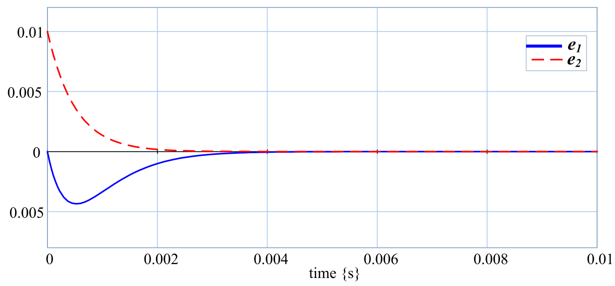

Fig. 11. Trajectories $e_i = x_i - \hat{x}_i, i = 1,2$ of the system

5. CONCLUSION

In this paper, a new input and state observer is proposed for linear systems. The classical assumption on the strong* detectability property is necessary. Two significant facts concern the simplicity of the observer synthesis and the efficiency of this observer. This observer is proposed in a SISO context, but can be easily extended to linear MIMO Systems.

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APPENDIX

Consider the quadruple $\Sigma(A^*,B^*,C^*,D^*)$, with $A^* = A^{-1}$, $B^* = -A^{-1}F$, $C^* = HA^{-1}$ and $D^* = -HA^{-1}F$. This model is controllable and observable, thus the invariant zeros of this model are the zeros of the Smith matrix defined in (14).

$$S(s) = \begin{pmatrix} sI - A^{-1} & A^{-1}F \\ HA^{-1} & -HA^{-1}F \end{pmatrix} \tag{14}$$

With the usual properties of matrix determinant, it comes $\det S(s) {=} \det HA^{-1}F.\det(sI - A^{-1} + A^{-1}F(HA^{-1}F)^{-1}HA^{-1})$, and thus $\det S(s) = \det(sI - N_{BO})$. The invariant zeros of the quadruple $\Sigma(A^*,B^*,C^*,D^*)$ are the poles of matrix N_{BO}.

With classical symbolic equivalent operation, it comes:

$$\begin{vmatrix} sI - A^{-1} & A^{-1}F \\ HA^{-1} & -HA^{-1}F \end{vmatrix} \sim \begin{vmatrix} sI - A^{-1} & (sI - A^{-1})F + A^{-1}F \\ HA^{-1} & HA^{-1}F - HA^{-1}F \end{vmatrix} \tag{15}$$

$$\det S(s) = \begin{vmatrix} sI - A^{-1} & sF \\ HA^{-1} & 0 \end{vmatrix} \sim \begin{vmatrix} sA - I & sF \\ H & 0 \end{vmatrix} \tag{16}$$

Thus $\det S(s) = \det(sA - I).\det(H(sA - I)^{-1}sF)$ and it comes $\det S(s) = s.\det(s^{-1}I - A).\det(H(s^{-1}I - A)^{-1}F)$. Thus the roots of $S(s)$ are all the inverse of the invariant zeros of system $\Sigma(H,A,F)$ with a root equal to 0.□