JACOBIAN BASED CONTROL OF WALKING ROBOT WITH COMPLIANT LEGS

M M Gor[(a)], P M Pathak[(b)], A K Samantaray[(c)], Jung-Ming Yang[(d)], Seong Woo Kwak[(e)]

[(a),(b)]Robotics & Control Lab, Indian Institute of Technology Roorkee, India
[(c)]Indian Institute of Technology, Kharagpur, India
[(d)]Catholic University of Daegu, South Korea
[(e)]Keimyung University, South Korea

[(a)]mehulmgor@rediffmail.com, [(b)]pushpfme@iitr.ernet.in, [(c)]samantaray@mech.iitkgp.ernet.in, [(d)]jmyang@cu.ac.kr, [(e)]ksw@kmu.ac.kr

ABSTRACT
Jacobian based control of walking robot is difficult as compared to terrestrial manipulator as walking robot's each leg can be considered as a terrestrial manipulator. Moreover each leg is not always in motion as it has to pass through stand and motion phase depending upon the gait pattern. This paper proposed an approach where first gait pattern is assumed and for this gait pattern joint motions are assumed. For these joint motions the leg tip velocity is simulated. For pattern generation this leg tip velocity is assumed as the reference velocity and given to leg tip during the motion of that particular leg. The actual velocity of leg is calculated from the joint velocity of leg. The error in leg tip velocity is evaluated in PID controller, calculates the corrective voltage required for the joint motors. For the said analysis, the bond graph has been adopted as the modeling tool.

Keywords: Walking Robot, Workspace, Jacobian, Bond Graph

1. INTRODUCTION

Research in legged robots started almost four decades ago with an attempt to realize rigid legged locomotion. Researchers have used either analytical based approaches or biologically inspired approaches for the accomplishment of legged robot locomotion control.

Research pertaining to four legged robots has started almost two decades ago. But the research has not yet matured enough to produce commercially exploitable robotic system.

Legged robot have great problem of trajectory planning and stability which demands for good kinematics and dynamics model of the system.

Bernardi and Da Cruz (2007) presented a kinematical and dynamical analysis of a quadruped robot in which robots behavior was considered as parallel chain during pushing stage of platform and then serial chain during leg motion seeking for the new point of grasping.

Angeles (2007) showed quadruped walking which includes kinematic loops that open when a leg takes off and open chains that close when a leg touches the ground. This fact implies time-varying in a degree of freedom. (Pfeiffer 1995).

Kolter (2008) presented the controller consisting of a high-level planner that plans a set of footsteps across the terrain, a low-level planner that plans trajectories for the robot's feet and center of gravity and a low-level controller that tracks these desired trajectories using a set of closed-loop mechanisms. Many researchers (Zhao 2002; Aclan 2009) used PID controller to control bipedal robot locomotion in which predefined trajectories are taken as input.

Research in the direction of use of compliance in legged robots started with a motivation to realize faster and efficient locomotion. Research in this field can be broadly categorized as studies focused on understanding the inherent passive dynamics phenomena associated with the compliance (in the form of leg muscles and tendons, at joints in the form of cartilages, viscous fluid etc.) in animals and human beings and the implementation of the principles through analytical and experimental models in legged robots ranging from one, four to six legged robots. Krishnan (2010) shows simulation study of compliant legged quadruped in joint space. Yet from literature review, it can be found that there are several issues to be addressed so that useful legged robots can be made. Control of walking robot in workspace is one of them.

The main idea of this article is to study and simulate four legged walking robot in workspace, where reference tip velocity pattern is generated which will be compared with the actual tip velocity of leg by PID controller. Using jacobian and calculated error by PID controller, corrective voltage is found for the joints.

2. MODELING OF COMPLIANT LEG WALKING ROBOT

It is assumed that the walking robot is performing locomotion through the bounding gait. Modeling of the quadruped robot with compliant leg in sagittal plane consists of modeling of translational and angular dynamics of robot legs and body. Each leg of the robot has been modeled as an open chain manipulator comprising of one compliant link and one rigid link connected through revolute joints (Krishnan 2011). As shown in Figure 1, frame $\{A\}$ is an inertial frame of reference and $\{V\}$ is the body frame.

A coordinate frame is also attached to each link. The link frames are named by number according to the link to which they are attached i.e. frame $\{i\}$ is rigidly attached to link i. The joint between links i and i+1 is numbered as i+1. Frame $\{0\}$ and $\{1\}$ are coincident, where frame $\{0\}$ is body fixed frame at the hip joint. The rotational inertia of a body is defined about a frame fixed at the centre of gravity (CG) of the body. The axis of the CG frame is fixed along the principal directions of the body. r_F and r_R is the distance of $\{0_F\}$ and $\{0_R\}$ frame respectively from $\{V\}$ frame, ϕ is body angular displacement, θ_{1F} and θ_{2F} are the joint angles of first and second joint of front leg, θ_{1R} and θ_{2R} are the joint angles of first and second joint of rear leg, l_1 and l_2 are the link lengths of the first and second links of front and rear leg.

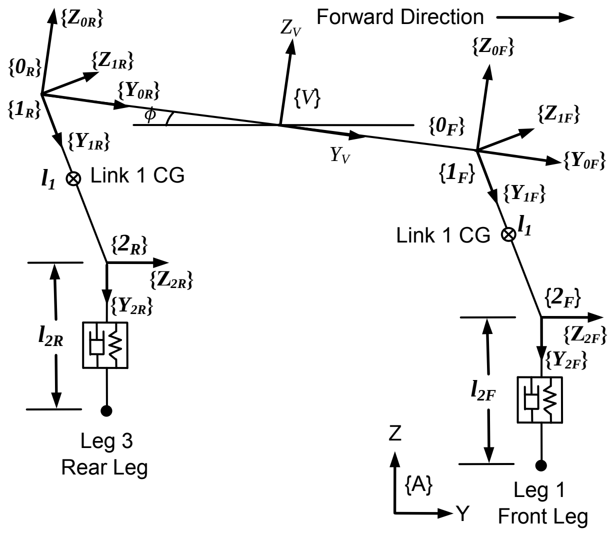

Figure 1: Compliant legged quadruped robot in Sagittal plane presentation

The position coordinate of the front leg tip can be expressed with respect to inertial frame as

$$\begin{bmatrix} Y_{Ftip} \\ Z_{Ftip} \end{bmatrix} = \begin{bmatrix} \begin{Bmatrix} Y_{CG} + r_F cos\phi \\ + l_1 \cos(\phi + \theta_{1F}) \\ + l_2 \cos(\phi + \theta_{1F} + \theta_{2F}) \end{Bmatrix} \\ \begin{Bmatrix} Z_{CG} + r_F sin\phi \\ + l_1 \sin(\phi + \theta_{1F}) \\ + l_2 \sin(\phi + \theta_{1F} + \theta_{2F}) \end{Bmatrix} \end{bmatrix} \tag{1}$$

The compliance in lower links of each leg is modeled on the basis of the following equations.

$$l_{2F}^2 = \left(Y_{Ftip} - Y_{2F}\right)^2 + \left(Z_{Ftip} - Z_{2F}\right)^2 \tag{2}$$

$$l_{2R}^2 = \left(Y_{Rtip} - Y_{2R}\right)^2 + \left(Z_{Rtip} - Z_{2R}\right)^2 \tag{3}$$

In above equations l_{2F} and l_{2R} are instantaneous lengths of link 2 of the front and rear leg respectively. Y_{Ftip} and Z_{Ftip} are the coordinate of toe of front leg and Y_{Rtip} and Z_{Rtip} are the coordinate of the rear leg. Y_{2F} and Z_{2F} are the coordinate of the frame $\{2_F\}$, and Y_{2R} and Z_{2R} are the coordinate of the frame $\{2_R\}$.

Taking derivative of equations (2) and (3)

$$\dot{l}_{2F} = \frac{(Y_{Ftip} - Y_{2F})}{l_{2F}}\left(\dot{Y}_{Ftip} - \dot{Y}_{2F}\right) + \frac{(Z_{Ftip} - Z_{2F})}{l_{2F}}\left(\dot{Z}_{Ftip} - \dot{Z}_{2F}\right) \tag{4}$$

$$\dot{l}_{2R} = \frac{(Y_{Rtip} - Y_{2R})}{l_{2R}}\left(\dot{Y}_{Rtip} - \dot{Y}_{2R}\right) + \frac{(Z_{Rtip} - Z_{2R})}{l_{2R}}\left(\dot{Z}_{Rtip} - \dot{Z}_{2R}\right) \tag{5}$$

Considering compliance in the lower links of the leg the velocity of the front leg tip can be given as

$$\begin{bmatrix} \dot{Y}_{Ftip} \\ \dot{Z}_{Ftip} \end{bmatrix} = \begin{bmatrix} \begin{Bmatrix} \dot{Y}_{CG} - (r_F \sin\phi)\dot{\phi} \\ -l_1 \sin(\phi + \theta_{1F})(\dot{\phi} + \dot{\theta_{1F}}) \\ - l_2 \sin(\phi + \theta_{1F} + \theta_{2F})(\dot{\phi} + \dot{\theta_{1F}} + \dot{\theta}_{2F}) \\ + \dot{l_{2F}} \cos(\phi + \theta_{1F} + \theta_{2F}) \end{Bmatrix} \\ \begin{Bmatrix} \dot{Z}_{CG} + (r_F cos\phi)\dot{\phi} \\ +l_1 \cos(\phi + \theta_{1F})(\dot{\phi} + \dot{\theta_{1F}}) \\ + l_2 \cos(\phi + \theta_{1F} + \theta_{2F})\,(\dot{\phi} + \dot{\theta_{1F}} + \dot{\theta}_{2F}) \\ + \dot{l_{2F}} \sin(\phi + \theta_{1F} + \theta_{2F}) \end{Bmatrix} \end{bmatrix} \tag{6}$$

Similarly Y_{Rtip} and Z_{Rtip} velocity equations for rear leg tip can be derived as

$$\begin{bmatrix} \dot{Y}_{Rtip} \\ \dot{Z}_{Rtip} \end{bmatrix} = \begin{bmatrix} \begin{Bmatrix} \dot{Y}_{CG} - r_R sin(\pi + \phi)\dot{\phi} \\ -l_1 \sin(\pi + \phi + \theta_{1R})(\dot{\phi} + \dot{\theta_{1R}}) \\ - l_2 \sin(\pi + \phi + \theta_{1R} + \theta_{2R})(\dot{\phi} + \dot{\theta_{1R}} + \dot{\theta}_{2R}) \\ + \dot{l_{2R}} \cos(\pi + \phi + \theta_{1R} + \theta_{2R}) \end{Bmatrix} \\ \begin{Bmatrix} \dot{Z}_{CG} + r_R cos(\pi + \phi)\dot{\phi} \\ +l_1 \cos(\pi + \phi + \theta_{1R})(\dot{\phi} + \dot{\theta_{1R}}) + \\ l_2 \cos(\pi + \phi + \theta_{1R} + \theta_{2R})\,(\dot{\phi} + \dot{\theta_{1R}} + \dot{\theta}_{2R}) \\ + \dot{l_{2R}} \sin(\pi + \phi + \theta_{1R} + \theta_{2R}) \end{Bmatrix} \end{bmatrix} \tag{7}$$

The bounding gait selected for locomotion enables us to study the locomotion dynamics in a sagittal plane. Using equations (6) and (7) bond graph model of four legged walking robot in sagittal plane can be drawn which is shown in Figure 2.

A soft pad is used to avoid differential causality. Soft pads are artificial compliances/lumped flexibilities (Ghosh 1991, Pathak 2005) that can be used in bond graph. In particular a soft pad is used instead of a pad in order to avoid algebraic loop while deriving equations. In bondgraph model, all bonds are not numbered to bring clarity in the figure. Transformer moduli for finding velocities at various points is shown in Table 1.

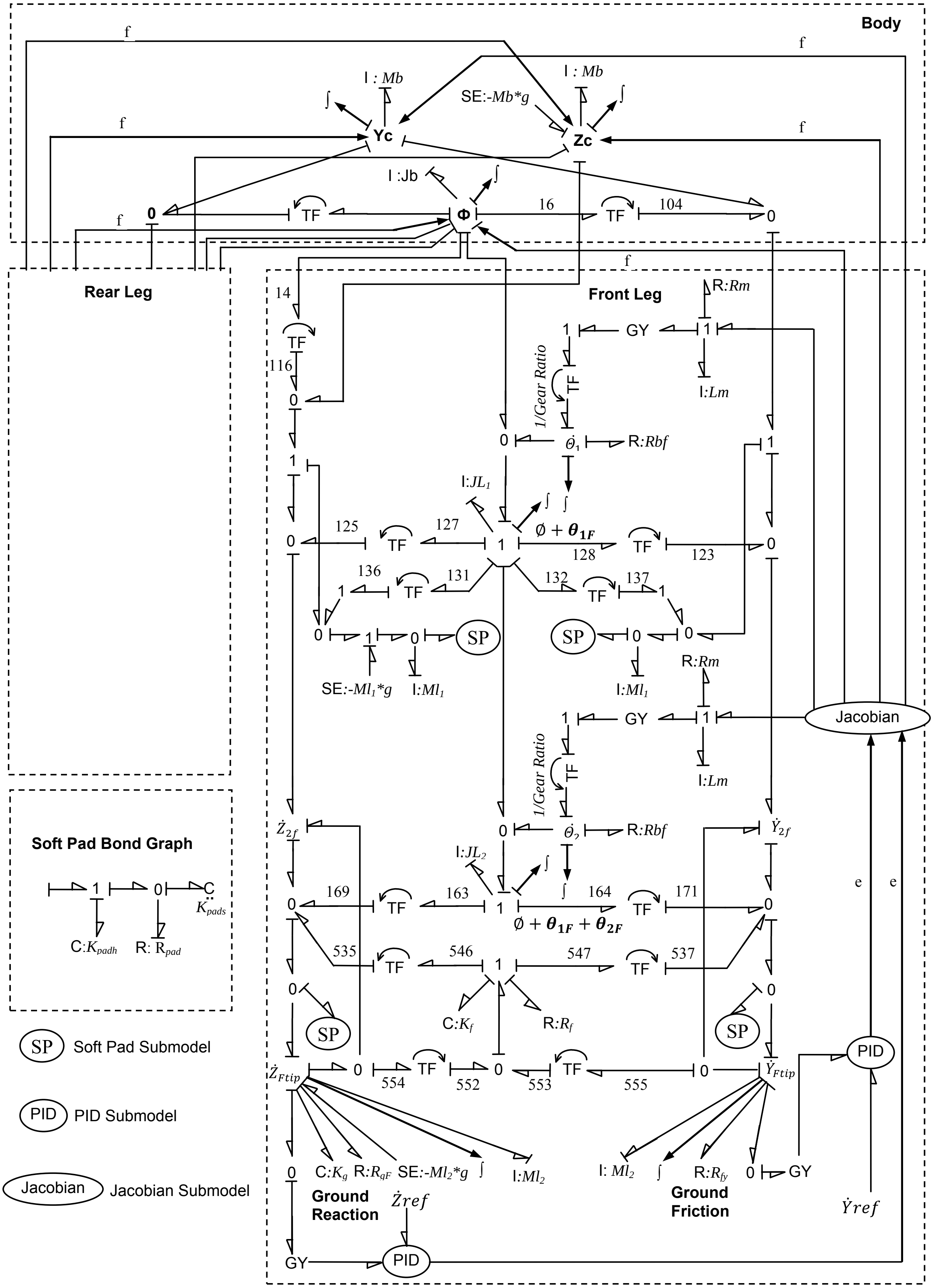

Figure 2: Bond Graph Model of Four Legged Walking Robot

Table 1: Transformer moduli for finding velocities at various points

	Y Direction	Z Direction
Platform	TF16-104=$-(r_F \sin\phi)$	TF14-16=$(r_F cos\phi)$
First Link Tip	TF128-123=$-l_1 \sin(\phi + \theta_{1F})$	TF127-125=$l_1 \cos(\phi + \theta_{1F})$
First Link CM	TF132-137=$-0.5 * l_1 \sin(\phi + \theta_{1F})$	TF131-136=$0.5 * l_1 \cos(\phi + \theta_{1F})$
Second Link Tip	TF164-171=$- l_2 \sin(\phi + \theta_{1F} + \theta_{2F})$ TF547-537=$\cos(\phi + \theta_{1F} + \theta_{2F})$ TF555-553=$\frac{(Y_{Ftip} - Y_{2F})}{l_{2F}}$	TF163-169=$l_2 \cos(\phi + \theta_{1F} + \theta_{2F})$ TF546-535=$\sin(\phi + \theta_{1F} + \theta_{2F})$ TF554-552=$\frac{(Z_{Ftip} - Z_{2F})}{l_{2F}}$

3. CONTROL IN WORKSPACE

3.1. Reference Velocity Pattern Generator

When compliant legged walking robot model works in joint space, it is found that foot tip velocity follows nearby sinusoidal curve. So here reference velocity pattern is generated using the same concept. It is assumed that robot will complete its walking cycle in 2.8 s and thus here all data are presented for 10 cycles i.e. for 28 s. Phase difference between front leg and rear leg reference velocity is kept as 0.2 s. Figure 3 and Figure 4 shows reference velocity of front leg and rear leg respectively.

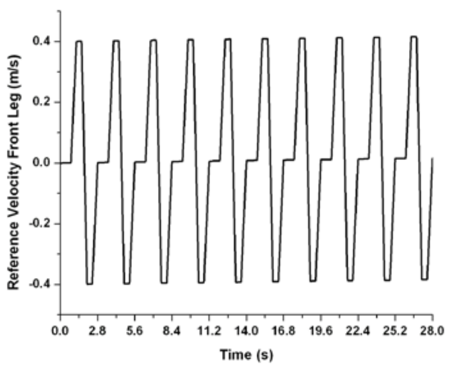

Figure 3: Reference Velocity of Front Leg

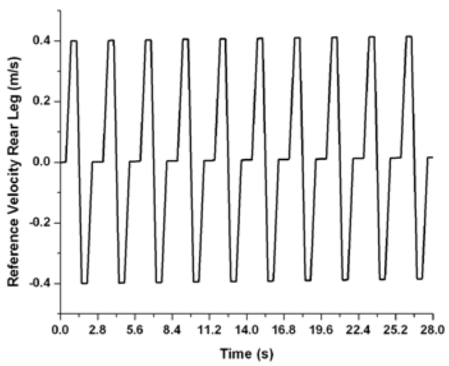

Figure 4: Reference Velocity of Rear Leg

3.2. PID Controller

The purpose of the use of PID controller is to make the leg movement in a desired way in terms of the reference pattern. PID controller has no offset error and reduces the tendency for oscillations. Controller is presented using bond-graph. The advantage of working in the bond graph domain is that a clear physical interpretation can be given to each controller coefficient. The PID controller discussed in the standard textbook (like Bolton 2007) gives a control signal

$$I_{out} = K_p e + K_I \int e \, dt + K_V \frac{de}{dt} + I_o \quad (8)$$

Where I_{out} is the output from the controller when there is an error e which is changing with time t, I_o is the set point output when there is no error, K_p is the proportionality constant, K_I is the integral constant and K_V is the derivative constant. Bond graph model of such controller can be represented as shown by Mukherjee (2006). Figure 5 shows the submodel of PID controller developed for this work.

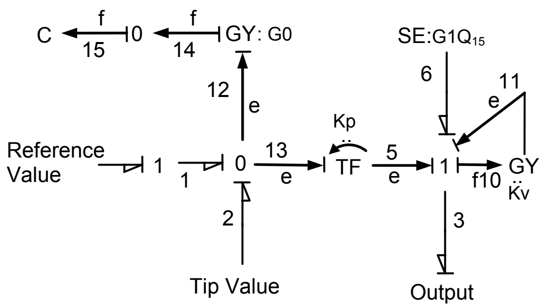

Figure 5: Bondgraph Submodel of PID Controller

3.3. Jacobian

In the bond graph modeling of the walking robot working in workspace with a PID controller, one needs to evaluate the jacobian of the manipulator. Jacobian is a linear mapping from velocities in joint space to Cartesian space. The inverse problem, where the joint velocities are to be determined for a given tip velocity, is practical importance and requires the inverse of the Jacobian.

The kinematic relations for the front leg tip displacements Y_{tip} and Z_{tip} in the Y and Z directions given with respect to inertial reference frame {A} is shown in equation (1). For jacobian model assuming link2 as rigid and differentiating equation (1), front leg tip velocity component can be derived as

$$\begin{bmatrix}\dot{Y}_{Ftip}\\ \dot{Z}_{Ftip}\end{bmatrix} = \begin{bmatrix}\left\{\begin{matrix}\dot{Y}_{CG} - (r_F \sin\phi)\dot{\phi}\\ -l_1 \sin(\phi + \theta_{1F})(\dot{\phi} + \dot{\theta_{1F}})\\ - l_2 \sin(\phi + \theta_{1F} + \theta_{2F})(\dot{\phi} + \dot{\theta_{1F}} + \dot{\theta}_{2F})\end{matrix}\right\}\\ \left\{\begin{matrix}\dot{Z}_{CG} + (r_F cos\phi)\dot{\phi}\\ +l_1 \cos(\phi + \theta_{1F})(\dot{\phi} + \dot{\theta_{1F}})\\ + l_2 \cos(\phi + \theta_{1F} + \theta_{2F})\,(\dot{\phi} + \dot{\theta_{1F}} + \dot{\theta}_{2F})\end{matrix}\right\}\end{bmatrix} \quad (9)$$

Similarly, velocity component of rear leg can be derived as

$$\begin{bmatrix}\dot{Y}_{Rtip}\\ \dot{Z}_{Rtip}\end{bmatrix} = \begin{bmatrix}\left\{\begin{matrix}\dot{Y}_{CG} - r_R sin(\pi + \phi)\dot{\phi}\\ -l_1 \sin(\pi + \phi + \theta_{1R})(\dot{\phi} + \dot{\theta_{1R}})\\ - l_2 \sin(\pi + \phi + \theta_{1R} + \theta_{2R})(\dot{\phi} + \dot{\theta_{1R}} + \dot{\theta}_{2R})\end{matrix}\right\}\\ \left\{\begin{matrix}\dot{Z}_{CG} + r_R cos(\pi + \phi)\dot{\phi}\\ +l_1 \cos(\pi + \phi + \theta_{1R})(\dot{\phi} + \dot{\theta_{1R}}) +\\ l_2 \cos(\pi + \phi + \theta_{1R} + \theta_{2R})\,(\dot{\phi} + \dot{\theta_{1R}} + \dot{\theta}_{2R})\end{matrix}\right\}\end{bmatrix} \quad (10)$$

Bondgraph presentation of the jacobian for front leg is shown in Figure 6. Similarly jacobian for rear leg is also developed.

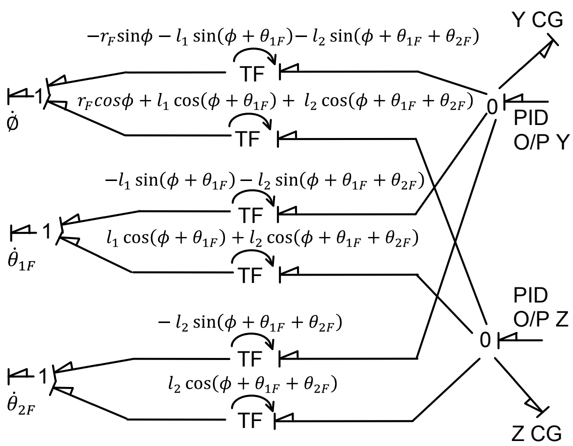

Figure 6: Bondgraph Submodel of Front Leg Jacobian

PID controller discussed in section 3.2 is used for trajectory control of walking robot. Generated reference flow velocity pattern is provided to the controller. The tip velocity of the leg is supplied to the controller. This input is compared with the reference flow input pattern and the error is processed in the controller, which in turn provides the corrective voltage as output of controller. This corrective voltage is fed to the Jacobian block, which evaluates the required voltage at the joints which will be supplied to the joints.

4. SIMULATION & RESULTS

To validate control strategy discussed above, the model has been simulated. Input parameters are listed in Table 2. Each cycle takes 2.8 s. Simulation is carried out for 10 complete cycle which takes 28 s. Animation result is shown in Figure 7. Figure 8 and 9 shows body CG displacement in Y and Z direction respectively. It can be seen that body propagates in positive Y direction. Figure 10 and 11 shows front leg tip movement in Y and Z direction respectively. Figure 12 and 13 shows rear leg tip movement in Y and Z direction respectively.

Table 2: Input parameters

Parameters	**Value**
Arm Parameters	
Front & Rear Leg Link 1 Length(l1)	0.065 m
Front & Rear Leg Link 2 Length(l2)	0.05 m
Front & Rear Leg Link 1 Mass(Ml1)	0.075 kg
Front & Rear Leg Link 2 Mass(Ml2)	0.065 kg
Location of Leg from body CG(rF/rR)	0.042 m
Compliant link 2 stiffness(Kf)	400 N/m
Compliant link 2 damping(Rf)	4 Ns/m
Inertia of Link 1 (Jl1)	0.00004 kgm^2
Inertia of Link 2 (Jl2)	0.00003 kgm^2
Common Parameter	
Mass of body(Mb)	0.28 kg
Inertia of body(Jb)	0.002 kgm^2
Ground Frictional Resistance(Rfy)	10 Ns/m
Ground Stiffness(Kg)	10000 N/m
Ground Damping(Rg)	10 Ns/m
Controller Parameter	
Proportional Gain of controller(Kp)	1.2
Derivative Gain of Controller(Kv)	-0.01
Integral Gain of Controller(G1)	5
Pad Parameter	
Stiffness of Spring (Kpads)	2000 N/m
Stiffness of Spring(Kpadh)	900 N/m
Damping Resistance(Rpad)	1 Ns/m
Actuator Parameter	
Motor Constant (Km)	0.01Nm/A
Motor Resistance(Rm)	0.1 Ohms
Motor Inductance(Lm)	0.001 H
Bearing Resistance(Rbf)	0.1 Nm/r/s
Gear Ratio	230

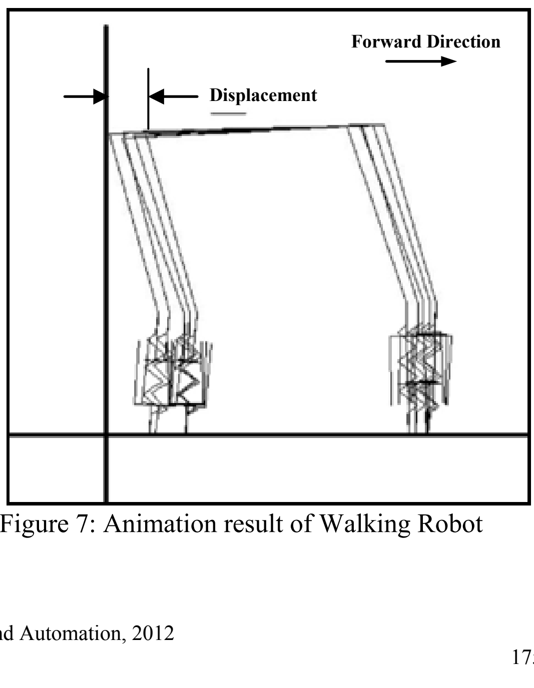

Figure 7: Animation result of Walking Robot

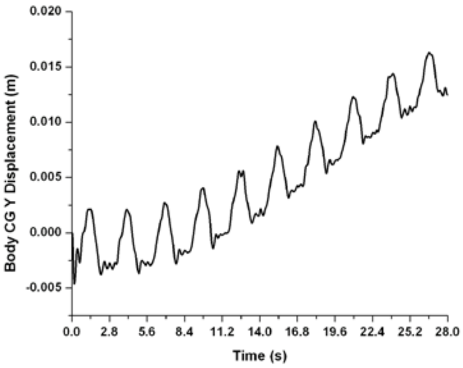

Figure 8: Body CG Y Displacement

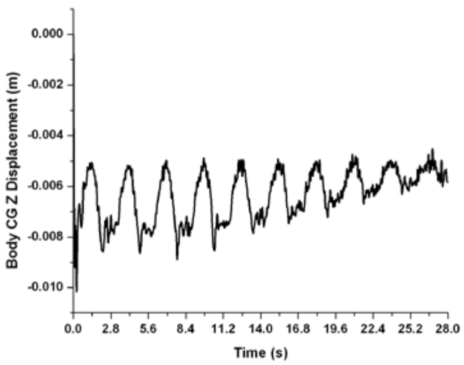

Figure 9: Body CG Z Displacement

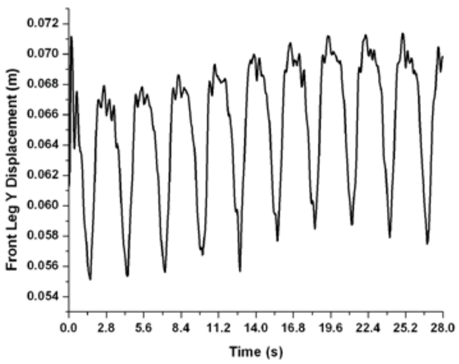

Figure 10: Front Leg Tip Y Displacement

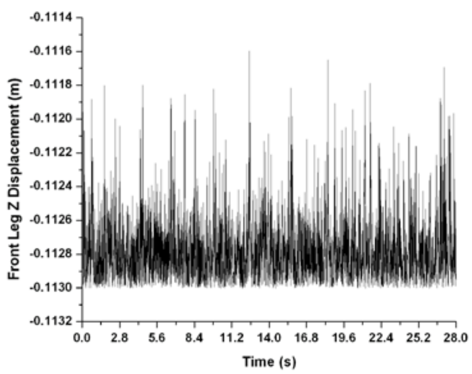

Figure 11: Front Leg Tip Z Displacement

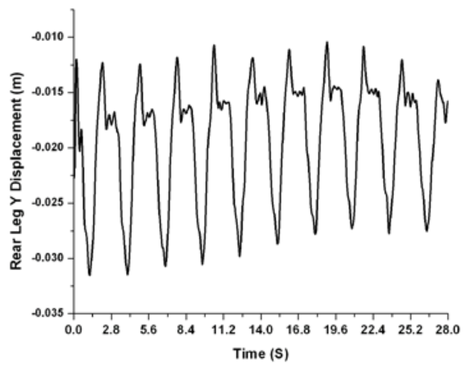

Figure 12: Rear Leg Tip Y Displacement

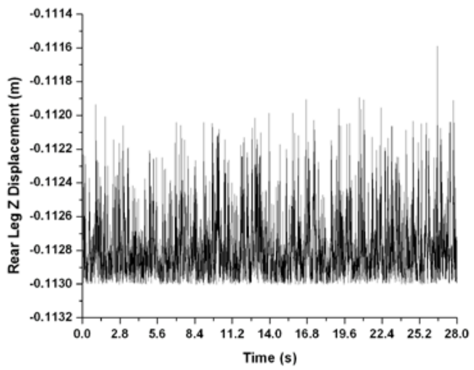

Figure 13: Rear Leg Tip Z Displacement

5. CONCLUSION

This work presents a method for trajectory control of a walking robot in workspace. The concept of using reference velocity pattern to evaluate Jacobian through PID controller is shown to be successful. The strategy can be extended for a control of walking robot in three dimensional models also.

REFERENCES

Aclan M.C., Ramos M.C., 2009, "Bipedal Robot Locomotion Using Multivariable Control", *TENCON 2009 - 2009 IEEE Region 10 Conference*, pp. 1-6.

Angeles, J., 2007, *"Fundamentals of Robotic Mechanical Systems. Theory, Methods, and Algorithms"*, Springer.

Bernardi, R. & Da Cruz, J. J., 2007, "Kamanbaré: A tree-climbing biomimetic robotic platform for environmental research", *International Conference on Informatics in Control, Automation and Robotics (ICINCO)*, pp. 478-484.

Bolton W., 2007, *"Mechatronics: Electronics Control Systems in Mechanical and Electrical Engineering"*, South Asia, Pearson Education.

Ghosh A. K., Mukherjee A., Faruqi M. A, 1991, "Computation of Driving Efforts for Mechanisms and Robots Using Bond Graphs", *Transactions of* *the ASME Journal of Dynamic, Systems, Measurement, and Control*, Vol. 113, pp. 744-748.

Kolter J. Z., Rodgers M. P., Ng A. Y., 2008, "A control architecture for quadruped locomotion over rough terrain," *in Proceedings IEEE Int. Conference on Robotics and Automation*, pp. 811-818.

Krishnan V. L, Pathak P. M., Sardana L., Jain S. C., 2010, "Simulation And Experimental Studies on Walking Robot with Flexible Legs", *4th International Conference on Integrated Modeling and Analysis in Applied Control and Automation (IMAACA 2010)*, Fes, Morocco, pp. 11-19.

Krishnan V. L., 2011, "*Modeling and Control of Quadruped Robot with Flexible Legs",* Ph.D thesis, IIT Roorkee, Roorkee, India.

Mukharjee A., Karmakar R., Samantaray A. K., 2006, "*Bond Graph in Modeling, Simulation and Fault Identification*", Indian, I. K. International.

Pathak P. M., Mukherjee A., Dasgupta A., 2005, "Impedance Control of Space Robots using Passive Degrees of Freedom in Controller Domain", *Transactions of ASME, Journal of Dynamic Systems, Measurement, and Control*., Vol. 127, pp. 564-578.

Pfeiffer F., Eltze J., Weidemann H. J., 1995, "The tum walking machine", *Intelligent Automation and Soft Computing. An International Journal* 1: pp. 307–323.

Zhao M., Liu L., Wanj J., Chen K., Zhao., Xu K., 2002, "Control system design of THBIP-I humanoid robot", *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*, Vol. 3, pp. 2253-2258.

ACKNOWLEDGEMENT

This work has been funded by Department of Science and Technology, Government of India (vide Grant No. INT/Korea/P-13) and National Research Foundation of Korea grant funded by the Korea government (MEST) (No. 2012-0006297), under Indo-Korea Joint Research in Science and Technology.

AUTHORS BIOGRAPHY

M M Gor received the B. E. -Production degree in 2000 and M. E. Mechanical-Machine Design degree in 2008, both from B. V. Mahavidyalaya, V. V. Nagar, India. He has joined the Department of Mechanical Engineering at G H Patel College of Engineering & Technology, V. V. Nagar, India in 2010.

Currently he is a Ph. D. candidate at the Department of Mechanical and Industrial Engineering, Indian Institute of Technology, Roorkee, India

P M Pathak received the B. Tech. degree in 1988 from NIT, Calicut, India and M. Tech. degree in 1998 from IIT, Kanpur, India both in Mechanical Engineering. He received Ph. D. degree in 2005 from IIT, Kharagpur, India.

Currently he is an Assistant Professor in the Department of Mechanical and Industrial Engineering, Indian Institute of Technology, Roorkee, India. His areas of interest are space robotics, walking robots, dynamics and control.

A K Samantaray received the Ph.D degrees in Mechanical Engineering, from Indian Institute of Technology, Kharagpur.

Currently he is an Associate Professor in Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, India. His areas of interest are Systems and Control, Nonlinear Mechanics, Vehicle system dynamics and Rotor dynamics.

Jung-Min Yang received the B.S., M.S., and Ph.D. degrees in Electrical Engineering from Korea Advanced Institute of Science & Technology (KAIST), Korea, in 1993, 1995, and 1999, respectively.

Currently he is an Associate Professor in Electrical Engineering Department of Catholic University of Daegu, Korea. His research interests are in gait study of walking machines and control of asynchronous sequential machines.

Seong Woo Kwak received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Korea Advanced Institute of Science & Technology (KASIT), Korea, in 1993, 1995 and 2000, respectively.

Currently he is an Associate Professor in Department of Electrical Engineering, Keimyung University, Korea. His research interests are in control of asynchronous sequential machines, space-borne digital electronics, and fault-tolerance in real time systems.