A NEW BOND GRAPH MODEL OF PHOTOVOLTAIC CELLS

S. Astier, G. Fontès, X. Roboam, C. Turpin, F. Gailly, L. Ménard

University of Toulouse; INPT, UPS; LAPLACE (Laboratoiry of Plasma and Conversion of Energy);
ENSEEIHT, 2 Rue Camichel, BP7122, 31071 Toulouse Cedex 7, FRANCE

Stephan.astier@laplace.univ-tlse.fr,

ABSTRACT

This paper presents the building of a Bond Graph model of photovoltaic cells based on PN junction by modeling directly the various phenomena involved in the photovoltaic conversion from incident sunlight to final electricity. The basics of photovoltaic conversion and the classical equivalent circuit modeling are presented. Then a new original Bond Graph model of a photovoltaic cell is elaborated. It is built with regards to the physical energy structure of the light and to the energy structure of the PN junction used for the conversion device i.e. the photovoltaic cell. A particular attention is paid to causality problem, as the light appears as a power source of energy which implies very particular constraints explained and leads to a model using bi-causality. Finally, it is shown how this model can be reduced to the classical equivalent circuit model.

Keywords: photovoltaic, modeling, Bond Graph, bi-causality

1. INTRODUCTION

Electricity generated from solar photovoltaics (PV), even if it is so far negligible with regards to the global electric power, with about 40 GW of peak power installed in the world at the end 2010, is characterised by a very significant growth of about 30% by year for the last ten years and more than 100% during the few last years (PV status report 2010, Eurobserv'er 2011). Indeed, in the context of sustainable development, this electricity generation directly from the solar radiation, the most abundant renewable source of energy, is expected to become one of the major sources of electricity in the middle of XXIst century (WWF energy report). Photovoltaic generators began to be used 50 years ago in space and 30 years ago on ground applications after the first oil crisis. Now well known, but not yet as largely developed as other types of electricity sources, the PV field is still strongly evolving with numerous researches and technological developments for new materials and new systems as well for small power remote applications as for grid-connected generators and for large power plants up to tens of MW.

The studies of electric systems are traditionally based on equivalent circuit models or their derivatives. Faithful equivalent electrical circuits of PV cells or arrays are well known. But other types of modeling are exploited, such as Bond Graph (BG), because they are better suited to studies of heterogeneous energy systems within different disciplinary fields (Karnopp 1991). Particularly, the use of Bond Graph representation has proved very fruitful in many domains as for example for the study of systems including PV and/or electrochemical components (Andoulsi 1999a and 1999b, Astier 2004, Saïsset 2006, Ménard 2010). However, this use relied on a direct translation of the electric circuits models of PV generators, not on modeling the PV conversion directly in Bond Graph.

In this paper, considering a solar PV cell classically based on a PN junction, we present the building of a Bond Graph model by modeling directly the various phenomena involved in the photovoltaic conversion from sunlight to electricity. First we recall some basics of photovoltaic conversion and the classical equivalent circuit model. Then we describe the construction of an original Bond Graph model of photovoltaic cell based on PN junction. We pay particular attention to causality problem and analyze it especially in comparison to those of other power converters as electrochemical ones. It is particularly shown that a bi-causal modeling is required. Finally, it is shown how this model can be reduced to the well-known equivalent circuit model.

2. PHOTOVOLTAIC CONVERSION AND CIRCUIT MODEL

2.1. Solar radiation and light energy carrying

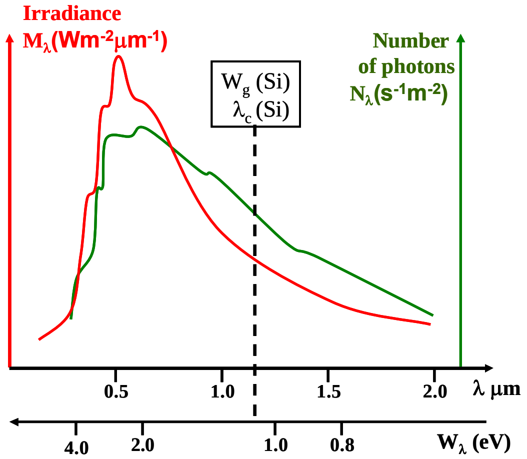

Figure 1: Solar radiation spectrum in space and associated flux of photons.

Solar light in the vacuum of space has a spectrum of electromagnetic radiation of wavelengths ranging continuously from 0.2 µm to 4 µm as shown on Fig. 1. This spectrum is very near from the one of the radiation emitted by a black body at 5800 K, the irradiance following the theoretical Planck law.

From the energy point of view, this electromagnetic radiation consists in a population of photons (Fig.1), each one carrying the energy w_λ associated with each radiation of wavelength λ and given by (1) where h is the Planck constant and c the speed of light in vacuum.

$$w_\lambda = h\frac{c}{\lambda} \qquad (1)$$

The surface density of the power carried by an electromagnetic radiation, called spectral irradiance M_λ [$Wm^{-2}\mu m^{-1}$] is therefore as well a surface density of photon flux (Fig. 1) N_λ [$s^{-1}m^{-2}$] given by (2) where $d\lambda$ represents a slice of spectral wave length of a given width.

$$N_\lambda = M_\lambda \frac{\lambda}{hc} d\lambda \qquad (2)$$

At the ends of the visible spectrum:

- for $\lambda = 0.40$ µm (UV); $w_\lambda = 3.10$ eV; N_λ varies between 1 and 3.10^{17} photons.$cm^{-2}s^{-1}$
- for $\lambda = 0.78$ µm (IR); $w_\lambda = 1.59$ eV; N_λ varies between 3 et 5.10^{17} photons.cm^{-2} s^{-1}

2.2. Photovoltaic conversion and materials

Generating electricity from electromagnetic radiation follows conditions already set by Einstein:

1. the photons must be absorbed by the material (optical absorption) by transmitting their energy to charge carriers of material (electrons);
2. the energy acquired by the excited carriers must be a potential energy recoverable as electrical work (voltaic) but not a thermal energy (kinetic energy);
3. the excited carriers (electrons) must be collected in the external circuit before returning to their initial energy level by recombination.

These conditions lead to define the criteria to realise an ideal photon-electron converter: a semiconductor material and a strong electric field at the location of excited electron-hole pairs. These conditions are particularly satisfied by a PV cell made from a PN junction structure with a semiconductor material. The Fig. 2 shows two classical structures of PV cells using either crystalline silicon or amorphous thin film silicon.

On the one hand the organisation of energy levels of electrons as a band structure with a w_g band gap in the semiconductor allows the photovoltaic conversion: an electron from the valence band jumps to the conduction band gaining the potential energy w_g.

On the other hand, the strong internal electric field, linked to the space charge around the PN interface, enables to collect the excited free carriers from within the material to the external electrical circuit connected to the front and rear faces of the cell. This function particularly appears on p-i-n structures of thin film amorphous silicon cells (Fig. 2).

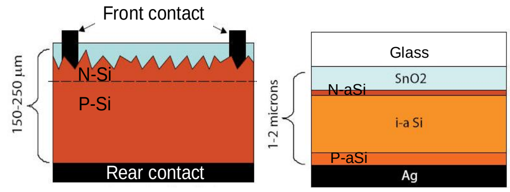

Figure 2: Structures of PV cells based on PN junction: crystalline and amorphous thin film silicon

According to their individual energy w_λ, photons are either reflected or absorbed or transmitted. Only the photons with an energy w_λ greater than the band gap w_g of the material are useful to the generation of photocurrent and to the photovoltaic effect. Thence a first basic necessary condition for the photovoltaic conversion to be realised is given by (3):

$$w_g \leq w_\lambda = h\frac{c}{\lambda} \qquad (3)$$

The energy of the photon is only partially absorbed in breaking a valence bond, which creates, thus, an electron-hole pair capable of mobility. The energy in excess is rapidly transferred to the lattice as heat (phonons). The electron and hole must be quickly released and collected to then participate in electrical conduction before their recombination. The pairs that reach the area of space charge are separated by the junction field and then collected. An absorption efficiency η_a can thus be defined: for crystalline silicon η_a is about 0.4.

In this complex process, we can consider a simple "two levels model of the photovoltaic conversion" so that:

- a photon with an energy w_λ less than the band gap energy w_g is not absorbed and passes through the material without transmitting any energy;
- a photon with an energy w_λ greater than the band gap energy w_g is completely absorbed and creates an excited electron-hole pair;
- excited electrons only acquire the potential energy equal to w_g. Some of them are involved in recombinations, the other ones participate to the generated photocurrent. However, for the following, the hypothesis of no recombination can be considered in order to simplify.

It results of these phenomena that the band gap energy w_g of the semi-conductor material is a major factor of the photovoltaic conversion.

On the one hand the band gap energy w_g fixes the only part of the spectrum that can be converted and shown on Fig.1, i.e. the radiations of shorter wave length less than λ_c (4):

$$\lambda \le \lambda_c = \frac{hc}{E_g} \tag{4}$$

Then, considering the actual solar spectrum on earth, at ground level, the ideal values of bandgap energy to do an efficient conversion with a “two level system” should be comprised between 0.9 and 1.5 eV. This is why, the most used material and technologies are, for example:

- Silicon with w_g (Si) = 1,11 eV
- Gallium Arsenide: w_g (AsGa) =1,35 eV
- Cadmium Telluride: w_g (CdTe) =1,45 eV

On the other hand, the band gap energy w_g also fixes the potential energy gained by the free photo-excited electrons and consequently the maximum output voltage V_g of the theoretical unit cell given by (5) where e ($e=1.6\ 10^{-19}$ C) is the absolute value of electric charge of electron:

$$V_g = \frac{w_g}{e} \tag{5}$$

As a consequence, any elementary photovoltaic converter, called PV cell, is a low voltage generator. As an example: V_g=1.1 V for silicon cells.

In addition, the energy of radiation being quantified, the number of photons for each wavelength is fixed by the irradiation M_λ and by the spectrum (Planck’s law, Fig.1). Consequently, this determines the number of photo-excited electrons and thus the photo-generated current I_{ph}.

In this process, such a theoretical photovoltaic converter appears to behave as a power source P_{ph} (6), imposing simultaneously and separately the voltage V_g AND the current I_{ph} delivered by the voltage source V_g.

$$P_{ph} = V_g I_{ph} \tag{6}$$

Such a lock is physically incompatible with the connexion to an actual electric circuit of given impedance, except if it can satisfy exactly the relation (6), i.e. with an equivalent resistance R_{eq} such that:

$$V_g = R_{eq} I_{ph} \tag{7}$$

Fortunately, this lock is broken thanks to the different losses that make the real semi-conductor converters imperfect, particularly with the diode effect linked to the PN junction, different voltage drops linked to conductions and different current leakages.

The well-known simplest equivalent circuit of the ideal PN junction PV cell, given on Fig.3, is deducted from the preceding physical considerations: it includes the current source I_{ph}, which models the photoelectric current, coupled with a diode in parallel which models the diode effect linked to PN junction.

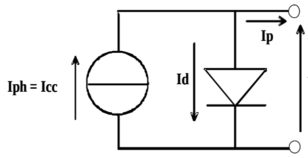

Figure 3: Equivalent circuit of an ideal PV cell based on PN junction.

Let’s notice that the current source is well related to the quantified nature of light which behaves as an imposed flow of photons. Many other power converters models involve a voltage source of energy such as for electromechanical or electrochemical ones as recalled in the analysis made further in this paper.

In practice, several factors reduce the efficiency of the PV conversion:

- the reflection of radiations on the cell surface;
- electron-hole recombination which reduces the output current (collecting efficiency);
- the actual output voltage V_p is lower than the theoretical voltage V_g
- the voltage drops at contacts and current leakage at the edges of the junction.

All these additional losses of energy, linked to voltage drops and leakage of current can be globally taken into account by adding to the ideal model of Fig. 3 the two resistors R_s and R_{sh} as indicated on Fig. 4.

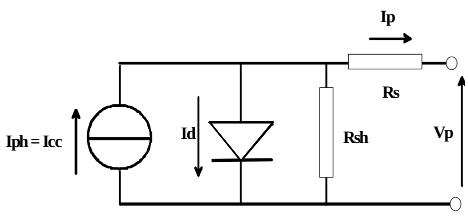

Figure 4: Equivalent circuit of a real PV cell based on PN junction with additional losses.

The equations of this model Fig.4 are (8), (9), (10):

$$I_d = I_s\left(\exp\frac{V_d}{n_D V_T} - 1\right) \tag{8}$$

$$I_P = I_{sc} - I_d - \frac{V_d}{R_{sh}} \tag{9}$$

$$V_P = V_d - Rs\ I_p \tag{10}$$

The values of the parameters of this equivalent circuit depend on the actual cell and determine its performance. For good quality cells, R_{sh} is more than 10 kΩ and R_s is less than 1 Ω.

A more accurate model can use two diodes connected in parallel, which better represents the mechanism of recombination of minority carriers near the middle of the band gap in the area of the space charge. Another simple way to improve the Fig.4 model is to introduce a diode factor n_D in the single diode model as done in (8). The diode factor is slightly greater than 1. Choosing n_D=1 neglects the recombination zone in the space charge.

A dynamic model can be obtained by putting a C_D capacitor representing the electric stored space charge associated to the PN junction across the diode on Fig. 4.

This static single diode model is very widely used in studies of PV systems with the well known shape of associated characteristics as indicated on Fig. 5.

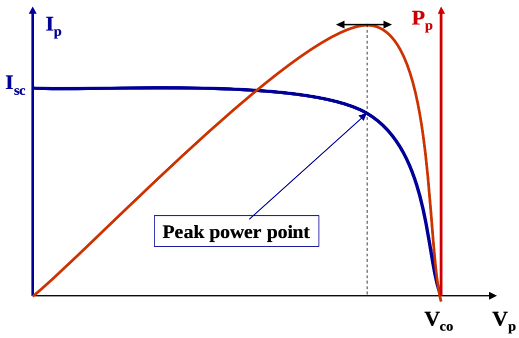

Figure 5: Typical electric characteristic of a PV cell based on PN junction.

The short circuit current I_{sc} is proportional to the power of the light (total irradiance). For a silicon cell, the open circuit voltage V_{oc} is about 0.6 V at 298 K. It varies with the temperature at a rate of about -0.4 % K^{-1}. Thence the basic PV cell is a DC current generator with a very low voltage of about 0.5V at peak power. Higher voltages can be obtained first by connecting a large number of PV cells in series as in commercialized PV modules and PV arrays, and second by means of well chosen static converter as a boost chopper for example.

3. BOND GRAPH MODELS OF PV CELLS

3.1. Bond graph model deducted from equivalent circuit model

A bond graph model can be easily deducted directly from the equivalent circuit given on Fig. 4.

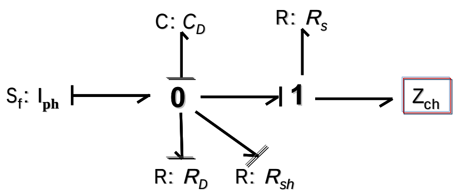

Figure 6: Bond Graph model of a PV cell deducted from the equivalent circuit of Fig. 4 (C_D is added).

The diode is represented by the nonlinear *R: R_D* element governed by the laws of electrical and thermal behavior of the PN junction. The equivalent capacitor of the junction is added and represented by element C: C_D which determines the causality on the zero junction. This type of model has been used efficiently in many studies on photovoltaic system with Bond Graph (Astier 2004, Andoulsi 1999a and 1999b).

The structure of this dynamic model can be simplified with no fundamental change of the behaviour by involving R_{sh} within R_D and R_s within the load Z_{ch}.

3.2. Bond graph model for monochromatic photovoltaic conversion (without causality)

Now, in order to obtain another model of photovoltaic conversion directly in Bond Graph, let's consider directly the conversion of the radiant energy.

At first let's consider a monochromatic radiation of wavelength λ carrying a power density P_λ. Each photon carries the energy w_λ given by (1). Thus the photons flux density N_λ is given by (11).

$$P_\lambda = N_\lambda \, w_\lambda \tag{11}$$

In order to use a normalized measure of collections of particles, we can consider molar flows densities (by m^2). Then the molar flow density of photons ξ_λ is given by (12) where N_A is the Avogadro number (N_A = 6.023 10^{23}).

$$N_\lambda = \xi_\lambda N_A \tag{12}$$

The energy W_λ of one mole of photons is given by (13) and the power density of the radiation P_λ by (14).

$$W_\lambda = N_A w_\lambda \tag{13}$$

$$P_\lambda = N_\lambda w_\lambda = \xi_\lambda W_\lambda \tag{14}$$

In the electric field, the photo-created current I_λ is therefore given by (15) where F = 96500 C is the Faraday i.e. the electric charge of one mole of electrons.

$$I_\lambda = k_{pe} N_\lambda e = k_{pe} \xi_\lambda F \tag{15}$$

k_{pe} < 1 takes into account the conversion rate and recombination before collecting the excited electrons. But in order to simplify it will be considered k_{pe} = 1 for the following which doesn't change the model.

Now, in a first step, if we consider the total transfer of the energy of each photon to each excited electron, the power conservation implies a theoretical voltage V_λ in the electric field depending only on λ by (13).

$$V_\lambda = \frac{P_\lambda}{I_\lambda} = \frac{\xi_\lambda W_\lambda}{I_\lambda} = \frac{W_\lambda}{F} = \frac{w_\lambda}{e} = \frac{hc}{\lambda e} \tag{16}$$

Then, with (15) and (16) it is easy to give the Bond Graph model of this theoretical ideal photovoltaic conversion from light field to electric field as on Fig. 7 with a TF element whose ratio is *F*.

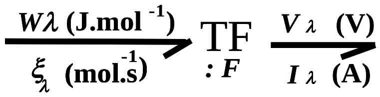

Figure 7: Basic Bond Graph model of an ideal monochromatic photovoltaic conversion.

In the light field the molar flux of photons ξ_λ is a Bond Graph flow while the energy W_λ is the effort. At this step it is independent from the actual semi-conductor material used by the conversion device.

3.3. About causality of PV model

Such a model of Fig.7 looks very similar to the one of the electrochemical conversion of power (Saisset, Ménard) which is recalled on Fig. 8. In this model ξ is the molar flow of the chemical reaction and ΔG the variation of the molar Gibbs free energy. Considering the causality, the molar flow ξ (i.e. the reactant consumption) is a direct consequence of the current *I* absorbed by the electric circuit. Thence the ξ flow is imposed by the electric side while the effort ΔG is imposed by the chemical side (molar free energy), thus imposing the voltage *E* in the electric field.

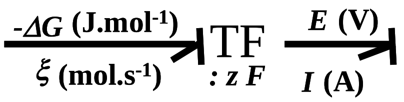

Figure 8: Basic Bond Graph model of an ideal electrochemical conversion.

Differently, the model presented on Fig. 7 does not enable to fix any causality at this step. Indeed the radiation appears as an input power source imposing simultaneously the current I_λ and voltage V_λ. This is the result of the microscopic quantification directly reflected at the macroscopic level by the mechanism of photon-electron conversion. Such a property will require a bi-causal formalism to be rightly represented. But, before, the material behavior also implied in the actual PV conversion has to be introduced.

3.4. Bi-causal BG model of PV conversion

The band energy structure of electrons in the semi conductor imposes to the excited electron the w_g energy instead of w_λ, so that the energy drop (w_λ - w_g) has to be represented. As all the energy w_λ is first transferred to electric charges, the exeeding power appears as a kinetic power P_k acquired by electric charges and lost by interaction with the lattice becoming heat. Thence, this energy dissipation can be expressed in the electric field as a voltage drop from V_λ down to V_g, as in (17).

$$P_k = W_\lambda - W_g = (V_\lambda - V_g).I_\lambda \qquad (17)$$

We have to represent this phenomenon both with the behavior of the light as a power source. The bi-causal model on Fig. 9 proposes a solution. In this model both W_λ and ξ_λ are imposed by the light on one side, which is represented by the bicausal source S_eS_f in the light field. On the other side, in the electric field, the potential energy in the semi-conductor is imposed equal to W_g, i.e. an electric potential Vg, which is imposed by the source of effort Se : Vg. The inserted bicausal element D_eD_f has the right constitutive law in order to dissipate precisely the power $P_k = (V_\lambda - V_g)I_\lambda$.

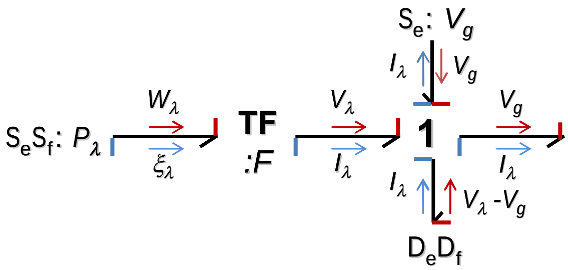

Figure 9: Bond graph model of a monochromatic of PV conversion using a material of V_g bandgap (part1).

At this step, just as before with the circuit modeling approach, the only possible working point in the electric domain is : (V_g, I_λ). The output of this model appears as current source I_λ flowing out from the voltage source V_g. So, all the model of Fig. 9 can be replaced by the equivalent bi-causal source: S_eS_f : V_g I_λ

Now, just as before, in order to get a more realistic model representing the actual process, it is necessary to introduce the diode effect tightly coupled to the photovoltaic conversion process for collecting excited carriers. Classically, the diode effect involves both a dissipative phenomenon and a capacitive one due to the space charge stored at PN interface. A solution is proposed on Fig. 10. The element R_D has the static constitutive law of a diode. The transition capacitor of the diode fixes the output causality by imposing the voltage V_c. Both with R_D and the electric load Z_{ch}, this constitutes the causal part of the model which has to be connected to the part 1 of Fig. 9, globally replaced on Fig. 10 by the bi-causal source S_eS_f: V_gI_λ.

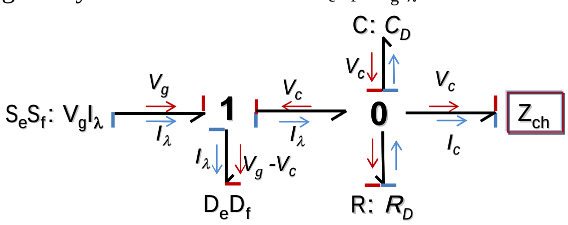

Figure 10: Bond graph model of a monochromatic of PV conversion using a material of V_g bandgap (part2).

On Fig.10, the inserted bi-causal element D_eD_f insures the transition between the bi-causal part and the causal part of the model representing a part of losses causing a voltage drop from V_g to V_c.

Indeed, even in open circuit the open circuit voltage (V_c= V_{oc}) is less than V_g. This because a potential barrier equal to the band gap V_g is not realistic in practice, corresponding to the extreme case of doping (p^+n^+) which would imply a too short lifetime of excited charges to be collected before recombination by tunnel effect. So another doping configuration is chosen in practice for PV cells, such as (pn^+), with the consequence of a voltage drop of the potential barrier down to a voltage V_{oc} in open circuit less than the theoretical V_g. More losses resulting in voltage drops and leakage currents can be modeled either with two more dissipative elements, R_s and R_{sh} as on Fig. 6 and not represented here or involved within R_D and Z_{ch}.

Finally, models of Fig. 9 and Fig. 10 can be associated together (part 1 + part 2) so as to constitute the Bond Graph dynamic model of a monochromatic PV conversion. This model satisfies the aim of the study, representing the different phenomena involved in the photovoltaic conversion from the light field to the electric field. But, at this step, it is only valid for monochromatic lights, which is not the case of the sunlight.

3.5. Bond Graph model of photovoltaic conversion with white sunlight

Now, with the white sunlight, a continuous multi-chromatic spectrum (Fig. 1) must be converted with a unique semi-conductor material of w_g band gap energy. In this case, the different values of energy w_λ of every convertible photon are all together downed to w_g when transferred to electrons, the surplus being kinetic energy dissipated as heat in the lattice as already seen.

Therefore as many legs as wavelengths have to be combined to give the global photo-current I_{ph} which is a discrete or a continuous summation of the various I_λ (considering slices $d\lambda$ around λ). And considering the different dissipative phenomena, the common voltage of photo-generated electrons is V_p. Then the model of the electric field is not changed and the Fig. 11 shows a model for a light spectrum with a discrete number n of monochromatic radiations. With a continuous spectrum, the number of legs would be infinite.

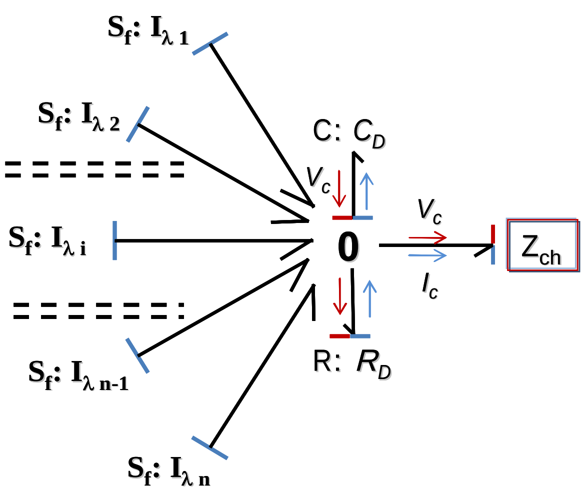

Figure 11: Bond graph model of a poly-chromatic photovoltaic conversion using a PN junction.

Now, the proposed model passing from the light field to the electric field represents the different phenomena involved in the photovoltaic conversion. But, it clearly appears complex, this complexity being related to the continuous spectrum of the white light. In order to avoid this great or infinite number of legs, inddeed not useful in practice, the part of the model at the left side of junction 0 can be replaced by a global flow source S_f: I_{ph} (current source). This operation leads us to the model of Fig. 6 which was directly deduced from the electric circuit model of Fig. 3 ... of course!

4. CONCLUSION

In this paper a new Bond Graph model of a photovoltaic cell based on PN junction, involving both light field and electric field, has been constructed by modeling directly the various phenomena involved in the photovoltaic conversion from sunlight to electricity, doing this instead of just translating the classical equivalent circuit in Bond Graph. If it is truly interesting considering physics, the continuous spectrum of white solar light leads to a model with an infinite number of legs, one for each slice of wavelength! Regrouping all the generated photocurrent in one current source I_{ph} finally leads to the well known model in the electric field. However, this approach showed that the model of PV conversion requires a bi-causal representation in the light field: this is very original among all well known models of power converters. Moreover, such an approach could be interesting for example to model the multi-junction cells (up to six junctions) which are developed for multispectral non linear responses under concentrated light with the aim to reach high efficiencies over 40%.

REFERENCES

Andoulsi, R., Mami, A., Dauphin-Tanguy, G., Annabi, M., 1999. Bond graph modelling of a photovoltaic generator coupled to a dc motor. Proceeding of ICBGM'99, pp 371-378.

Andoulsi, R., Mami, A., Dauphin-Tanguy, G., Annabi, M., 1999. Modelling and simulation by bond graph technique of a DC motor fed from a photovoltaic source via MPPT boost converter. Proceeding of CSSC'99, pp4181-4187.

Astier, S., Saïsset, R., Roboam, X. 2004. Modelling and study of a solar car with embedded photovoltaic array and Li-ion storage Bond. I3M-IMAACA Gènes, October 28-30, 2004.

Saïsset, R., Fontès, G., Turpin, C., Astier, S. 2006, Bond – graph model of PEM fuel cell, Journal of Power Sources 156, pp 100 - 107.

Karnopp, D., Rosenberg, R., 1991. Systems dynamics: a unified approach. John Wiley and Sons.

Ménard, L., Fontès, G., Astier, S., 2010. Dynamic energy model of a lithium-ion battery. Mathematics and computer in simulation, vol 81, issue 2, pp 327-339.

Observ'er 2011 : on website

PV status report 2010, Joint Research Center of European Commission. : on website

WWF, 2010, The energy report : on website.