Autofocusing Slow-moving Objects Sensed by an Airborne Synthetic Aperture Radar *SAR*

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Abstract– During the processing of synthetic aperture radar (SAR) images, unknown-moving objects cause phase modulations in their phase histories. Depending on the motion, such modulations cause undesirable image distortions such as blurring and object-displacement artifacts. This paper discusses the estimation of time-frequency representations of non-stationary signals, by means of autoregressive moving-average (ARMA) model with time-dependent coefficients. This estimate will determine the object's motion law and consequently the possibility of reconstructing and plotting the true trajectory.

Keywords– Syntethic aperture radar, Time-varying auto-regressive moving average models, Kalman filters, Phase estimation.

1. Introduction

In this section we describe some basics of the *SAR* principle. As shown in Figure 1, a pulsed airborne radar moves in the indicated flight path, transmitting pulses and receiving echoes reflected from the scene. Suppose the cycle starts at point A where the radar transmits a pulse, then others at x_2, x_3,...,x_{n-1} and so on until reaching point B in x_n. What defines the interval size between A and B is the first and last contact of beam with the object (indicated in the figure as range-A and range-B). During that period the object is being iluminated by the radar lobe, receiving x_n pulses. This distance is what is called synthetic aperture. Moreover, the returns of each pulse which contain range information (fast-time), are stored by columns in the *SAR* data matrix. The cells of each column is called range-increments, where his size is defined by the pulse-width. Thus, in cell C_{11} is stored the farther return in distance and the nearest in cell C_{m1}. The same applies to the transmitted pulse in x_2, but their returns are now stored in column 2 (cells C_{12} to C_{m2}).

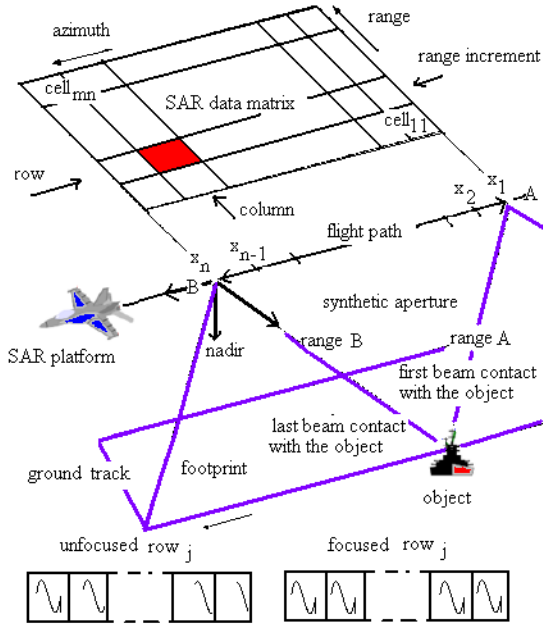

Figure 1: SAR Geometry

This is repeated until the column n (cells C_{1n} to C_{mn}) is reached. Also, the typical assumption is that the platform moves with a constant and known speed and the escene objects is static, which ultimately implies the existence of a Doppler effect. Row cells (slow-time) store "versions" of the returns but shifted in phase (phase history), as shown for the case of row_j. Optimized algorithms focus the distributed energy of the raw data for all scene locations simultaneously. However, if marine or ground objects have an unknown motion they will cause a unknown phase modulation in their phase histories, precisely because of the unknown relative motion between sensor and object (section 2.). In turn, these modulations cause *SAR* image distortions such as blurring and smearing (Sparr and Krane 2003).

In order to develop this work (Figure 2), we consider: 1) the time serie (TS), comprised of the sequencing radar returns, is modeled by a time-varying autoregressive moving average (TV-$ARMA$) model, 2) the $TV-ARMA$ coefficients are updated by a Kalman filter (KF), 3) using the $ARMA$ coefficients updated at each iteration we obtain the instantaneous estimated power spectral density (PSD), 4) the maximum value of PSD provides us with the instantaneous frequency IF, 5) finally, the last objective is to analyze and to compare the performance of a phase $ARMA-KF$ estimator, operating in a non-stationary simulated environment which has a moving object, using as framework the WignerVille distribution WVD.

2. SAR simulated scenario whith a dynamic object

Here we define the scenario that contains an object with rotational movement around a fixed point as shown in Figure 3, which considers the projection of this movement over the line of sight (LOS).

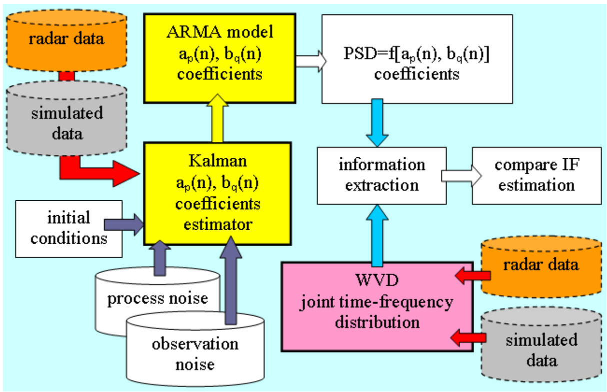

Figure 2: $ARMA-KF$ and WVD processes block diagram

Let the freedom to choose the location of the reference system so that: 1) the radar platform is placed at $[0, 0, h]$, 2) the object has a initial coordinates $[x_0, y_0, 0]$ and a absolute position $[x(n), y(n), 0)]$, determined by $\mathbf{R}(0)$ plus the phasor $\mathbf{r}$ (whose initial phase is φ_0), 3) The relative coordinates are the projections of $\mathbf{r}$ on the x and y axis, i.e. with a oscillatory variation proportional to $\sin(n\omega_m)$ and $\cos(n\omega_m)$ respectively. The term ω_m represent the object rotational frequency. The magnitude of range $\mathbf{R}(n)$ measured from the radar to the object is

$$|R(n)| = \sqrt{(x(n) - v_{pl}n)^2 + h^2 + y(n)^2} \quad (1)$$

The Figure 4 shows the successive positions of the platform. As we have stated, it is necessary to apply a phase correction (applying a rotation to the appropriate phasor) to each return received. The error and therefore the correction phase, is directly proportional to d_j^2, the square of the j-th instantaneous position relative to the center of the aperture. But more important here is that it is inversely proportional to $|\mathbf{R}(n)|^2$. Namely, the correction is (Stimson 1998)

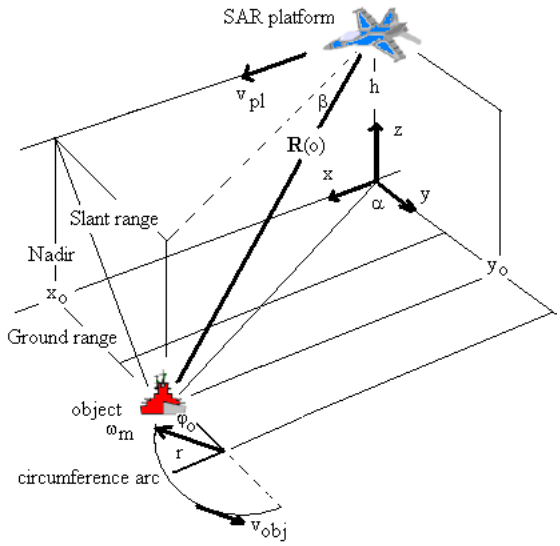

Figure 3: Scenario and coordinate system

$$\varphi_j \approx \frac{2\pi}{\lambda|\mathbf{R}(n)|^2} d_j^2 \quad (2)$$

It is obvious that lack of knowledge of true coordinates $x(n), y(n)$ of the object, preventing an effectively phase correction. Let's see in the next section how this scenario affects the signal model.

3. Physical signal model

As we will see in the section 4. this scenario allows us to form a TS $\{x(n)\}$ of the trajectory. Then, each transmitted linear frequency modulated LFM pulse is expressed by the complex exponential $s_t(\tau) = g(\tau)e^{j2\pi(\varphi_0+f_0\tau+\alpha\tau^2)}$, where τ is the pulse round-trip time, $g(\tau)$ is a square pulse of $T_w[\mu sec]$ wide, α the slope of the modulating frequency, φ_0 an arbitrary initial phase, f_0 the carrier frequency, so $(\varphi_0+f_0\tau+\alpha\tau^2)$ is the phase $\varphi(\tau)$ [rad]. The signal $s_t(\tau)$ is valid in the interval $[-T_w/2, T_w/2]$. Therefore, the received echo from an object at the distance R will be

$$s_t(\tau) = g(\tau)e^{j2\pi(\varphi_0+f_0(\tau-2R/c)+\alpha(\tau-2R/c)^2)} \quad (3)$$

In this work we are only interested in the resulting signal at the end of the reception chain, i.e. after pulse compression processing. As the SAR platform moves along a hypothetical circumference arc flight track, the radar emits and receives pulses. Considering one object of interest, the difference between pulses will be determined by the attenuation of the signal from the path loss and the antenna pattern as well as by the change in distance to the object. Based on this, the pulse compressed signal before SAR processing is

$$s_{pc}(n,\tau) = T_w(1 - \frac{|2\mathbf{R}(n)/c - \tau|}{T_w})e^{j2\pi f_0(2\mathbf{R}(n)/c-\tau)}$$

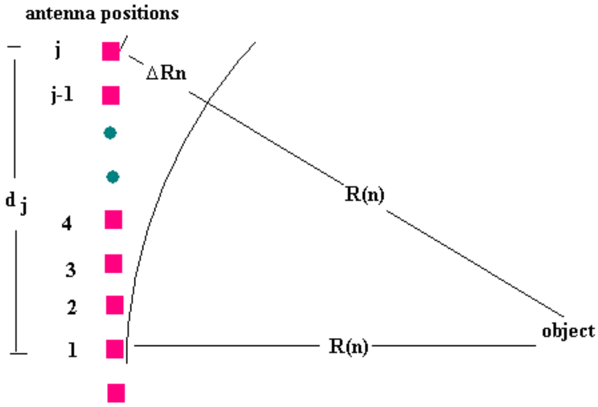

Figure 4: Focused Array

$$\left[sinc(2\pi\alpha T_w(2\mathbf{R}(n)/c-\tau)(1-\frac{|2\mathbf{R}(n)/c-\tau|}{T_w})\right] \quad (4)$$

function of n, and τ and valid in the interval $[2|\mathbf{R}(n)|/c - T_w, 2|\mathbf{R}(n)|/c + T_w]$. Using the expression above, renamed as $x(n)$ we generate a set $s_{pc}(n,\tau) \quad n = 0,\ldots,N-1$, where each element is a complex-value signal per pulse. N is the number of pulses emitted during the aperture time. We will use that set to test the proposed $ARMA-KF$ methods and reference WVD time-frequency representation.

4. Building a mathematical model for TS

Now we will develop a mathematical model of the received SAR signal

$$x(n) = -\sum_{k=1}^{p} a_k(n)x(n-k) + \sum_{k=0}^{q} b_k(n)w(n-k) + n(n) \quad (5)$$

where $x(n)$ is considered as one that describes the system and the measurement process, with coefficients $\{a_k, b_k\}$ and $\mathbf{w}(n)$ the clutter noise input driving sequence, independent w.r.t. past values $x(n-k)$. The values of p and q are the order of autoregressive $ARMA$ model. Now, if we consider that the TS samples has varying spectral properties, it is also natural to assume the coefficients to be time dependent. So, it is more realistic: 1) to model the a_k, b_k with a degree of randomness i.e. $a_k(n+1) = a_k(n) + v_a(n)$ and $b_k(n+1) = b_k(n) + v_b(n)$, where the zero-*th* order random processes $\mathbf{v}(n) = \{v_a(n), v_b(n)\}$ can be interpreted as the uncertainty of the prediction of the next coefficient value and 2) accept the measurements as being noisy. To denote this new condition, the $x(n)$ expression is modified by the adittion of the observation noise $\mathbf{n}(n)$, as independent of $AWGN$ coefficient noise $\mathbf{v}(n)$ and clutter $\mathbf{w}(n)$ processes.

5. TS expressed as state-space model (SSM)

As we saw, $\{x(n)\}$ is an *uni*-variate TS represented as an $ARMA$ model with coefficient $a_k(n)$, $b_k(n)$. Now

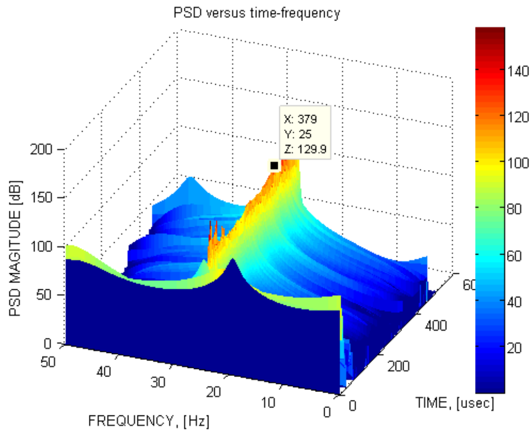

Figure 5: PSD versus time-frequency

we pose our problem of spectral estimation in SSM term. For this, let the $ARMA$ coefficients represent the state of the system

$$\mathbf{s}(n+1) = \mathbf{s}(n) + \mathbf{v}(n) \quad (6)$$

where $\mathbf{s}(n) = [a_1(n),\ldots,a_p(n), b_0(n),\ldots,b_q(n)]$ is the (m)-dimension (m=p+q+1), unobservable state vector and $\mathbf{v}^T(n) = [v_1(n),\ldots,v_{p+q}(n)]$ is the $(m-1)$-dimension noise vector, that as mentioned above, models the unknown statistics of the coefficients. So, get the state vector $\mathbf{s}(n)$ is to get the TV $ARMA$ coefficients. On the other hand, if we define the (m)-dimension vector of returns and noise process as $\mathbf{C}(n) = \{-x(n-1),\ldots,-x(n-p), w(n), w(n-1),\ldots,w(n-q)\}$, we can write the measurement equation as

$$x(n) = \mathbf{C}(n)\mathbf{s}(n) + n(n) \quad (7)$$

6. State estimation via the AKF

Comparing equations (6) and (7) with the conventional SSM model equations, we can conclude that, in the problem under study: 1) the transition matrix Φ is the identity matrix I, 2) the measurement matrix H is represented by $\mathbf{C}$ and 3) the estimate of state vector $\mathbf{s}(n)$ (or $ARMA$ coefficients $\{a_k(n), b_k(n)\}$), can be gotten from the observed data $\{x(n)\}$ using a KF (Harashima, Ferrari and Sankar 1996). To illustrate this "recast", let $(p,q) = 2$, $\mathbf{C}(n) = [-x(n-1), -x(n-2), w(n), w(n-1), w(n-2)]$ and $\mathbf{s}(n) = [a_1(n), a_2(n), b_0(n), b_1(n), b_2(n)]^T$, then we can express $x(n)$ in matrix form as

$$x(n) = \begin{vmatrix} -x(n-1) \\ -x(n-2) \\ w(n) \\ w(n-1) \\ w(n-2) \end{vmatrix}^T \begin{vmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{vmatrix} + n(n) \quad (8)$$

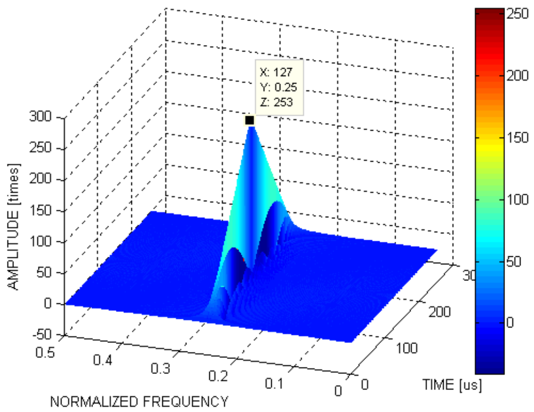

Figure 6: WVD versus time normalized-frequency

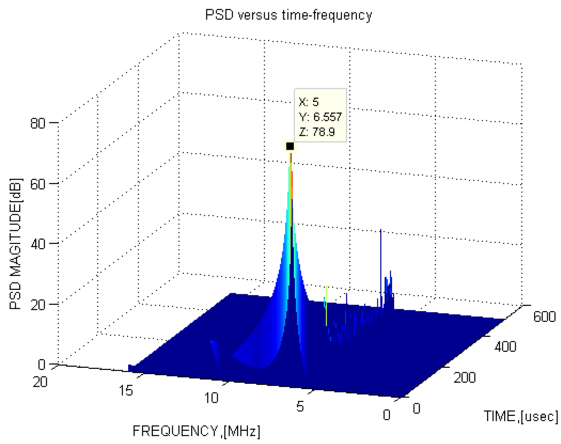

Figure 7: PSD of a signal received from a moving object

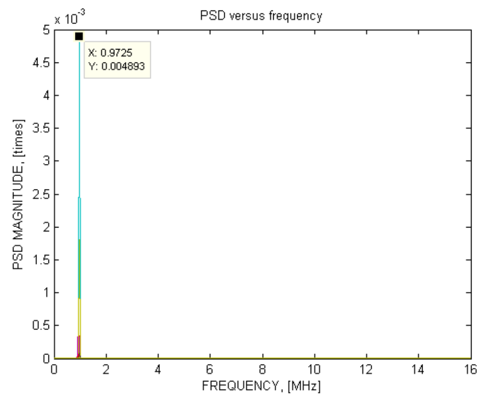

Figure 8: $ARMA - KF$ processes to radar return signal

Moreover, the state $\mathbf{s}(n)$ of the system and the gain matrix $\mathbf{K}(n)$ can be estimated recursively, with the proper interpretation of stochastic characteristics of both, the input driving signal $\{\mathbf{v}(n);{\sigma_v}^2\}$ and measurement noise $\{\mathbf{n}(n);{\sigma_n}^2\}$. To apply the AKF, we also assume that $a_k(0)$ and $b_k(0)$ are Gaussian random variables.

7. Initializing the augmented KF

In this section we define the set of parameters required to initialize the AKF algorithms, considering that the only information available is the noisy measure of range represented by the TS $\mathbf{x}(n)$ and covariances of the noises $\mathbf{R}(n)$ and $\mathbf{U}(n)$, as we will see later. Moreover, we will not repeat here the recursive set of equations that make up the AKF, except those related to the initialization process. We only recall that one way of expressing an estimated $\hat{\mathbf{s}}(n)$ of $\mathbf{s}(n)$ that is as close to it as possible in a mean-squared sense is $\hat{\mathbf{s}}(n) = \mathbf{I}\hat{\mathbf{s}}(n-1) + \mathbf{K}(n)[\mathbf{x}(n) - \mathbf{C}\mathbf{I}\hat{\mathbf{s}}(n-1)]$. We know that the recursive computation of the variable-gain matrix $\mathbf{K}(n)$ involves three matrix equations (Auger, Flandrin, Gonçalvès and Lemoine 2002)

$$\begin{aligned}
\mathbf{K}(n) &= \mathbf{J}(n|n-1)\mathbf{C}(n)^T[\mathbf{C}(n)\mathbf{J}(n|n-1)\mathbf{C}(n)^T + \\
&+ \mathbf{R}(n)]^{-1} \\
\mathbf{J}(n|n) &= \mathbf{J}(n|n-1) - \mathbf{K}(n)\mathbf{C}(n)\mathbf{J}(n|n-1) \\
\mathbf{J}_{n+1|n} &= \mathbf{I}\mathbf{J}(n|n)\mathbf{I}^T + \mathbf{U}(n)
\end{aligned}$$

where $\mathbf{J}(n|n) = \mathbf{E}[[\mathbf{s}(n) - \hat{\mathbf{s}}(n)][\mathbf{s}(n) - \hat{\mathbf{s}}(n)]^T]$ is just the covariance matrix of estimation errors. Note that in our time-varying case the matrices $\mathbf{J}(n|n)$ and $\mathbf{K}(n)$ can not be computed a priori because of the dependence of $\mathbf{C}(n)$ on the actual system measurements. Moreover, as the inspection of the Kalman gain reveals, we must consider that if the measurement noise is small, then $\mathbf{K}(n)$ will be large, i.e. a lot of credibility will be given to the measurement. In contrast, if the measurement noise is large, then $\mathbf{K}(n)$ will be small, i.e. very little credibility will be given to the measurement, in both cases when computing the next value of $x(n)$.

Now, the key issue is to assume values for the variances ${\sigma_n}^2$, ${\sigma_w}^2$ and ${\sigma_v}^2$. In the case of ${\sigma_n}^2$, it should be assigned on the basis of our knowledge of the noise introduced by measurement sensors.

8. Phase estimation

From the moment that we have the time-varying coefficients of the $ARMA$ model, is possible to obtain the unknown phase from the PSD, expressed as a function of that coefficient a_k, b_k. The ridge-peak of this function is the instantaneous frequency.

$$\hat{P}_x(e^{j\omega}) = \frac{|\sum_{k=0}^{q} \hat{b}_q(k)e^{-jk\omega}|^2}{|1 + \sum_{k=0}^{p} \hat{a}_p(k)e^{-jk\omega}|^2} \tag{9}$$

9. Comparison between the $ARMA - KF$ estimator and the WVD

In this section, we compare the performance of the $ARMA - KF$ estimator against WVD. To do this we consider the following design parameters. With respect to the platform: 1) The antenna has a physical inclination (look-angle) of $76[degrees]$ 2) we assume a pulsed linear frequency modulation (LFM) transmitted signal, of $T_w = 10[\mu s]$ wide and slope of the modulating frequency $\alpha = 40[MHz/s]$, 3) a pulse repetition frequency $PRF = 500[Hz]$, frequencying a duty cycle or interval between pulses IP of 1% or $\approx 2.0[msec]$ of maximum processing-time, 4) the transmiter has a carrier frequency $f_0 = 1.275[GHz]$ ($\lambda = 0.0235[m]$), and a intermediate frequency stage FI of $25[MHz]$, where the radar video is sampled at a frequency $f_s = 100[MHz]$, with a sample interval $\Delta t = 1/f_s = 10[ns]$ and time of the sample n defined by $tn = n\Delta t$, 5) the platform operates in stripmap mode, with a spatial and azimuth resolution of $6[m]$ and $1[m]$ respectively (depend on the size of the antenna), 6) besides that exposed, the platform has a speed of $100[m/s]$ and a height of $4000[m]$ results a ground-range value of $1800[m]$ (see x_0 in section 2.), 7) moreover, from the supposed speed of the object and also from an arbitrary value of $\mathbf{d}\sin(\theta) = 50[m]$, we then obtain a simulated signal with frequency $f_m = 0.5[Hz]$, 8) finally, the number of samples N depends on the range-cell resolution (RCR), which is proportional in turn to the compressed pulse wide. If the compression ratio is 20 then the $RCR = 75[m]$ or $500[ns]$. Since $f_s = 100[MHz]$, then an acceptable minimum is $N = 64$. With respect to the $ARMA$ and kalman filter: 1) we use $p = q = 2$, 2) for the variances ${\sigma_n}^2$, ${\sigma_w}^2$ and ${\sigma_v}^2$ we consider a reception chain $SNR \geq 10[dB]$ and measurement errors of 1%, then we can use ${\sigma_n}^2 = 1000$, 3) in the case of ${\sigma_w}^2$, after an assessment of the way the $\mathbf{w}(n)$, representing the clutter, is likely to vary (Anderson and Moore 1979). Again, if the signal to clutter ratio $SCR \geq 10[dB]$, with values of radar video signal on the order of $[mV]$, we consider then ${\sigma_w}^2 = 1$, 4) moreover, for ${\sigma_v}^2$, we consider a trial an error approach method, 5) finally, we need to assume a priori initial values for each $a_k(n)$, $b_k(n)$ of these coefficients, in that case, we assume complete ignorance of them, so $a_k(0)$, $b_k(0)$. We test three case studies, all of them related to the scenario of section 2.

CASE A– Scenary with a constant frequency signal. Is a signal with $f_m = 25[Hz]$, $SNR = 15[dB]$, with amplitude of $\mp 15[mV]$. This signal is processed by the $ARMA - KF$ algorithm proposed. Figure 5 shows a three-dimensional representation of the PSD in $[dB]$ versus time-frequency. The crest of the surface manifests a maximum value of PSD for $25[Hz]$. The WVD representation using a Matlab® Time-Frequency Toolbox (Auger, Flandrin, Gonçalvès and Lemoine 2002) is shown in Figure 6. At a sampling

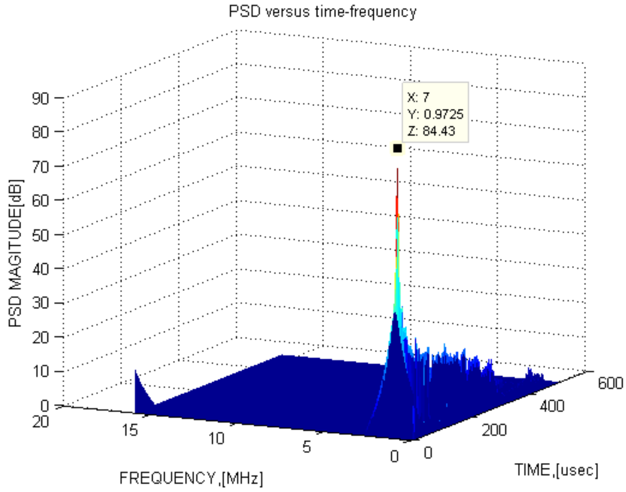

Figure 9: PSD of radar return signal

frequency $f_s = 100[MHz]$, the normalized frequency of 0.25 represents the $25[Hz]$ searched by estimation.

CASE B– Scenary with a object moving at constant speed. The example plots the PSD calculated using equation (9), received from a object moving at the speed $v = 10[m/s]$ ($36[km/h]$, as shown in Figure 7. The rotating frequency is $f_0 = 6.5[Hz]$. The object is moving along a straight line, which gets closer to the observer up to a distance $d = 5000[m]$ and then moves away.

CASE C– Scenary with a radar return signal.
The example uses 128 samples of the test radar return signal with doppler frequency $f_m = 0.9578[Hz]$. The estimated instantaneous frequency calculated by the $ARMA - KF$ procedure is represented as the maximum of PSD as shown in Figure 8. The Figure 9 shows a PSD time-frequency representation of the same signal.

10. Results, conclusions and future works

The study aims to estimate the phase caused by the unknown dynamics of an object moving relatively slowly with respect to the platform. To estimate this phase we use an $ARMA - KF$ model and as frame of reference a non-parametric time-frequency representation. The $ARMA - KF$ model is very good in resolution, but totally dependent on the model order, that is, a higher order $ARMA$ would show unnecessary peaks and a lower order may miss the peaks in the clutter spectrum. However, WVD has the problem of cross-spectral components and needs some insight in understanding the distribution, as can be seen clearly in the Figure 10, where we have applied the Hough transform on the WVD image to produce a representation with peaks, whose coordinates give estimates of the linear frequency modulation parameters. This is in contrast to the immediate interpretation of the PSD of the proposed method, Figure 9. The time progression of radar signal return in complex form, in the presence of additive measurement $AWGN$ and $AWGN$-clutter, was evaluated. This procedure utilizes the adaptive feature of the KF which is carried out recursively for each sample, and offers sufficient resolution in time-frequency domain, in spite of the higher computational severity compared to other parametric or non parametric methods. The first experimental results show that the resolution of the introduced method is equal or higher than other usual time-frequency techniques. The reliability of the procedure was tested using Matlab® programs on simulated data. Compared with standard methods, the $ARMA-KF$-based estimation responded most quickly to parameter changes. The usefulness of our approach in the analysis of Radar returns (SAR) was introduced by an example. Referring to this application, from a general point of view, the use of Kalman filters for the estimation procedure has some advantages compared to other approaches: Kalman filters can be constructed for multivariate systems with stochastic variation of the parameters and the properties of the resulting estimates can be described theoretically. Particularly, this is not necessary to search for a suitable set of base functions to model the temporal evolution of coefficients or to implement procedures for the detection of change points. Furthermore, the Kalman algorithm is appropriate for implementation on microcomputers due to its recursive structure that allows on-line processing, even of huge data sets.

The disadvantage of this procedure is the afore mentioned strong dependence on the order of $ARMA$ model and the establishment of the initial conditions of the variances ${\sigma_n}^2$, ${\sigma_w}^2$ and ${\sigma_v}^2$. It also depends on the initial covariance of the estimation error P and the initial values of the coefficients $a_k(0)$, $b_k(0)$. On the other hand, since the model of system represented by equations (6) and (7) is conditionally Gaussian with covariance $\mathbf{J}(n|n)$, the estimates can be of high variance, thereby a smoothing procedure for the estimates should be involved. Therefore, as future work, to maintain the adaptation speed a nonlinear recursive lowpass filter $\hat{a}_k$ and $\hat{b}_k$ should be used for each component a_k and b_k of the estimated coefficient vector $\mathbf{s}(n)$.

On the other hand, as well as tasks to develop in the future, we list: 1) compare the performace with an AR model in place of the WVD, 2) implement the reconstruction of trajectory as suggested in section 8.3) improvement the solution of the KF Ricatti equation trought the use of Cholesky factors (known as squared-root filter), 4) use other KF to identify the initial $ARMA-KF$ values, 5) also in the KF, the use of an alternative implementation of state vector, called information filter to improve the numerical stability, specially in cases of very large uncertainties of initial condition estimations, 6) design other types of trajectory dynamics such as the aforementioned rotational translation, 7) non-Gaussian clutter with varying SCR, 8) other model for the unknown statistics of the coefficients, 9) analysis and comparison of the computational complexity between $ARMA-KF$ and WVD, 10) extraction of information from a time-frequency image.

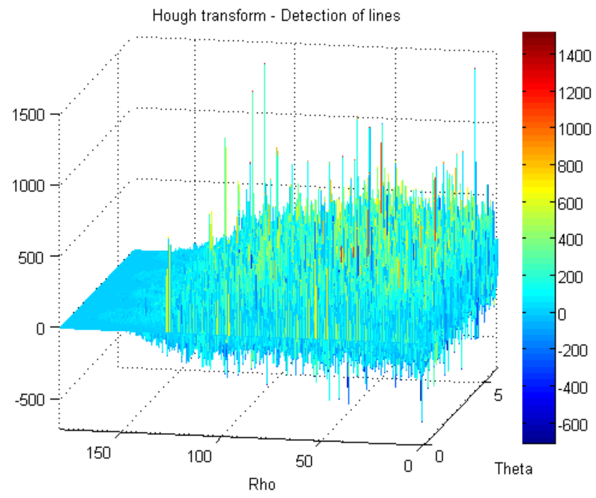

Figure 10: Hough transform to WVD

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