

INTEGRATED DAMPING PARAMETERS AND SERVO CONTROLLER DESIGN FOR OPTIMAL H_2 PERFORMANCE IN HARD DISK DRIVES

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ABSTRACT

Achieving the highest achievable tracks per inch (TPI) in hard disk drives (HDDs) is equivalent to minimize the H_2 norm from the disturbances to the position error signals (PES). Thus the H_2 optimal performance is a matter of great significance. This paper presents an integrated method of the plant and controller design sequentially to achieve an optimal H_2 performance for voice coil motor (VCM) plant. The VCM plant is redesigned first to guarantee that the modified plant has a better H_2 performance by using the linear matrix inequality (LMI) based approach. Then the H_2 optimal controllers are designed for the modified plant. Finally, simulation results are presented to validate the effectiveness of the proposed modified plant redesign and the H_2 optimal controller design method. It is found that the modified plant has a better H_2 performance, higher bandwidth and phase margin than the original plant.

Keywords: damping ratio, H_2 performance, integrated design, H_2 optimal controller design

1. INTRODUCTION

Traditionally, people employ the sequential design method in the mechanical control design problem, where the mechanical plant satisfying the requirements of stiffness, strength, weight, etc., is first designed and then the controller design follows. For the controller design step, it is known that the change in the controller topology and/or the application of the advanced controller algorithm can improve the controlled performance of a given plant. In particular, the key role of advanced control techniques in improving precision and accuracy has been well recognized in the field of nanopositioning (Devasia, Eleftheriou, and Moheimani 2007; Du, Xie, Guo, and Teoh 2007; Du, Xie, Lewis, and Wang 2009). However, this sequential design method does not take the advantage of the freedom in plant design to help the mechanical controlled plant achieve a much better overall mechanical control performance. It fails to meet the practical requirements as systems demand higher performance (Joshi 1999), for example the ever increasing high precision control of the HDDs.

In order to tackle this problem, the simultaneous design method which aims to numerically optimize the parameters of both plant and controller is proposed (Hiramoto, and Grigoriadis 2006; Lu, and Skelton 2000; Grigoriadis, and Wu 1997). And it has been known that the system changes such as structural parameters can influence the ultimate performance. Unfortunately, this scheme has its inherent significant limitation since it has been known that such an integrated design method for most of the mechanical plant falls into a bilinear matrix inequality (BMI) problem, which requires iterative algorithm and intractable computation leading to a local optimal solution. The optimal solution depends on the initial conditions and conservative of the adopted iterative algorithm. Besides, this method does not provide us a deep understanding of how the plant parameters have an impact on the plant property.

There is another mechanical control design method proposed by Iwasaki, Hara, and Yamauchi 2003. In this scheme, they consider the plant design to guarantee the existence of a controller that achieves a good closed-loop performance and argue that the mechanical plant which is designed to be positive real in the desired control bandwidth is the so-called good plant. An LMI approach is given to verify this finite frequency positive real (FFPR) condition. Based on the concept of finite frequency property, the condition Π and finite frequency high-gain (FFHG) are further proposed to clarify the so-called easily controllable plants (Kanno, Hara, and Onishi 2007; Hara, Iwasaki, and Shimizu 2002). It seems that all of these three concepts of defining a good plant, i.e., FFPR, condition Π and FFHG, point to a fact that a good or easily controllable plant is the one that is designed to have the performance as close as possible to the open-loop shaping specifications of controller design, whose relationship we have shown in the Figure 1. In fact, this inherent relationship is not hard to intuitively understand since it is obvious that a plant having performance close to the open-loop shaping specifications may make the further controller design easier and even improve the overall controlled performance to a large extent. Although there is not much theoretical justification for these three concepts, they have been proven to be efficient in tackling the placement problem of actuator and sensor

to enhance the controlled performance (Iwasaki, Hara, and Yamauchi 2003; Kanno, Hara, and Onishi 2007; Hara, Iwasaki, and Shimizu 2002). However, when these concepts come to the design problem of the stiffness and damping parameters, they also result in a BMI problem.

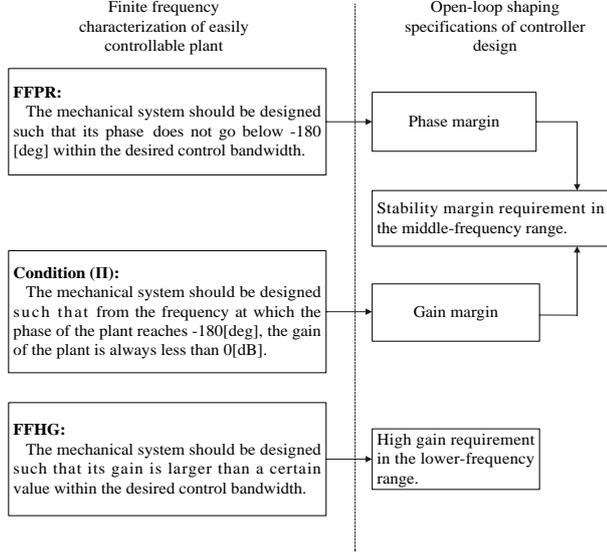


Figure 1: Relationship between Finite Frequency Plant Property and Typical Open-loop Shaping Specifications of Controller Design

Recently, the damping parameter design in structural systems such as the vector second order systems with collocated sensors and actuators has been paid more and more attention (Hiramoto, and Grigoriadis 2008; Mohammadpour, Meisami-Azad, and Grigoriadis 2008; Bai, and Grigoriadis 2005; Bai, Grigoriadis, and Demetriou 2006). Different from the simultaneous design method and the FFPR condition, the damping parameter design of such structural systems can be formulated as a convex optimization problem. This method has advantage over the other integrated design methods in the computational efficiency, especially for large scale structures.

In this paper, we will study the damping parameter design for a class of non-collocated vector second order systems. The static output feedback is used to modify the damping ratios of the plant, then the control design will be discussed. Furthermore, the result will be applied into the VCM plant model in HDD, which shows the efficiency of the proposed method.

2. SYSTEM MODELING

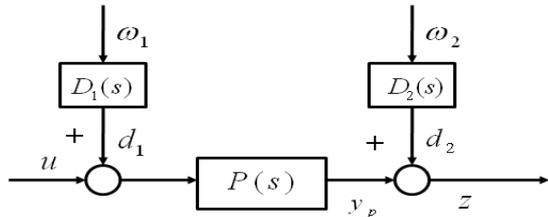


Figure 2: A Dynamic System $P(s)$ with Disturbances

Consider a dynamic system with disturbances as shown in Figure 2, where P is the plant of the dynamic system, d_1 is the input disturbance, d_2 is the output disturbance. D_1 and D_2 are the input and output disturbance models, and ω_1, ω_2 are white noises with zero mean and unit variance. The plant model of the system is easily identified using the curve fitting method. Generally, the plant can be represented as the following transfer function:

$$P(s) = \sum_{i=0}^m \frac{k_i}{s^2 + 2\zeta_i \bar{\omega}_i s + \bar{\omega}_i^2}, \quad (1)$$

where $\zeta_i > 0$ is the damping ratio, $\bar{\omega}_i$ is the resonance frequency, k_i is the residue of the resonance mode and m is the number of the resonance modes. This transfer function can be realized by the following vector second order system with non-collocated actuators and sensors

$$\begin{aligned} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) &= B_u d_1(t) + B_u u(t), \\ y_p(t) &= C_0 q(t), \\ z(t) &= y_p(t) + d_2(t), \end{aligned} \quad (2)$$

where, $q(t)$, $u(t)$, $y_p(t)$ and $z(t)$ are the generalized coordinate vector, control input, plant output and measured output vector, respectively. Matrices M , D and K are symmetric positive definite matrices that represent the structural system mass, damping and stiffness matrices of the mechanical system, respectively. Matrices B_u and C_0 are the distribution matrices on the control input and the control output, respectively.

It is easy to know that (1) can be described in the form of (2), where

$$\begin{aligned} M &= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 2\zeta_0 \bar{\omega}_0 & 0 & \cdots & 0 \\ 0 & 2\zeta_1 \bar{\omega}_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\zeta_m \bar{\omega}_m \end{bmatrix}, \\ K &= \begin{bmatrix} \bar{\omega}_0^2 & 0 & \cdots & 0 \\ 0 & \bar{\omega}_1^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{\omega}_m^2 \end{bmatrix}, \quad B_u = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, \\ C_0 &= [k_0 \quad k_1 \quad \cdots \quad k_m]. \end{aligned}$$

The disturbance models D_1 and D_2 can be modeled based on the power spectrum of the PES. Assume that the input disturbance model D_1 is a constant gain G_D and the output disturbance model has the form

$$D_2(s) = \sum_{i=1}^r \frac{k_{d_i}}{s^2 + 2\zeta_{d_i} \omega_{d_i} s + \omega_{d_i}^2}, \quad (3)$$

where $\zeta_{d_i} > 0$, ω_{d_i} , k_{d_i} and r is the damping ratio, resonance frequency, residue and number of the output disturbance resonance modes, respectively, then d_2 can also be generated by the following vector second order system

$$\begin{aligned} M_d \ddot{q}_d(t) + D_d \dot{q}_d(t) + K_d q_d(t) &= B_\omega \omega_2(t), \\ d_2(t) &= C_\omega q_d(t), \end{aligned} \quad (4)$$

where q_d is the coordinate vector of output disturbance model. Matrices M_d , D_d and K_d are the symmetric positive definite matrices that represent the mass, damping and stiffness matrices of the output disturbance model, respectively. Matrices B_ω and C_ω are the distribution matrices on the output disturbance input and the output disturbance output, respectively.

Denote $\bar{x}(t) = [q(t) \ q_d(t) \ \dot{q}(t) \ \dot{q}_d(t)]$ and $\omega(t) = [\omega_1(t) \ \omega_2(t)]^T$, then the state-space representation is

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}_1\omega(t) + \bar{B}_2u(t), \\ z(t) &= \bar{C}\bar{x}(t), \end{aligned} \quad (5)$$

$$\text{where } \bar{A} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -M^{-1}K & 0 & -M^{-1}D & 0 \\ 0 & 0 & -M_d^{-1}K_d & 0 \\ & & & -M_d^{-1}D_d \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M^{-1}B_u G_D & 0 \\ 0 & M_d^{-1}B_\omega \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ 0 \\ M^{-1}B_u \\ 0 \end{bmatrix}, \text{ and}$$

$$\bar{C} = [C_0 \ C_\omega \ 0 \ 0].$$

Remark 2.1: When $B_u = C_0^T$, the vector second order system corresponds to the one with collocated actuator and sensor placement. In particular, for a single-input-single-output (SISO) system, this would lead to an in-phase property of the mechanical systems in the whole frequency range. The mechanical plant with such an in-phase property has been claimed to be helpful to expect a good controlled performance (Ono and Teramoto 1992), however, it is a rather ideal case since the actuators and sensors are not perfect and such a requirement in the mechanical design may be practically infeasible (Iwasaki, Hara, and Yamauchi 2003) due to the unavoidable limitation on the control bandwidth. The integrated design for such a collocated vector second order system has been extensively studied in several papers, see Hiramoto, and Grigoriadis (2008); Mohammadpour, Meisami-Azad, and Grigoriadis (2008); Bai, and Grigoriadis (2005); Bai, Grigoriadis, and Demetriou (2006) and the references therein. What we consider here is more general than the previous discussed ones in that our formulation allows the case when $B_u \neq C_0^T$ and furthermore we will incorporate the output disturbance model. It is in this sense that we call our problem as integrated design for the non-collocated vector second order systems.

3. INTEGRATED DAMPING PARAMETERS AND CONTROLLER DESIGN FOR OPTIMAL H_2 PERFORMANCE

This section provides an LMI expression for the integrated damping parameters and controller design. The plant is designed first by using the output feedback to change the damping parameters.

Consider the vector second order system (5) with the measured output equation

$$v = C_0\dot{q}(t) + C_\omega\dot{q}_d(t). \quad (6)$$

Our objective is to design the static output feedback control law

$$u_G(t) = -Gv(t), \quad (7)$$

such that the closed-loop system is stable with its H_2 norm satisfying $\|T_{z\omega}\|_2 \leq \gamma$ for a given γ , as shown in Figure 3.

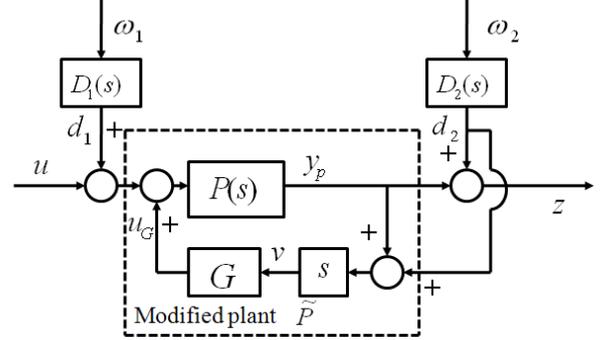


Figure 3: Structure of Vector Second Order System with Output Feedback Loop

It is easy to obtain the closed-loop system

$$\begin{aligned} \dot{\bar{x}}(t) &= A_{cl}\bar{x}(t) + \bar{B}_1\omega(t) + \bar{B}_2u(t), \\ z(t) &= \bar{C}\bar{x}(t), \end{aligned} \quad (8)$$

where,

$$A_{cl} = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -M^{-1}K & 0 & -M^{-1}\tilde{D} & -M^{-1}B_u G C_\omega \\ 0 & -M_d^{-1}K_d & 0 & -M_d^{-1}D_d \end{bmatrix},$$

$$\tilde{D} = D + B_u G C_0.$$

It has been shown that an upper bound of the H_2 performance for the collocated vector second order system can be explicitly given using a solution for the linear matrix inequality formulation of the norm analysis conditions (Mohammadpour, Meisami-Azad, and Grigoriadis 2007; Mona, Javad, and Karolos 2009; Mohammadpour, Meisami-Azad, and Grigoriadis 2008). In this paper, we will study the case of the non-collocated vector second order systems and apply the result into the analysis of VCM plant model. For this purpose, we first introduce the following lemmas.

Lemma 3.1 (Scherer, Gahinet, and Chilali 1997): Consider the stable system (5) with no control input, i.e. $u = 0$, then the system can achieve an H_2 norm γ if and only if there exist symmetric positive matrices Q and Z such that

$$\begin{bmatrix} \bar{A}^T Q + Q \bar{A} & Q \bar{B}_1 \\ \bar{B}_1^T Q & -I \end{bmatrix} < 0, \quad (9)$$

$$\begin{bmatrix} Q & \bar{C}^T \\ \bar{C} & Z \end{bmatrix} > 0, \quad (10)$$

$$\text{trace}(Z) < \gamma^2. \quad (11)$$

Lemma 3.2 (Boyd 1994): The block matrix

$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, where S_{11} and S_{22} are symmetric, is positive definite if and only if $S_{11} > 0$ and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} > 0$; or $S_{22} > 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T > 0$.

Using the above formulation and lemmas, the solution of the integrated damping parameters and output feedback controller design to satisfy higher closed loop H_2 performance is obtained.

Theorem 3.1 For the closed-loop system (8), if the following matrix inequalities are satisfied,

$$\begin{bmatrix} -(\tilde{D} + \tilde{D}^T) + \alpha B_0 B_0^T & -B_u G C_\omega \\ - (B_u G C_\omega)^T & -2D_d + \alpha B_\omega B_\omega^T \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} \alpha K & 0 & C_0^T \\ 0 & \alpha K_d & C_\omega^T \\ C_0 & C_\omega & Z \end{bmatrix} > 0, \quad (13)$$

$$\text{trace}(Z) < \gamma^2, \quad (14)$$

then the integrated design problem of designing the output feedback gain G and the damping parameters $\zeta_i, i = 0, 1, \dots, m$ in D is solvable such that the H_2 norm of the closed-loop system (8) satisfies $\|T_{z\omega}\|_2 \leq \gamma$ for a given γ .

Proof: From Lemma 3.1, the closed loop system (8) with no input can achieve an H_2 norm γ if and only if there exist symmetric positive matrices Q and Z such that LMI (9)-(11) is feasible.

Choosing the candidate Lyapunov matrix Q as

$$Q = \alpha \begin{bmatrix} K & 0 & 0 & 0 \\ 0 & K_d & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & M_d \end{bmatrix},$$

where α is a positive scalar to be used as an additional degree of freedom in order to reduce the conservativeness of the H_2 norm bound. Then the LMI (9) can be simplified as follows,

$$\begin{bmatrix} -\alpha(\tilde{D} + \tilde{D}^T) & -\alpha B_u G C_\omega & \alpha B_0 & 0 \\ -(\alpha B_u G C_\omega)^T & -2\alpha D_d & 0 & \alpha B_\omega \\ \alpha B_0^T & 0 & -I & 0 \\ 0 & \alpha B_\omega^T & 0 & -I \end{bmatrix} < 0.$$

By using the Lemma 3.2, we can obtain

$$\begin{bmatrix} -(\tilde{D} + \tilde{D}^T) + \alpha B_0 B_0^T & -B_u G C_\omega \\ - (B_u G C_\omega)^T & -2D_d + \alpha B_\omega B_\omega^T \end{bmatrix} < 0$$

yielding the LMI (12) with respect to α, G and D .

Furthermore, $\begin{bmatrix} Q & \tilde{C}^T \\ \tilde{C} & Z \end{bmatrix} > 0$ is equivalent to LMI (13) by using Lemma 3.2 again.

Remark 3.1 The integrated damping parameter and output feedback gain design problem can be solved as an H_2 optimization problem as follows:

$$\min_{\alpha, \beta, \zeta_i, G} \gamma^2 \quad (15)$$

Subject to (12)-(14)

Remark 3.2 In fact, the closed-loop system (8) after integrated damping parameters and output feedback gain design in this section can be seen as a modified

plant (see Figure 3). Based on the observation in Figure 1, we have concluded that the plant having a good open-loop specification may be expected to ease the control and improve the overall performance. Then the advanced control techniques can be further applied to achieve possible better control performance.

4. APPLICATION IN HDD

In this section, we will apply the result obtained in the above sections into the damping parameters design of the VCM plant in HDDs. It has been shown in the mechanical engineering community that the damping configuration for the head actuator can influence its tracking dynamics (Jiang and Miles 1999).

In order to achieve higher TPI, one would have to minimize track misregistration (TMR) caused by the disturbances. TMR is defined as three times of the standard deviation of the PES (z in Figure 7), i.e. $3\sigma_{PES}$. σ_{PES} is defined as

$$\sigma_{PES} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n PES(i)^2}, \quad (16)$$

where n is the number of the PES samples.

Let $T_{z\omega}$ denote the transfer function from the disturbances to the PES. When n is large enough, the H_2 norm of $T_{z\omega}$ is given by

$$\|T_{z\omega}\|_2 = \sqrt{\frac{1}{n-1} \sum_{i=1}^n PES(i)^2}. \quad (17)$$

Therefore, the control problem to increase TPI can be formulated as the H_2 optimization problem.

Consider the typical servo loop of HDDs as shown in Figure 2, where P is the VCM plant, d_1 is the input disturbance representing all torque disturbances, d_2 is the output disturbance including disturbances due to disk motion, motor vibration, suspension and slider vibrations.

Table 1: Parameters of the Original Plant

mode i	k_i	ζ_i	ω_i (rad/s)
0	1	0.2	$2\pi 100$
1	-0.8	0.02	$2\pi 9700$
2	-0.5	0.03	$2\pi 14000$
3	-1	0.03	$2\pi 20500$

We consider the VCM plant in a 2.5 inch HDD with three resonance modes, i.e. $= 3$, to show the efficiency of the integrated design method. The frequency response of the original plant is shown in Figure 4. The dash line stands for the measured data from the experiment and the solid line represents the modeled frequency response using the resonance

parameters in Table 1. Here, we omit a resonance mode at 7k Hz because of the large damping ratio.

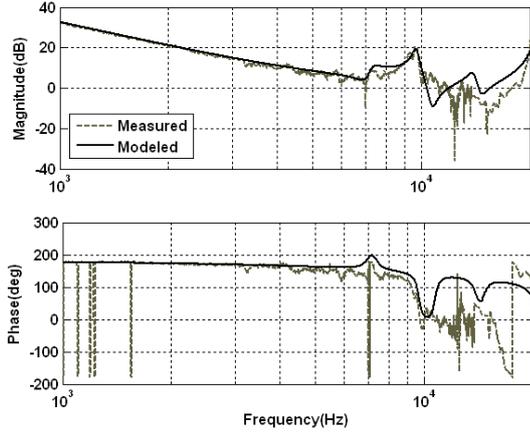


Figure 4: Frequency Response of the VCM Plant

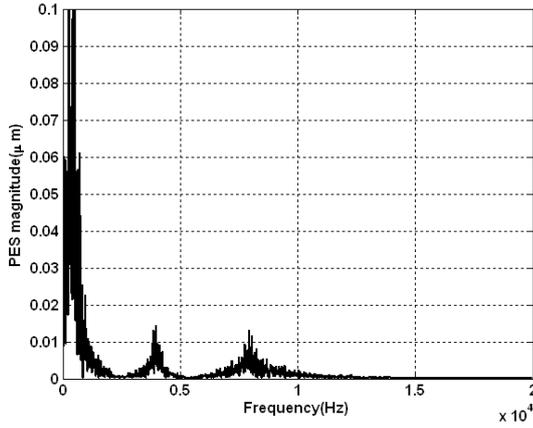


Figure 5: Power Spectrum of PES NRRO without Servo Control

Based on the power spectrum of PES NRRO data without servo control as shown in Figure 5, we can model the input disturbance model D_1 as the constant gain and the output disturbance model D_2 as the structure in Equation (3) with $r = 3$. The parameters of the output disturbance model are shown in Table 2, where three main resonance modes are consider.

Table 2: Parameters of the Output Disturbance

mode i	k_{d_i}	ζ_{d_i}	ω_{d_i} (rad/s)
1	0.04	0.3	$2\pi 500$
2	0.005	0.03	$2\pi 4000$
3	0.01	0.05	$2\pi 8000$

The integrated design is proposed for the HDD VCM plant model to determine the damping ratio ζ_i and feedback gain G such that the H_2 performance of the closed-loop system (8) would satisfy $\|T_{z\omega}\|_2 \leq \gamma$ for some positive scalar γ . Such an LMI problem can be solved by (15).

After obtaining the damping parameters and the feedback gain of the plant, we can get the modified

plant model $\tilde{P}(s)$. Here we just take two cases for example, as shown in Figure 6. The modified plant₁ and the modified plant₂ has the H_2 norm 4.4103nm and 3.9424nm, respectively. On the contrary, the original plant has the H_2 norm of 4.9655 nm. Obviously, the modified plant which is closer to the rigid mode will have a better control performance than the original plant.

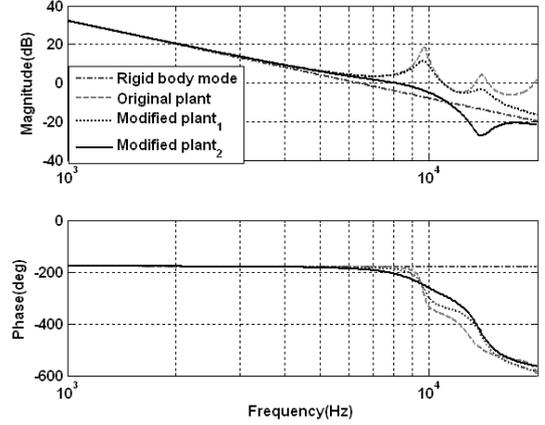


Figure 6: Comparison of the Frequency Responses of the Rigid Body Mode, Original Plant and Modified Plants

A block diagram presentation of a typical HDD servo loop with disturbances and measurement noise is shown in Figure 7. Then the optimal H_2 controller \tilde{K} is designed for the original plant $P(s)$ and the modified plant $\tilde{P}(s)$ using algebraic Riccati equations (Li, Guo, Chen and Lee 2001), respectively.

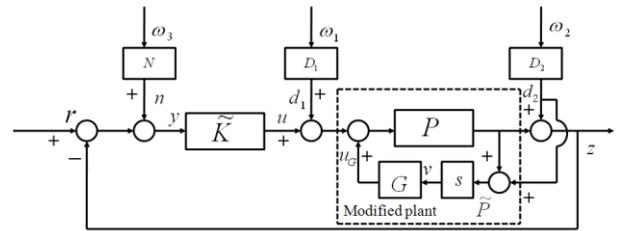


Figure 7: H_2 Control Scheme for HDD Servo Loop with Disturbance Models

Remark 4.1 In fact, the H_2 optimal controller \tilde{K} and the output feedback G can be considered together as the combined controller K_{cc} , as shown in the Figure 8. Let $P: (A_p, B_p, C_p, 0)$ with the state $x_p = [q^T \dot{q}^T]^T$, $D_2: (A_d, B_d, C_d, 0)$ with the state $x_d = [q_d^T \dot{q}_d^T]^T$, $\tilde{K}: (A_c, B_c, C_c, 0)$ with the state x_c , $v = \tilde{C}_p x_p + \tilde{C}_\omega x_d$ with $\tilde{C}_p = [0 \ C_0]$, $\tilde{C}_\omega = [0 \ C_\omega]$. Define the state of the combined controller as $x_{cc} = [x_p^T \ x_d^T \ x_c^T]^T$, then the combined controller K_{cc} is

$$x_{cc}(k+1) = A_{cc}x_{cc}(k) + B_{cc}y(k),$$

$$\tilde{u}(k) = C_{cc}x_{cc}(k),$$

$$\text{where } A_{cc} = \begin{bmatrix} A_p - B_p G \tilde{C}_p & -B_p G \tilde{C}_\omega & B_p C_c \\ 0 & A_d & 0 \\ 0 & 0 & A_c \end{bmatrix},$$

$$B_{cc} = \begin{bmatrix} 0 \\ 0 \\ B_c \end{bmatrix}, C_{cc} = [-G \tilde{C}_p \quad -G \tilde{C}_\omega \quad C_c],$$

$$A_p = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, B_p = \begin{bmatrix} 0 \\ M^{-1}B_u \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & I \\ -M_d^{-1}K_d & -M_d^{-1}D_d \end{bmatrix}.$$

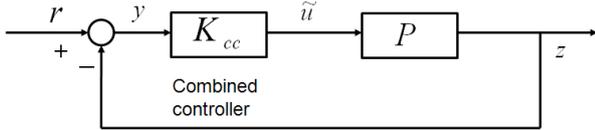


Figure 8: Equivalent Servo Control Systems of Figure 7

Table 3: Control Performance Comparison

Performance	Rigid body mode	Original plant	Modified plant ₁	Modified plant ₂
Open loop 0 dB crossover frequency (Hz)	1.82	1.45	1.55	1.74
Gain margin (dB)	7.93	7.93	7.93	7.93
Phase margin (deg)	30.3	28.1	28.3	28.6
H ₂ performance (σ ₂)(nm)	2.4803	2.8222	2.6985	2.5482

The comparison of the control performances is summarized in Table 3. The modified plants have the higher phase margin, higher bandwidth and the same gain margin. The H₂ performance of the closed loop systems with modified plants is lower than the original plant. The sensitivity functions are also compared in Figure 9. The modified plants have the higher bandwidth, but the sensitivity hump is almost the same. The power spectrum of PES NRRO data (z in Figure 7) of the original plant and the modified plant₂ are compared in Figure 10, from which we can observe that the modified plant₂ yields a considerable improvement in the low frequency up to 2k Hz. Above 8k Hz, the modified plant₂ leads to a bit small degradation. Overall, the H₂ norm of the modified is lower than the original plant, as shown in Table 3.

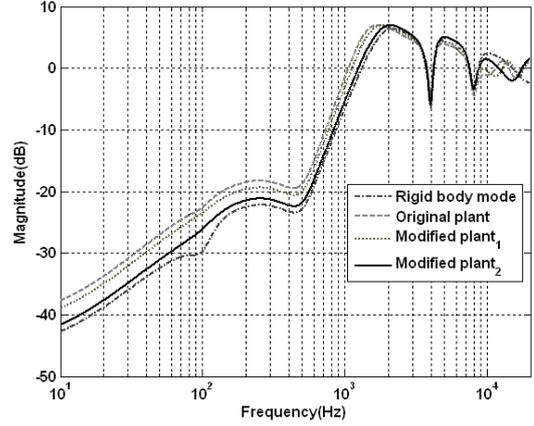


Figure 9: Sensitivity Function Comparison of the Rigid Body Mode, Original Plant and Modified Plants

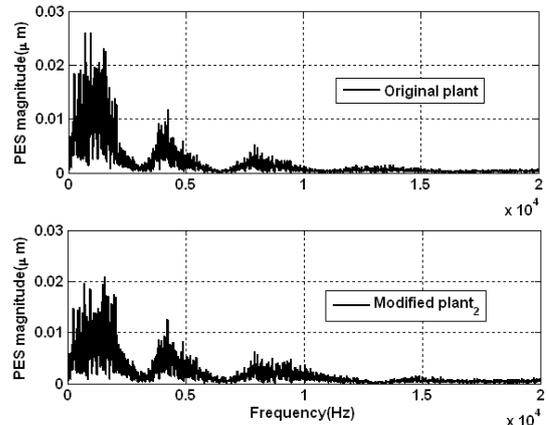


Figure 10: Power Spectrum Comparison of the PES NRRO with the Original Plant and Modified Plant₂

5. CONCLUSIONS

We have developed an LMI based approach to design the damping ratios of the plant and the output feedback gain to guarantee that the modified plant has a better H₂ optimal performance. The corresponding H₂ optimal controllers are designed for each case. A numerical example of the VCM plant in HDD is presented to show the validity and effectiveness of the proposed method. Two modified plants are proposed and compared with the original VCM plant. From the simulation, the proposed modified VCM plants have the lower H₂ performance, higher phase margin and bandwidth.

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