Diodes and an alternative Procedure to derive equations from Bond Graph models by exploting a Linear Graph approach.

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Keywords- Algebraic equations, State Space Equations, Bond Graph Modelling, Linear Graph Modelling, Tree Theory, Causal Paths, Shockley Model of the diode.

***Abstract*— This paper deal with methods to get equations from a Bond Graph (BG) Model. Mixed Techniques to get equations from both Linear Graph (LG) and BG Models are employed. A criterion that allows to identify the tree and the co-tree in a BG model is reviewed. Also, the advantages of causal paths inherent to BG are exploited. The equivalence between a LG model and its respective BG model is also used. A procedure to get equations from a given BG model is proposed. To show the functionality of the proposed methods, examples with diodes are solved.**

I. Introduction

Graphs are simple geometrical figures consisting of nodes and lines that connect some of these nodes; they are sometimes called "Linear Graphs" [1]. One of the most important applications of Graph Theory is its use in the formulation and solution of the electrical network problem by Kirchhoff [1]. Equations can be determined from a LG by finding a normal form based on an appropriate tree [1].

Bond graph is a highly structured modelling technique that allows to analyze different kinds of physical systems in an unique way determined by a basis of unified description [2]. This technique allows the determination of the State Space Equation (SSE) in several different ways by reviewing the interactions of the dynamic elements and the causal paths they determine [3], [4].

Both BG and LG treats with spatially discrete physical systems [5] even when only LG modelling preserves the spatial visualization related to the model being analyzed. The spatial constraints in a BG model are implicit in the location of the 0 and 1 junctions [5].

There exists several ways to get the SSE from a BG model [6], some of them exploiting the concept of causality and causal paths.

Mathematical manipulation of non linear semiconductor models could be difficult. Here, diodes are reviewed and modeled with a non linear constitutive relation.

In this paper a procedure to obtain equations from a BG model by exploiting the LG tree Theory is proposed. The proposed procedure can be particularly useful in the analysis of some systems involving algebraic loops.

The contents are ordered as follows:

Section II shows a reviewing of the Shockley diode model . Section III presents some current techniques to derive equations from BG and LG models. Section IV shows the main result of this work, First a criterion that allow to identify the LG related tree in a BG model is discussed.Then, a procedure that allow to derive the SSE from a BG model by employing the LG related tree theory is presented. Finally three examples are analyzed and discussed while they are solved with the proposed approach. In section V, the conclusions and final comments are written.

In the next Section a short review of the Shocley diode model is presented.

II. The diode model

The diode can be considered as a non controlled switching device [7]. Switched systems can be viewed from several approaches and two main classifications can be made: The variant topology approach and the invariant topology approach [6].

Most texts deal with the idealized model of the diode, i.e. a variant topology model whose representation changes between a short circuit and an open circuit, depending on its on-off state.

The Shockley model is a non linear invariant topology model that is presented in several power electronics textbooks. However, a procedure to derive equations from systems involving this Shockley model is rarely presented and authors start their analysis with the idealized on-off-switch approach. In this paper the Shockley model is used.

The constitutive relation of the Shockley model as appears in [8] is

$$I_D = I_S \left(e^{\frac{V_D}{nV_T}} - 1 \right)$$

where:

- I_D = Current through the diode.

- V_D = Voltage in the diode in the anode referred to the cathode.
- I_S = Reverse saturation current with a value of $1 \times 10^{-12} A$.
- n = Empirical constant known as emission coefficient, that depends on the construction of the element.
- V_T = Thermal voltage given by

$$V_T = \frac{kT}{q}$$

Where q is the electron charge equal to $1.6022 \times 10^{-19} C$; T is the absolute temperature in kelvins and k is the Boltzmann constant equal to $1.3806 \times 10 - 23 J/K$.

For simplicity, some values will be fixed and it will be assumed that

$$I_D = I_S \left(e^{g V_D} - 1 \right) \tag{1}$$

considering a $27°C$ temperature and $n = 2$ that is a value that can be assigned to silicon diodes. With these considerations the g value is

$$g = \frac{1}{\eta v_T} \simeq 19.2308$$

The characteristic $v - i$ curve for this constitutive relation is shown in Figure 1.

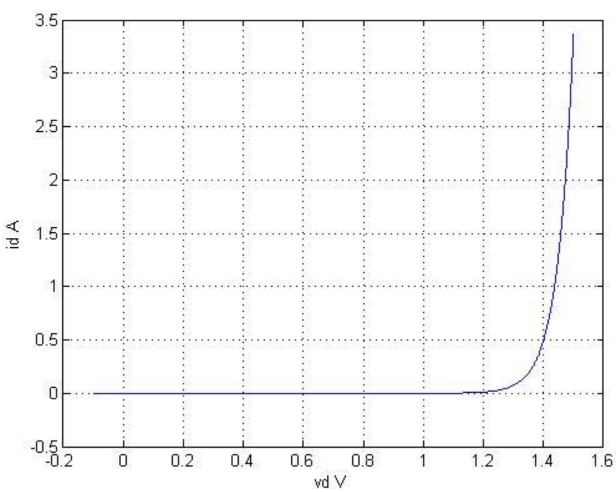

Fig. 1. The caracteristic $v - i$ curve resulting of the constitutive relation (1).

The BG model that will be used in this paper is a resistive non linear R element whose constitutive relation is that presented in equation (1).This model appears in figure 2.

$\rightharpoonup$ R:D

Fig. 2. A resistive non linear diode model.

In the next section a reviewing of some of the current techniques to get the equations from both BG and LG models is performed.

III. Current methods of equations derivation.

There exist several techniques in order to derive equations from both LG and BG models, and some of them are reviewed in the following paragraphs.

A. SSE derivation from a BG models through junction structure matrix

There exists several approaches to develop the SSE from a BG model [6], one of the most powerful is the related with the junction structure matrix [3]. In Figure 3 The key vectors and the junction structure relationships are shown.

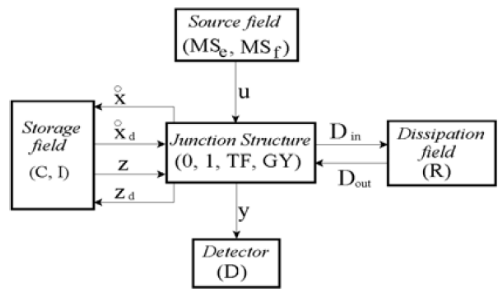

Fig. 3. Key vectors and the junction structure.

In figure 3, the key vectors are formed as follows: (MS_e, MS_f) is the sources field; (L, C) is the storage field; (R) is the dissipative field; (D_e) is the field of detectors; finally, $(0, 1, TF, GY)$ are the elements of the junction structure. The vectors x and x_d represent the states of the system at integral and derivative causality, respectively. z denotes the co-energy vector and z_d the derivative co-energy vector. The vectors u and y are the input and the output, respectively. D_{in} and D_{out} show the relationships between efforts and flows in the dissipative field. The constitutive relationships are given by,

$$z = Fx \tag{2}$$

$$D_{out} = L D_{in} \tag{3}$$

$$z_d = F_d x_d \tag{4}$$

Also the relationships of the junction structure are specified by,

$$\begin{bmatrix} \dot{x} & D_{in} & y \end{bmatrix}^T = S \begin{bmatrix} z & D_{out} & u & \dot{x}_d \end{bmatrix}^T \tag{5}$$

where the matrix of junction structure S is given by,

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \end{bmatrix} \tag{6}$$

$$z_d = G_1 - S_{14}^T z$$

By employing the junction structure matrix S and the relationships between the different fields it is possible to write the model as a space-state equation of the form

$$\dot{x} = Ax + Bu \tag{7}$$

$$y = Cx + Du \quad (8)$$

with

$$A = E^{-1}\left(S_{11} + S_{12}MS_{21}\right)F \quad (9)$$

$$B = E^{-1}\left(S_{13} + S_{12}MS_{23}\right) \quad (10)$$

$$C = \left(S_{31} + S_{32}MS_{21}\right)F \quad (11)$$

$$D = S_{33} + S_{32}MS_{23} \quad (12)$$

where

$$E = I + S_{14}F_d^{-1}S_{14}^{T}F \quad (13)$$

$$M = \left(I - LS_{22}\right)^{-1}L \quad (14)$$

The dynamic elements being placed in vectors x and x_d are selected depending on its causality assignment.
One disadvantage of this method arises in non linear systems modelling, since this matrix representation is not always possible to obtain.

In the next subsection, other approach employed to get the SSE from a BG model is cited.

B. Other SSE derivation method from Bond Graph

According to [6] and [2], given a BG model, the SSE can be achieved by proceeding in a systematic way, as follows:

1) Write the structure laws in the junctions by considering causality.
2) Write the constitutive relationships of the elements by considering causality.
3) Combine these different laws in order to put on explicitly the derivatives of the state variables as function of the state variables and the inputs.

The key concept in both methods is causality, because the formulation of S matrix (in the first method) depends totally on causal paths as well as the systematic (second) method needs the causality assignment already performed. Once the S matrix has been constructed, the procedure to derive the SSE of the model is just a step-by-step algorithm. In the next subsection a short review on causality and causal paths is done.

C. Causality and causal paths

One of the most powerful characteristics of bond graph is that a lot of information can be retrieved without writing any equations, just by analyzing the causality. Physical systems are full of interacting variable pairs [2]. If two elements are bonded, the effort causes one element to respond with flow, while the flow causes the first element to respond with effort. Thus, the cause-effect relationships for efforts and flows are represented in opposite directions.

A detailed description of causal paths can be found in [9] from where the next has been extracted:

Definition 1: A causal path in bond graph is an alternation of bonds and basic elements, called "nodes" such that:

1) For the acausal graph (before establishing causality), the sequence forms a single chain.
2) All of the nodes in the path have complete and correct causality.
3) Two bonds in a causal path have in the same node opposite causal orientations.

According to the variable being followed, there are two kinds of causal paths. The causal path is simple if it can be crossed by following always the same variable and the causal path is mixed if it is necessary to perform a variable change while the graphic is crossed. In addition, two elements P_1 and P_2, belonging to the set $\{R, C, I, S_e, S_f, D_e, D_f\}$ are *causally connected* if the input variable of one is influenced by the output variable of the other.

In the next section some of the techniques used to develop equations from a LG model are mentioned.

D. SEE derived from a LG model

The tree theory developed originally by Kirchhoff [1] is very useful in the formulation of equations in electrical networks. In purely resistive linear networks, two general approaches can lead to the formulation of a minimal set of equations: general node analysis and general loop analysis [10]. If a given network has dynamic elements, there exists procedures to derive the SSE by using the basis of a normal tree. According to [1] a normal tree can be defined as follows:

Definition 2: A normal tree of a connected directed graph representing a network is a tree that contains all the independent voltage (effort) sources edges, the maximum number (all) of capacitive edges, the minimum number (none) of inductive edges, and none of the independent current (flow) source edges.

The SSE equation of any linear time-invariant system represented by a LG model can be derived by following a systematic precedure [1]. In the case when a normal tree as described in definition 2 cannot be constructed, the LG model includes dynamical elements that cannot be expressed independently in the resulting SSE.

In the next section a mixed approach that exploits the LG tree theory is presented. This proposed approach works directly on the BG model.

IV. Deriving the Equations from a BG model by using a LG tree approach

The equivalence between a LG model and a BG model is reviewed in [12]. Authors establishes a relation between a BG model an the tree of the related LG model by considering causality and adyacent causal strokes. A slightly different cualitative analysis can be performed. Consider the electrical network shown in Figure 4.

In Figure 4 (a) an electric network is drawn. In Figure 4 (b) the corresponding tree (the only possible normal tree) of the system is shown. In Figure 4 (c) the unique related Bond Graph model with integral preferred causality (BGI) can be seen. It is very important to notice the branches of the tree and the orientation of the causal strokes in the corresponding elements on the BG model. The following concepts allow to

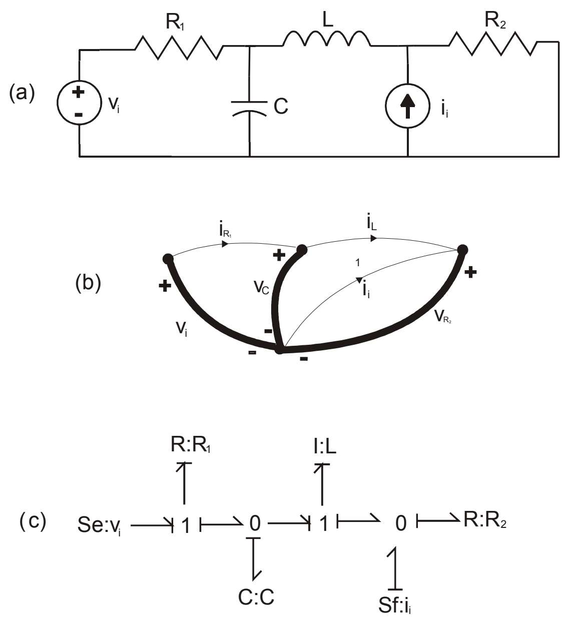

Fig. 4. Electrical network (a) and their LG (b) and BGI (c) representation.

formalize the criterion employed to identify a tree on a BG model.

When a port-1 element is causally connected to a BG junction structure two situations are possible: (i) The causal stroke (effort) go from the junction structure toward the element or *outwards* as in Figure 5 (a), or (ii) the causal stroke go from the element toward the junction structure or *inwards*, as in Figure 5 (b). In Figure 5, the X element is assumed to be a port-1 element.

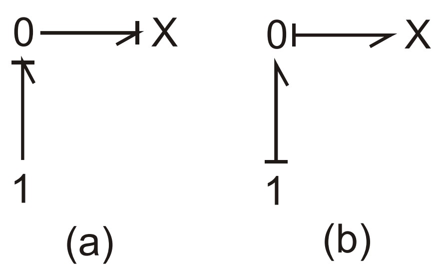

Fig. 5. The two possible causal orientations of a port-1 element.

This direction on the casual strokes is important, because it bring the conditions to identify the spanning tree of a system in a BG model. This is summarized in the next criterion:

Criterion 1: In a BGI model, each port-1 element whose causal stroke is inwards (junction adjacent) determine a branch of the related spanning tree. The set of all these port-1 elements spans the related tree. In the other hand, each port-1 element whose causal stroke is outwards (element adjacent) determine a chord of the related cotree.

This criterion allow to find out a one to one relation between a BG model and its associated LG model.

In LG theory, Kirchhoff Current Law (KCL) is applied over tree branches in order to get their currents as a sum of currents of the cotree chords. This is completely equivalent to analyze the causal paths related to each "tree branch" on the BG model after Criterion 1 has been applied. With this consideration, a direct procedure to derive the SSE from a BG model can be written as follows:

Procedure 1:

1) Given an acausal BG model, assign causality as BGI.
2) Determine the tree branches and the cotree chords according to criterion 1.
3) Set the effort sources and capacitive elements in the tree as tree-main-effort (TME) variables. If resistive elements belong to the tree, set them as tree-helper-effort (THE) variables.
4) Set the flow sources and inductive elements in the cotree as key-flow (KF) variables. If resistive elements belog to the cotree, set them as nonkey-flow (NF) variables
5) By using the element constitutive relation, write each NF variable as a combination of all the TME variables that hold a direct causal path with it.
6) By using the element constitutive relation, write the each THE variable as a combination of all the flow variables that hold a direct causal path with it.
7) Write the state equations by determining each capacitor flow in the tree as a combination of the cotree flows that hold a direct causal path with it. And by determining each inductive effort in the cotree as a combination of the tree efforts that hold a direct causal path with it. Finally, write the state equation by using (in the linear case) the relations $f_C = C\dot{e}_C$ and $e_I = I\dot{f}_I$

Remark 1: If in step 1, Integral causality cannot be assigned in all elements, it implies that a normal tree cannot be constructed. In step 5 and 6 if it is found that direct causal paths are held between THE and NF elements, then algebraic loops will arise in the case of linear systems or differential algebraic equations must be constructed in the case of non linear systems.

The preceding Procedure allows to derive the SSE from a BG model. If only algebraic equations are needed (resistive systems), the general node analisys can be performed. General node analysis mixed with BG analysis brings a very systematic procedure to derive equations. Once a normal tree is established, it is always possible to find causal paths between tree elements and cotree elements. The key of the procedure is to determine the flows in the tree elements in terms of cotree variables by following the (unique) causal paths that are formed, at the same time cotree elements can be determined by their constitutive relationship and the (also unique) causal paths to tree elements. The next example show how this Procedure can be applied.

Example 1: Consider again the system in Figure 4.

In Figure 6 the BGI model of Figure 4 has been redrawn and the bonds are now numbered, as well as the tree branches are

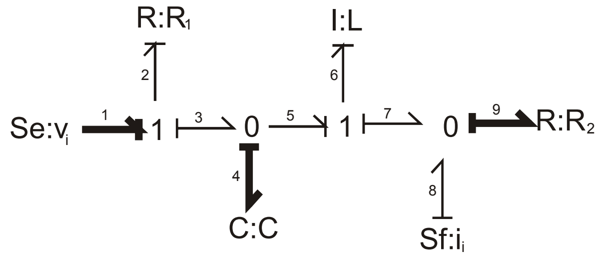

Fig. 6. The BGI model of system in Figure 4.

now remarked as black bonds. According to Procedure 1, the first step is already done. According to criterion 1, the second step is also performed and the bonds corresponding to the tree branches are darkened. In this terms, the bonds belonging to the tree are 1, 4 and 9. the bonds of the cotree are 2, 6 and 8. According with step 3, TME variables are $e_1$and $e_4$whereas e_9.is the unique THE variable. Step 4 allow to identify f_6 and f_8 as KF variables and f_2 as NF variable.Step 5 of Procedure 1 ask to write any cotree R element flow (NF variable) as a combination of main efforts on the tree and the related constitutive relationship. This is the case of R_1. The flow on R_1 can be written as,

$$\begin{aligned} f_2 &= \frac{1}{R_1} e_2 \\ f_2 &= \frac{1}{R_1}(e_1 - e_3) \\ f_2 &= \frac{1}{R_1}(e_1 - e_4) \end{aligned} \quad (15)$$

Step 6 requires to write e_9 as a combination of flows in the cotree. It can be seen that R_2 holds direct causal paths with i_1 through$\{9, 8\}$ and with L through $\{9, 7, 6\}$, therefore it can be written

$$\begin{aligned} e_9 &= R_2 f_9 \\ e_9 &= R_2 (f_7 + f_8) \\ e_9 &= R_2(f_6 + f_8) \end{aligned} \quad (16)$$

Step 7 requires to write the flow in the capacitor as a combination of cotree variables causally connected with it. Reviewing the causal paths, it can be noticed that

$$\begin{aligned} f_4 &= f_3 - f_5 \\ f_4 &= f_2 - f_6 \end{aligned} \quad (17)$$

Replacing (15) into (17) the result is,

$$\begin{aligned} f_4 &= \frac{1}{R_2}(e_1 - e_4) - f_6 \\ f_4 &= -\frac{1}{R_2} e_4 - f_6 + \frac{1}{R_2} e_1 \end{aligned} \quad (18)$$

Following step 7 I effort equation can also be derived as

$$\begin{aligned} e_6 &= e_5 - e_7 = \\ e_6 &= e_4 - e_9 \end{aligned} \quad (19)$$

Replacing (16) into (19) the result is

$$\begin{aligned} e_6 &= e_4 - R_2(f_6 + f_8) \\ e_6 &= e_4 - R_2 f_6 - R_2 f_8 \end{aligned} \quad (20)$$

Finally, equations (18) and (20) can be modified as

$$\begin{aligned} \dot{e}_4 &= -\frac{1}{R_2 C} e_4 - \frac{1}{C} f_6 + \frac{1}{R_2 C} e_1 \\ \dot{f}_6 &= \frac{1}{L} e_4 - \frac{R_2}{L} f_6 - \frac{R_2}{L} f_8 \end{aligned} \quad (21)$$

Equation (21) is the SSE corresponding to model in Figure 4. In the next example, a diode circuit is analyzed.

Example 2: A purely resistive system: The central tap full wave rectifier with resistive load

Consider the electrical network shown in Figure 7.

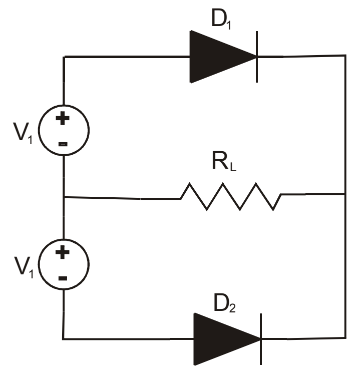

Fig. 7. A full wave rectifier with resistive load.

In Figure 8 the model with causality assigned is shown

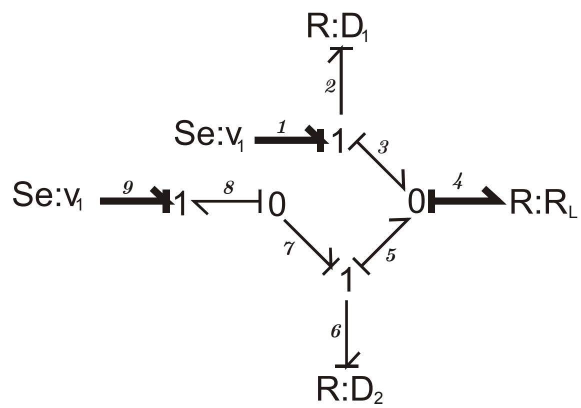

Fig. 8. Bond Graph model of the system in figure 7 with causality assigned.

Notice in figure 8 that bonds 1, 4 and 9 belong to the tree. The R elements in the cotree are D_1 and D_2. Their flows depending on the tree efforts and the constitutive relation (1) are as follows:

$$f_2 = I_s\left(e^{g e_2} - 1\right)$$

or

$$f_2 = I_s\left(e^{g(e_1 - e_4)} - 1\right) \quad (22)$$

and

$$f_6 = I_s \left(e^{g e_6} - 1\right)$$

or

$$f_6 = I_s \left(e^{g(-e_9 - e_4)} - 1\right) \tag{23}$$

Writing one equation in this example can lead to the solution of the entire model. This equation is

$$f_4 = f_2 + f_6 \tag{24}$$

Substituting equations (22) and (23) in equation (24) it results

$$f_4 = I_s \left(e^{g(e_1 - e_4)} + e^{g(-e_9 - e_4)} - 2\right) \tag{25}$$

Also the effort in R_L can be expressed as

$$e_4 = R_L f_4 \tag{26}$$

Substituting equation (26) in equation (25), the following equation can be written

$$f_4 = I_s \left(e^{g(e_1 - R_L f_4)} + e^{g(-e_9 - R_L f_4)} - 2\right) \tag{27}$$

Equation (27) is an implicit function of the flow f_4 in R_L that can be simulated in Matlab as follows

```
t=[0:1e-4:32e-3];
ei=[36*sin(377*t)];
e9=ei;
n=2;
vt=26e-3; %@T=27 celsius
is=1e-12;
g=1/(n*vt)
R=500;
sixe=size(ei);
sais=sixe(:,2);
fl4=zeros(1,sais);
e2=zeros(1,sais);
e6=zeros(1,sais);
ye=zeros(1,sais);
for j=1:sais
funk=@(f4)((is*(exp(g*(ei(1,j)-(R*f4)))+exp(g*(-e7(1,j)-
(R*f4)))-2))-f4);
varr=fzero(funk,[-.0010 .1]);
fl4(1,j)=double(varr);
ye(1,j)=R*fl4(1,j);
end
plot(t,ei,t,ye)
xlabel('time, s');
ylabel('in and out Voltages, V');
```

And the graphic of the Figure 9 is obtained

The preceding example has shown how to apply the proposed Procedure in purely resistive systems containing diodes represented by the Shockley model (1).

Now a dynamic system is analized. Consider the electric network shown in Figure 10.

The corresponding BGI is shown in Figure 11.

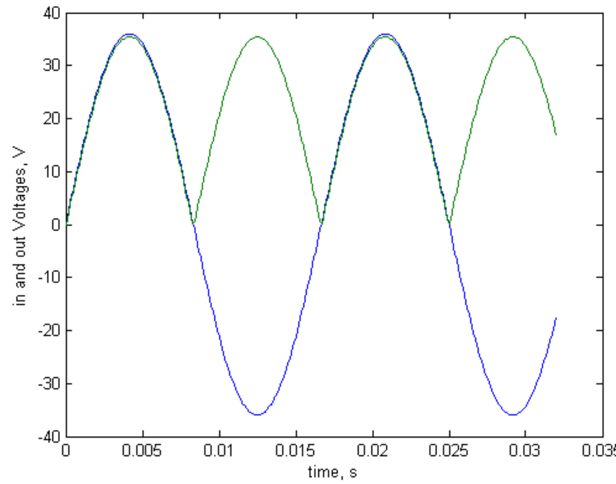

Fig. 9. The input and output voltages of the system in Figure 7.

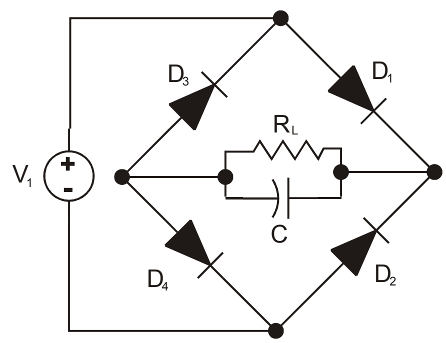

Fig. 10. A full wave rectifier with RC load.

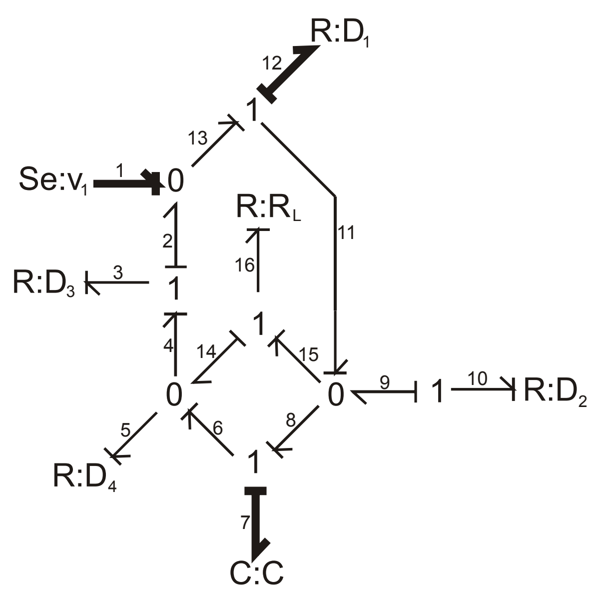

Fig. 11. BGI model of the system in Figure 10.

The tree is conformed by 2 TME variables an 1 THE varaible, (e_1, e_7 and e_{12}, respectively). All elements in the cotree are causally conected with elements in the tree, D_2 is causally connected with D_1through the causal path $\{10, 9, 11, 12\}$, and with V_1 through the causal path $\{10, 9, 11, 13, 1\}$, and effort e_{10} can be expressed as

$$e_{10} = -e_1 + e_{12} \tag{28}$$

Similarly, R is causally connected with C, D_1 and V_1, and

$$e_{16} = e_7 \tag{29}$$

In a similar way, the following cotree efforts can be expressed in terms of the tree variables as:

$$e_3 = -e_{12} - e_7 \tag{30}$$
$$e_5 = e_1 - e_{12} - e_7 \tag{31}$$

According to general node analysis, and the given procedure, KCL must be applied twice, and in the BG model this means that the flow in the unknowns must be expressed in terms of the flows in the cotree. Starting with the flow in D_1, it can be seen that this element is causally connected with D_2, R_L, D_3 and D_4. This dependence can be written as

$$f_{12} = f_3 + f_5 - f_{10} \tag{32}$$

due to the non linear nature of the system, the e_{12} tree variable is very difficult to eliminate, so a differential algebraic equation must me constructed, and equation 32 shall be rewritten as

$$-f_{12} + f_3 + f_5 - f_{10} = 0$$

by applying the diode constitutive relations, it can be seen that

$$I_s \left[-e^{ge_{12}} + e^{ge_3} + e^{ge_5} - e^{ge_{10}}\right] = 0 \tag{33}$$

Substituting 28, 30 and 31 into 33, the first system equation is obtained as:

$$I_s \left[-e^{ge_{12}} + e^{g(-e_{12}-e_7)} + e^{g(e_1-e_{12}-e_7)} - e^{g(-e_1+e_{12})}\right] = 0 \tag{34}$$

Now, considering the flow in C, this element is causally connected with D_4, R_L and D_3, and its flow can be written as

$$f_7 = f_5 + f_3 - f_{16}$$

or

$$f_7 = I_s\left(e^{ge_5} - 1\right) + I_s\left(e^{ge_3} - 1\right) - \frac{1}{R_L} e_7 \tag{35}$$

Substituting 29, 30 and 31 into 35, and isolating the derivative of the state, the second system equation can be written as

$$\overset{\bullet}{e}_7 = \frac{I_s}{C}\left(e^{g(e_1-e_{12}-e_7)} + e^{g(-e_{12}-e_7)} - 2\right) - \frac{1}{R_L C} e_7 \tag{36}$$

equations 34 and 36 conform a differential algebraic system. This system is simulated in Matlab with the next code lines:

```
clear
clc
masa=[1 0;...
0 0];%mass matrix to indicate DAE
options = odeset('Initialstep',1e-30,...
'MStateDependence','none','Mass',masa,...
'RelTol',1e-10,'AbsTol',[1e-12 1e-12]);
[T,Y] = ode23t(@full_RC,[0:0.1e-5:32e-3],...
[0 0],options);
ent=100*sin(377*T);
plot(T,ent,T,Y(:,1),'-',T,Y(:,2),'-.')
%plotting the input, the D1 voltage and the output
```

With the fucntion full_RC defined as

```
function dy = full_RC(t,y)
%Shockley model parameters
n=2; %silice 2, germanium 1
vt=26e-3; %@T=27 degrees celsius
is=1e-12;
g=1/(n*vt);
%component values
c=47e-6;%load capacitance
r=100;%load resistence
ei=100.*sin(377*t);%input voltage
dy = zeros(2,1);
dy(1)=((is/c)*(exp(g*(-y(2)-y(1)))+...
exp(g*(ei-y(2)-y(1)))-2))-((1/(r*c))*y(1));
dy(2)=-exp(g*y(2))+exp(g*(-y(2)-y(1)))+...
exp(g*(ei-y(2)-y(1)))-exp(g*(y(2)-ei));
```

Once these codes are implemented in Matlab, the plotting in the following Figure is obtained:

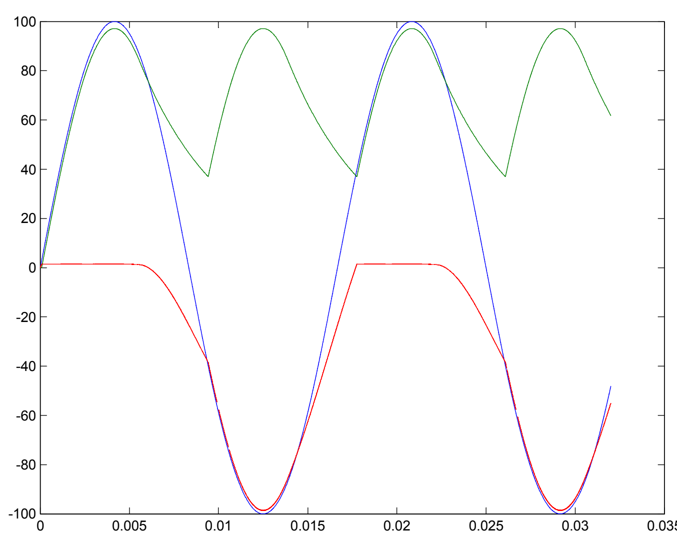

Fig. 12. Simulation results of the system in Figure 11.

V. Conclusions

An alternative approach to derive equations from a BG model has been presented. Two important tools are employed: The causality and causal paths involved in BG theory and the tree formulation of SSE employed in LG theory.
The proposed approach allow to derive the SSE in a systematic way by following a step by step procedure.

Causal connections between tree elements and cotree elements can be directly found with the suggested Procedure, allowing the equation derivation of systems containing elements with non linear constitutive relations, such as the well known Shockley diode model. General node analysis allows to analyze even purely resistive (algebraic) systems, and also dynamic systems with the proposed modification in BG models.

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