MODELING MULTIPHASE FLOW DYNAMICS IN OFFSHORE PETROLEUM PRODUCTION SYSTEMS

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ABSTRACT

In this study, a mathematical model is proposed to investigate the dynamics of gas, oil and water flow in a pipeline-riser system. The pipeline is modeled as a lumped parameter system and considers two switchable states, one in which the gas is able to penetrate into the riser, and another in which there is a liquid accumulation front, preventing the gas from penetrating the riser. The riser model considers a distributed parameter system, in which movable nodes are used to evaluate local conditions along the subsystem. The method of characteristics allows the simplification of the differentiation in the hyperbolic system of equations obtained. The resulting equations are discretized and integrated using an implicit method with a predictor-corrector scheme for the treatment of the nonlinearities. Simulations corresponding to severe slugging conditions are presented. As result of the simulations, pressure, volume fractions and superficial velocities for the phases along the tubes are calculated.

Keywords: multiphase flow, petroleum production systems, lumped and distributed parameter systems, switched systems.

1. INTRODUCTION

In offshore petroleum production systems, the fluids that leave the well are often transported to platforms by means of flexible pipes. The pipes are composed of a pipeline (or flowline), which conducts the fluids over the seabed topography, and a riser, which elevates the fluids to the separator vessel located at the platform, as shown in Fig. 1. Usually, the transported fluids are composed of gas, oil and water, but due to the severe conditions of pressure and temperature, it is possible the formation of emulsions, hydrates and wax. These features make the modeling of the multiphase flow dynamics a complex task.

Some models were proposed for the air-water flow in a pipeline-riser system. Schmidt *et al.* (1979), Fabre *et al.* (1987), Taitel *et al.* (1990), Sarica and Shoham (1991) and Baliño *et al.* (2010) are some of the authors that researched the behavior of this biphasic flow and proposed different methods to determine the system stability. Using air-water as flowing fluids, it is possible to investigate basic mechanisms of the flow behavior; however, there are many limitations when trying to extrapolate these results to petroleum production systems.

Pipeline lengths and riser heights in petroleum production systems are much larger (order of kilometers long) than the values for air-water experimental facilities. The high pressure ratios between the bottom and top of the riser give rise to important expansion effects in the gas phase, invalidating models based on the assumption of a mean void fraction.

Petroleum is a multicomponent system in which both liquid and gas phases coexist at operating conditions (McCain, 1990). Mass transfer between the phases are dependent on pressure and temperature through the pressure, volume and temperature (PVT) data. With the high pressure variations in the riser, mass transfer effects cannot be ignored. Besides, the fluid coming from the reservoir has a water content, so three phases can coexist in the general case.

In the present work, a mathematical model is proposed to investigate the dynamics of gas, oil and water flow in a pipeline-riser system. The pipeline is modeled as a lumped parameter system and considers two switchable states, one in which the gas is able to penetrate into the riser, and another in which there is a liquid accumulation front, preventing the gas from penetrating the riser. The riser model considers a distributed parameter system, in which movable nodes are used to evaluate local conditions along the subsystem. For both subsystems continuity equation for gas, oil and water phases are taken into account. Oil and water phases are considered to have the same velocity and are homogenized. Mass transfer between the oil and gas phases is calculated using the black oil approximation. The properties of fluids are calculated by analytical correlations based on experimental results and field data.

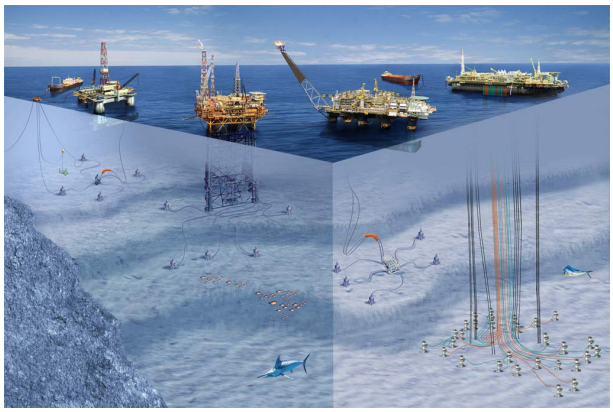


Figure 1: Typical offshore petroleum production systems (source: Petrobras).

The pipeline model considers a stationary linear mo-

mentum balance for the smooth stratified flow regime and the riser model uses a simplified momentum equation without inertia terms for the mixture. A drift-flux model, evaluated for the local conditions in the riser, is used as a closure law. The method of characteristics allows the simplification of the differentiation in the hyperbolic system of equations obtained. The resulting equations are discretized and integrated using an implicit method with a predictor-corrector scheme for the treatment of the nonlinearities.

2. Riser model

The model is based on a distributed parameter system, in which nodes are used to evaluate the local condition along the subsystem. It considers a one-dimensional three-phase isothermal flow. Continuity equation for gas, oil and water phases and a simplified momentum equation without inertia terms for the phases flowing together are the governing equations. Oil and water phases are considered to have the same velocity and are homogenized. Slip between the liquid and gas phases are taken into account by using a drift flux model. Mass transfer between the oil and gas phases are calculated using the black oil model. The liquid and gas phases are assumed to be compressible and the gas behaves as a real gas. Solubility of gas and vaporization are neglected for water.

2.1 Riser geometry

The riser geometry is characterized by a function expressing the coordinates of the points belonging to the riser; from this, it is possible to determine the local position along the riser and the local inclination angle. For a catenary riser, geometry is characterized by the coordinates X and Z, corresponding to the abscissa and the height of the top of the riser (see Fig. 2). It is assumed that the inclination angle at the bottom is zero.

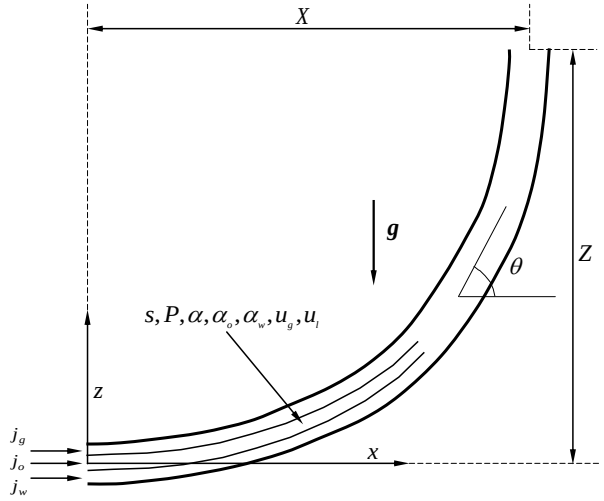


Figure 2: Variables at the riser.

The local height z of a point belonging to catenary can be written as:

$$z = \varphi \left[\cosh\left(\frac{x}{\varphi}\right) - 1 \right] \tag{1}$$

where the dimensional catenary constant φ is obtained from the solution of the following transcendental equation:

$$Z = \varphi \left[\cosh\left(\frac{X}{\varphi}\right) - 1 \right] \tag{2}$$

The local position s along the catenary results:

$$s = \varphi \sinh\left(\frac{x}{\varphi}\right) \tag{3}$$

The local inclination angle θ can be written as:

$$\theta = \arctan\left[\sinh\left(\frac{x}{\varphi}\right) \right] \tag{4}$$

Knowing the position s, the local abscissa x can be calculated from Eq. (3):

$$x = \varphi \operatorname{arcsinh}\left(\frac{s}{\varphi}\right) \tag{5}$$

2.2 Conservation equations

Considering continuity equations for the phases gas, oil and water, we get:

$$\frac{\partial}{\partial t}(\rho_g\,\alpha) + \frac{\partial j_g}{\partial s} = \Gamma \tag{6}$$

$$\frac{\partial}{\partial t}(\rho_o\,\alpha_o) + \frac{\partial j_o}{\partial s} = -\Gamma \tag{7}$$

$$\frac{\partial}{\partial t}(\rho_w\,\alpha_w) + \frac{\partial j_w}{\partial s} = 0 \tag{8}$$

where s is the coordinate along the flow direction, t is time, ρ_g, ρ_o and ρ_w are the densities of the phases (correspondingly gas, oil and water), j_g, j_o and j_w are the superficial velocities, α, α_o and α_w are the volume fractions and Γ is the vaporization source term.

In most of the transients occurred in oil and gas transport, for instance in severe slugging, the response of the system proves to be relatively slow, showing that pressure waves do not have a strong effect on the initiation and transport of void waves. In the no-pressure-wave (NPW) model (Masella *et al.*, 1998), acoustic waves are ruled out by neglecting inertia terms from the momentum equation, resulting an algebraic relation for the pressure gradient:

$$\frac{\partial P}{\partial s} = -\frac{4\,\tau_w}{D} + \rho_m\,g_s \tag{9}$$

$$\rho_m = \rho_g\,\alpha + \rho_o\,\alpha_o + \rho_w\,\alpha_w \tag{10}$$

where P is pressure, ρ_m is the density of the mixture, D is the pipe diameter, g_s is the gravity component in the s-direction and τ_w is the mean shear stress at the pipe wall. The volume fractions are related by:

$$\alpha_o + \alpha_w + \alpha = 1 \tag{11}$$

2.3 Closure laws

In order to close mathematically the problem, some simplifications must be made.

2.3.1 Homogenization of liquid phases

Assuming equal velocities for oil and water, we obtain:

$$j_o = j_l \frac{\alpha_o}{1-\alpha} \tag{12}$$

$$j_w = j_l \frac{\alpha_w}{1-\alpha} \tag{13}$$

$$j_l = j_o + j_w = u_l (1-\alpha) \tag{14}$$

where j_l and u_l are correspondingly the superficial velocity and the velocity of the liquid (oil plus water) phase.

2.3.2 Shear stress at the wall

The shear stress at the wall is estimated using a homogeneous two-phase model and the correlation from Chen (1979) for the Fanning friction factor f.

2.3.3 Real gas

As the pressures involved are high, the constitutive relation for the gas phase is considered as:

$$\rho_g = \frac{\gamma_g M_a}{\Lambda T} \frac{P}{Z} \tag{15}$$

where $\gamma_g = M_g/M_a$ is the gas specific gravity, M_g and $M_a = 28.966$ are respectively the molar masses of gas and air, Z is the gas compressibility factor (dependent on pressure, temperature and gas composition) and $\Lambda = 8.314\, m^2 s^{-2} K^{-1}$ is the gas universal constant.

2.3.4 Drift flux model

The superficial velocities for the liquid and gas phases are determined by using a drift flux model (Zuber and Findlay, 1965):

$$j_g = \alpha (C_d\, j + U_d) \tag{16}$$

$$j_l = (1 - \alpha\, C_d)\, j - \alpha\, U_d \tag{17}$$

$$j = j_l + j_g \tag{18}$$

where the parameters C_d and U_d depend on the local geometric and flow conditions (Bendiksen, 1984; Chexal *et al.*, 1992). In a general form, it will be assumed that $C_d = C_d(\alpha, P, j, \theta)$ and $U_d = U_d(\alpha, P, j, \theta)$.

2.3.5 Black oil model

The vaporization term can be calculated by using the black oil model (McCain, 1990). According to this model, the gas specific gravity does not change with variations of pressure and temperature:

$$\gamma_g \cong \gamma_{g0} \tag{19}$$

$$\gamma_{dg} \cong \gamma_{g0} \tag{20}$$

where γ_g is the gas specific gravity at local conditions, γ_{g0} is the gas specific gravity at standard conditions and γ_{dg} is the dissolved gas specific gravity.

In this way, many properties corresponding to the phases at operating conditions can be estimated based on parameters at standard condition (1 atm and 60 ^oF for API, American Petroleum Institute) and a set of correlations depending on pressure, temperature and composition, which will be considered as locally and instantaneously valid.

The vaporization term can be expressed as:

$$\Gamma = -\frac{\rho_{g0}\, \alpha_o}{B_o} \left(\frac{\partial R_s}{\partial t} + \frac{j_o}{\alpha_o} \frac{\partial R_s}{\partial s} \right) \tag{21}$$

where ρ_{g0} is the gas density at standard condition, B_o is the oil formation volume factor and R_s is the solution gas-oil ratio. It is worth noting that for $\Gamma > 0$ must be $\alpha_o > 0$ (there must exist oil for vaporization), while for $\Gamma < 0$ must be $\alpha > 0$ (there must exist gas for condensation).

2.4 Well-posedness and method of characteristics

For a model to describe physical phenomena correctly it must be well-posed, this is, the solution must exist, must be uniquely determined and must depend in a continuous fashion on the initial and boundary conditions (Drew and Passman, 1999). This property is particularly important in multiphase flows, where partial differential equations of hyperbolic nature can be found; in this case, well-posedness implies that the characteristic values (eigenvalues or characteristic wave velocities) must be real.

The characteristic values of the presented system of conservation equations are given by Nemoto and Baliño (2009):

$$e_1 = \frac{\partial j_g}{\partial \alpha} \quad e_2 = \frac{j_o}{\alpha_o} = u_l \quad e_3 = \infty \quad e_4 = \infty \tag{22}$$

where u_l is the liquid velocity. If the parameters C_d and U_d are not dependent of α, i.e. $C_d = C_d(P, j, \theta)$ and $U_d = U_d(P, j, \theta)$ (as in the correlation developed by Bendiksen (1984)) we have:

$$\frac{\partial j_g}{\partial \alpha} = \frac{j_g}{\alpha} = u_g \tag{23}$$

where u_g is the gas velocity.

There exists an algebraically-double eigenvalue equal to ∞, these eigenvalues are related to the pressure wave velocities. The pressure wave is propagated in negative and positive directions with an infinite velocity, meaning that any pressure change is felt by the entire system instantaneously.

The method of characteristics will be applied to solve the system of equations. This method is the natural numerical procedure for hyperbolic systems. By an appropriate choice of coordinates, the original system of hyperbolic partial differential equations can be replaced by a system of ordinary differential equations expressed in the characteristic coordinates. Characteristic coordinates are the natural coordinates of the system in the sense that, in terms of these coordinates, differentiation is simpler (Ames, 1992).

The resulting system of equations in the characteristic coordinates, or compatibility conditions, is given by:

$$b_{11}^* \frac{D_g \alpha}{Dt} + b_{13}^* \frac{D_g P}{Dt} + d_1^* = 0 \tag{24}$$

$$b_{21}^{*}\,\frac{D_l\alpha}{Dt} + b_{22}^{*}\,\frac{D_l\alpha_o}{Dt} + b_{23}^{*}\,\frac{D_lP}{Dt} = 0 \tag{25}$$

$$\frac{\partial j}{\partial s} = f_1\,\frac{D_gP}{Dt} + f_2\,\frac{D_lP}{Dt} \tag{26}$$

$$\frac{\partial P}{\partial s} = -\frac{4\,\tau_w}{D} + \rho_m\,g_s \tag{27}$$

where the coefficients b_{11}^{*}, b_{13}^{*}, b_{21}^{*}, b_{22}^{*}, b_{23}^{*}, d_1^{*}, f_1 and f_2 are function of the state variables and dependent variables.

The directional derivatives are defined as:

$$\frac{D_g}{Dt} = \frac{\partial}{\partial t} + u_g\,\frac{\partial}{\partial s} \tag{28}$$

$$\frac{D_l}{Dt} = \frac{\partial}{\partial t} + u_l\,\frac{\partial}{\partial s} \tag{29}$$

3. Pipeline model

The pipeline model is based on a lumped parameter system, assuming two switchable states, one in which the gas is able to penetrate into the riser, and another in which there is a liquid accumulation front, preventing the gas from penetrating the riser. The model considers one-dimensional three-phase isothermal flow. The governing equations are the continuity equation for gas, oil and water phases and a momentum balance equation evaluated at the stationary state (Taitel and Dukler, 1976), which is used to determine the void fraction in the pipeline.

It is assumed that water and oil phases have the same velocity and are homogenized, the flow pattern is the smooth stratified, the pressure is constant in the gas cavity, mass transfer between the oil and gas phases is calculated using the black oil model and variations in the void fraction α_p are neglected during the transient.

The pipeline model is composed of two control volumes (see Fig. 3). The control volume $\forall_1$ comprises the gas and liquid in the stratified region, while the control volume $\forall_2$ comprises the liquid penetration region.

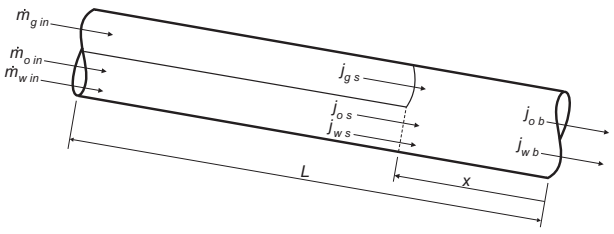


Figure 3: Control volumes in the pipeline model.

3.1 State $x = 0$

The state $x = 0$ is observed when there is no liquid penetration in the pipeline. In this case, there exist only one control volume in the pipeline, the control volume $\forall_1$, and the pressure in the gas cavity is constant and equal to the pressure at the bottom of the riser (P_b).

The state equations are obtained by applying continuity equations for the gas, oil and water phases, resulting in expressions that allows the calculation of the superficial velocity of gas (j_{gb}), oil (j_{ob}) and water (j_{wb}) at the bottom of the riser.

3.2 State $x > 0$

In this state, the liquid front penetrates the pipeline and two control volumes are taken into account. The pressure in the control volume $\forall_1$, P_s, is constant throughout its length and the pressure in the control volume $\forall_2$ depends on position. In order to evaluate fluid properties and its derivatives, a representative pressure P_m for the control volume $\forall_2$ is used:

$$P_s = P_b - \rho_l\,g\,x\,\sin\,\beta \tag{30}$$

$$P_m = \frac{P_b + P_s}{2} \tag{31}$$

where ρ_l is the density of the liquid phase, g is the gravity, β is the pipeline inclination, x is the liquid penetration length, the index s refers to the interface between the control volumes and the index m refers to the mean value at the control volume $\forall_2$.

3.2.1 Volume control $\forall_1$

From the continuity equations for the gas, oil and water, it is possible to obtain expressions for the velocity of the liquid accumulation front (dx/dt) and superficial velocity of oil (j_{os}) and water (j_{ws}) at the interface of the control volumes.

3.2.2 Volume control $\forall_2$

From the continuity equations for the gas, oil and water, we obtain the superficial velocity of oil (j_{ob}) and water (j_{wb}) at the bottom of the riser.

3.3 Switching condition

Whenever the gas superficial velocity at the bottom of the riser becomes zero, commutation exists between the set of equations determined by the state $x = 0$ to the set of equations determined by the state $x > 0$. And the commutation from the state $x > 0$ to the state $x = 0$ happens when the liquid penetration length becomes zero.

4. FLUID PROPERTIES

The properties of fluids are calculated by analytical correlations based on experimental results and field data.

4.1 Gas formation volume factor and gas density

The gas formation volume factor is calculated by the following expression:

$$B_g = \frac{P_0}{T_0}\,\frac{Z\,T}{P} \tag{32}$$

where P_0 is the pressure at standard conditions, T_0 is the absolute temperature at standard conditions, Z is the compressibility factor and P and T are the pressure and the absolute temperature at local conditions.

The compressibility factor is determined using the correlation of Dranchuk and Abu-Kassem (1975). For the evaluation of the compressibility factor is also necessary to calculate the pseudocritical temperature and pressure, which can be determined using the correlation of Standing (1981).

Considering the black oil approximation, which assumes a approximately constant gas specific gravity, it can be shown that Eq. (15) reduces to:

$$\rho_g \cong \frac{\rho_{g0}}{B_g} \tag{33}$$

4.2 Water formation volume factor and water density

The correlation for water formation volume factor is presented in the work of McCain (1990). Water density at local condition is determined by:

$$\rho_w = \frac{\rho_{w0}}{B_w} \tag{34}$$

4.3 Gas-oil solubility and bubble point pressure

If the local pressure is above the bubble point pressure, the gas-oil solubility is equal to the gas-oil ratio GOR, otherwise the gas-oil solubility is calculate according to the correlation of Standing (1981).

The bubble point pressure is determined based on the correlation of Velarde *et al.* (1999).

4.4 Oil formation volume factor and oil density

Based on the definition of oil formation volume factor:

$$B_o = \frac{\upsilon_o}{\upsilon_{o0}} \tag{35}$$

where υ_o and υ_{o0} are, respectively, the oil volume at local conditions and at standard conditions, the following material balance relation results:

$$B_o = \frac{\rho_{o0} + \dfrac{P_0\, M_a}{\Lambda\, T_0}\, R_s\, \gamma_{dg}}{\rho_o} \tag{36}$$

Assuming that the black oil approximation is valid and substituting Eq. (20) in Eq. (36), we obtain:

$$B_o \cong \frac{\rho_{o0} + \rho_{g0}\, R_s}{\rho_o} \tag{37}$$

The oil density is calculated based on the correlation of Velarde *et al.* (1999).

4.5 Gas, oil and water viscosities

The gas viscosity is calculated using the correlation of Lee *et al.* (1966).

The dead oil viscosity at standard pressure is calculated using the correlation of Ng and Egboah (1994). The dead oil viscosity is necessary to calculate the saturated and subsaturated oil viscosity. The former is calculated using the correlation of Beggs and Robinson (1975), while the latter is calculated using the correlation of Vasquez and Beggs (1980).

The water viscosity is calculated using the results of Collins (1987). The first step is the determination of the water viscosity at standard pressure, then it is possible to evaluate the water viscosity at local conditions.

4.6 Vaporization term

Considering isothermal flow, B_o, α_o and R_s depend only on pressure, so Eq. (21) reduces to:

$$\Gamma = -\frac{\rho_{g0}\,\alpha_o}{B_o}\frac{\partial R_s}{\partial P}\frac{D_l P}{Dt} \tag{38}$$

5. STATIONARY STATE

The stationary state is important since it is used as the initial condition for the transient simulations. The stationary state can be obtained by setting to zero the time derivatives in the dynamic equations. Applying this procedure to Eqs. (6), (7) and (8), we obtain respectively:

$$j_g = \frac{Q_{o0}\ (GOR - R_s)\ B_g}{A} \tag{39}$$

$$j_o = \frac{Q_{o0}\, B_o}{A} \tag{40}$$

$$j_w = \frac{Q_{o0}\, WOR\, B_w}{A} \tag{41}$$

where Q_{o0} is the oil flow rate at standard conditions, R_s is the gas-oil solubility ratio, B_o, B_w and B_g are the formation factor for the corresponding fluids, A is the flow passage area of the riser tube and WOR is the water-oil ratio.

The void fraction α along the riser is calculated using Eq. (16). The oil volume fraction is calculated using the following expression, obtained from Eqs. (11), (12) and (13):

$$\alpha_o = \frac{1-\alpha}{\frac{j_w}{j_o} + 1} \tag{42}$$

The water volume fraction is calculated from Eq. (11):

$$\alpha_w = 1 - \alpha - \alpha_o \tag{43}$$

6. TRANSIENT STATE

To evaluate the transient state, the initial conditions are taken from the stationary state results. A moving grid method was adopted, in which node i ($1 \leq i \leq N-1$) moves with the gas characteristic velocity. Last node N moves with the liquid velocity if the liquid level falls bellow the top of the riser, or remain fixed at the top otherwise. The time step (Δt) is calculated as the time step such that the node $N-1$ intersects the node N. As the gas velocity is positive, a node disappears at the liquid level or top of the riser and a node is created at the bottom of the riser, keeping constant the number of nodes.

As the nodes move with the gas velocity, lines are defined in the space-time plane (s, t), as shown in Fig. 4. These lines are the gas characteristic path (red lines) on which the integration of the equations projected in the gas direction is performed. The liquid characteristic path (blue lines) must also be known in order to evaluate the integration of the equations projected in the liquid direction. For this purpose, it is considered that at time $t_2 = t_1 + \Delta t$ the liquid characteristic path crosses the nodes. The characteristic velocities for the gas and liquid and the time step Δt are determined from the nodal values at time t_2, as an implicit scheme is used. Once the liquid characteristic path is known, it is possible to interpolate the values for the liquid directional derivatives from the known solution at time t_1.

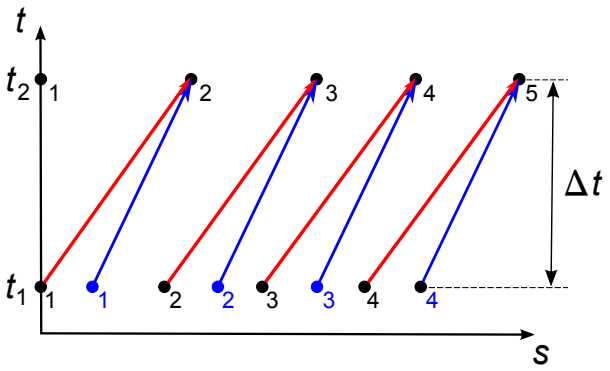


Figure 4: Characteristic lines in the (s, t) plane.

The transient calculations can be summarized as follows:

1. The predictor values of the variables are assumed as the corresponding values obtained at time t_1.

2. The time step Δt is calculated.

3. The fluid properties are calculated at the nodal positions.

4. The predictor values of the fluids superficial velocities at the bottom of the riser are used to evaluate the pressure and gas, oil and water volume fractions along the riser at time t_2, using an implicit scheme.

5. The pressure at the bottom of the riser is used to determine the fluids superficial velocity at the riser base, using the state equations of the pipeline model.

6. The convergence of the variables is verified and in the case it is not achieved, new predictor values for the variables are determined using an under-relaxation factor. A new iteration starts with step 2.

7. If convergence is achieved, time is increased and a new time step is calculated, beginning with step 1.

7. SIMULATIONS

Based on the presented model, a computational program for transient simulations was developed using MATLAB (MATLAB, 2011).

Table 1 presents the input data used to simulate a case. The following figures show the transient response of important variables: void fraction at the bottom of the riser (Fig. 5(a)), oil volume fraction at the bottom of the riser (Fig. 5(b)), water volume fraction at the bottom of the riser (Fig. 5(c)), gas (red), oil (black) and water (blue) superficial velocity at the bottom of the riser (Fig. 5(d)), gas (red), oil (black) and water (blue) superficial velocity at the liquid level in the riser (Fig. 5(e)), pressure at the bottom of the riser (Fig. 5(f)), position of liquid accumulation front at pipeline (Fig. 5(g)) and height of the liquid level at the riser (Fig. 5(h)).

Table 1: Input data for simulation.

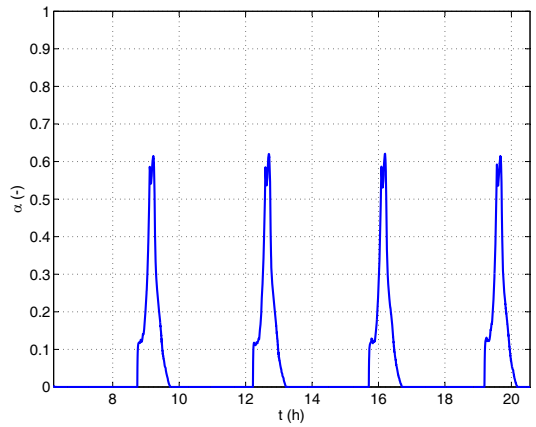
Symbol	Variable	Values
API	API-grad	19
γ_g	Gas specific gravity	0.6602
Q_{g0}	Gas volumetric flow rate at standard conditions	$0.1\ Sm^3/s$
GOR	Gas-oil ratio	145
WOR	Water-oil ratio	0.3
T	Temperature	$323K$
D	Inner diameter	4”
X	Horizontal length of the top of the riser	$845\ m$
Z	Height of the top of the riser	$1300\ m$
ϵ	Roughness	$4,6\ .\ 10^{-5}\ m$
P_{sep}	Pressure at the separator	$25\ bara$
Y	Salinity	0
N	Number of nodes	101
L	Pipeline length	$1000\ m$
β	Pipeline inclination angle	2^o

The simulations correspond to a phenomenon known in petroleum technology as severe slugging (Taitel *et al.*, 1990), in which the flow destabilizes and reaches a limit-cycle. The flow destabilization results from two competing mechanisms: pressure drop across the riser (mainly influenced by the volume fraction distribution) and pipeline compressibility.

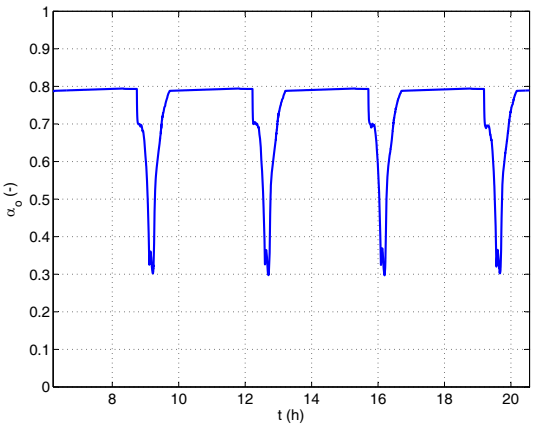
The cycle begins with the blockage of the gas passage at the bottom of the riser, that is represented by a void fraction at the bottom of the riser (α) equals to zero. As the liquid coming from pipeline continues to flow in, as show the oil and water superficial velocities at the bottom of the riser (j_{ob} and j_{wb} respectively), and the gas that was already in the riser continues to flow out, as show the gas superficial velocity at the liquid level (j_{gu}), the riser column becomes heavier. Thus, the pressure at the bottom of the riser (P_b) increases continuously, causing the same behavior to the liquid penetration length (x), compressing the gas in the pipeline. The liquid level at the riser (z_u), which was at the lowest level of the cycle, also begins to increase since the liquid supply to the riser remains approximately constant. This stage is known as slug formation.

As the liquid level (z_u) reaches the top while the gas passage is kept blocked at the bottom, pressure reaches a maximum and there is only liquid and vaporized gas flowing in the riser. This is the slug production stage.

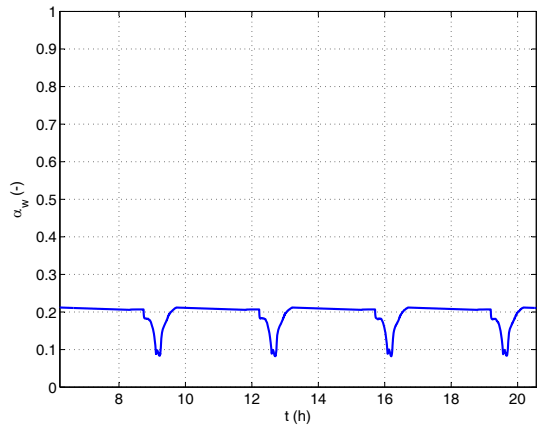
Observe that during the slug formation stage no vaporization occurs until the liquid level reaches the top, as can be observed from the gas superficial velocity at the liquid level (j_{gu}). From Eq. (38), as $\partial R_s/\partial P > 0$, the necessary condition for vaporization is $D_l P/Dt < 0$. As the inclination angle increases with position in the riser, the vertical velocity component of the liquid in the riser is smaller than the vertical velocity component of the liquid level. As a consequence, the liquid particles in the riser undergo higher pressures (mainly hydrostatic) as they flow upwards. When the liquid level reaches the top of the riser, the liquid particles undergo lower pressures as they flow upwards and vaporization occurs in the riser. The gas vaporization leads to a pressure decrease at the bottom of the riser due to the decrease in hydrostatic pressure drop; thus, in the slug production stage, vapor flowing out the riser comes from vaporization in the riser. This phenomenon prevents the pressure at the



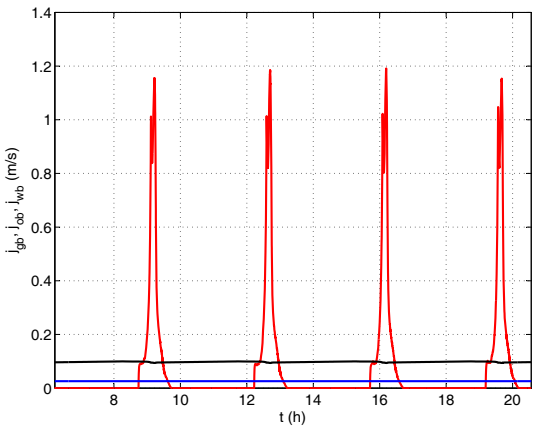
(a)



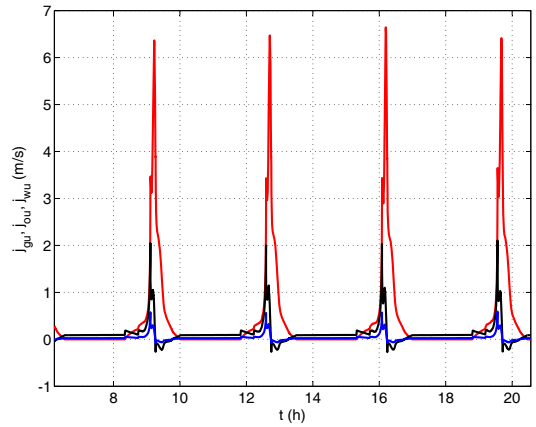
(b)



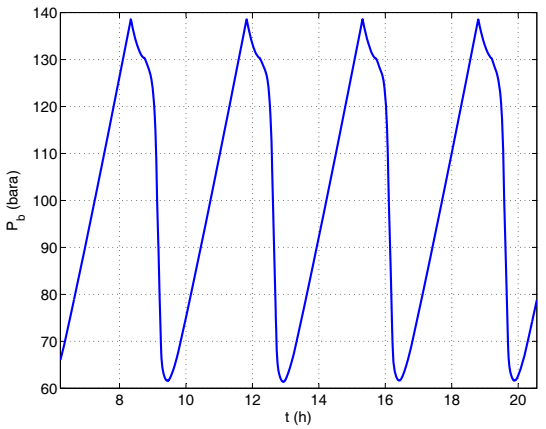
(c)



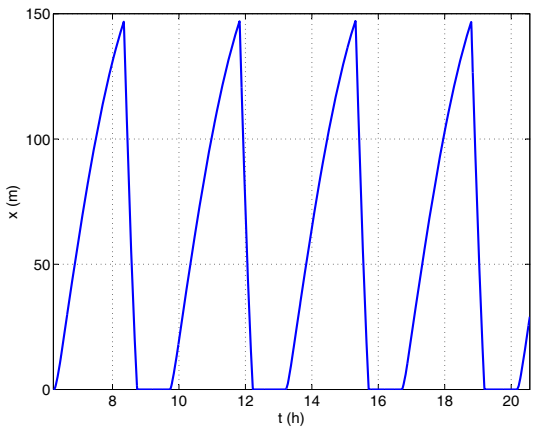
(d)



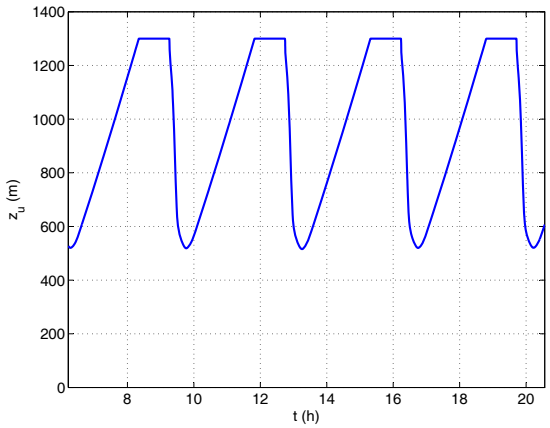
(e)



(f)



(g)



(h)

Figure 5: Simulation results.

bottom of the riser from keeping the maximum value during the production stage, as observed in severe slugging cycles for air-water systems.

As gas keeps flowing in the pipeline, the gas pressure eventually equals and overcomes the riser column pressure, so that the liquid accumulation front is pushed back until it reaches the bottom of the riser, starting the blowout stage. This stage is characterized by a peak in the gas superficial velocity (j_{gb}) and in the void fraction at the bottom of the riser (α).

As the gas penetrates into the riser the column becomes lighter, decreasing the pressure and then rising the gas flow. When gas reaches the top of the riser, gas passage is free through the stratified flow pattern in the pipeline and the intermittent/annular flow pattern in the riser, causing a violent expulsion and rapid decompression that brings the process to the slug formation again. This stage is known as gas blowdown. Observe that at the end of the decompression the pressure at the bottom of the riser (P_b) and the liquid level at the riser (z_u) reach the lowest levels in the cycle.

As can be seen from the figures, the periods associated to severe slugging are of the order of hours in offshore facilities; besides, fluctuations of all the variables are usually very high, compared to the stationary values. These long periods in which there is only liquid phase flowing, followed by relatively shorter periods in which there is a gas blowdown, cause instabilities in the liquid control systems of the separators and usually lead to platform shutdown. It is interesting to notice that the stationary state used as the initial condition does not exist, so any prediction made by a steady state program would be useless.

The simulation results shown in Fig. 5 characterize a severe slugging type 1 (SS1), in which the liquid slug has a length equal or greater than the riser length. Depending on the input parameters, other less dangerous severe slugging types exist: SS2 (in which the liquid slug length is smaller than the riser length) and SS3 (in which there is a continuous gas penetration at the bottom of the riser). The existence of SS1, SS2 and SS3 show a richness in the configuration states that can be attained by a pipeline-riser system, as well as poses a challenge in the model simulation capabilities.

Simulations with the same input variables shown in Table 1 were performed using the comercial software OLGA (OLGA, 2011). OLGA is a computational program developed to simulate multiphase flow in pipeline networks, with processing equipment included. The program solves separate continuity equations for the gas, liquid bulk and liquid droplets, two momentum equations, one for the continuous liquid phase and one for the combination of gas and possible liquid droplets and one mixture energy equation, considering that both phases are at same temperature. The equation are solved using the finite volume method and a semi-implicit time integration. Although the results obtained from the simulations using OLGA are not presented in this paper, they show a very good agreement with the ones obtained with the present, simpler model.

8. CONCLUSIONS

The proposed pipeline-riser model was able to simulate the dynamics of the gas, oil and water flow in a typical offshore petroleum production system. It is important to notice that the model captures important physical effects in a highly nonlinear system such as moving liquid level at the riser and liquid penetration front at the pipeline. The model represents an advance compared to other works in literature (for air-water systems) in the sense that it considers oil-gas mass transfer, real gas behavior, fluids compressibility and local thermodynamic properties based on the black oil model. Further developments are being researched, such as the consideration of a variable void fraction in the pipeline, the inclusion of a choke valve at the top of the riser, the insertion of inertia terms in the momentum equations and the modeling of the gas lift. This is a work in progress.

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