MONITORING STEADY-STATE GAINS IN LARGE PROCESS SYSTEMS

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ABSTRACT
Stationary process gains are critical model parameters to determine targets in commercial MPC technologies. Consequently, important savings can be reached by acceding to an early prevention system capable of indicating when the actual process moves away from the modeled dynamics, particularly when the actual process gains are no more represented by those included in the model identified during commissioning stages. A subspace identification method is used here to determine the state-space model matrices that help to define a gain matrix estimator. The pursued goal is a monitoring method capable to identify those gains of a multivariable model that start a mismatching condition, or that show tendencies to change already known mismatches with the actual process. The anticipated knowledge of these events should prevent process engineers about the eventual reliability of targeting optimal process conditions with wrong gain estimations, and thus, help to localize the dynamic relationships for which an updating identification would be necessary.

Keywords: Steady-State Gains, Multivariable Estimator, Subspace Identification, LP-MPC

1. INTRODUCTION

Model predictive control (MPC) has a wide application in the chemical process industry and other industrial sectors. Commercial MPC systems are typically implemented in conjunction with a steady-state linear (LP) or quadratic programming (QP) optimizers (Ying and Joseph 1999), whose main function is to track the economic optimum and provide feasible set-points or targets to the predictive controller. However, despite the widespread adoption of these two-level control systems, occurrences of poor performance have been reported. A frequent claim has been that model mismatches lead the operation away from the real optimum, and that large variations in the computed input and output targets have been observed (Nikandrov and Swartz, 2009). Since the stationary process gains are critical model parameters to determine the MPC targets, important savings can be obtained by acceding to an early prevention system capable to indicate when the actual process moves away from the modeled behavior, particularly by indicating when the actual process gains are no more represented by those included in the model identified during early commissioning stages.

This work is the first stage of a research project attempting to contribute to the development of a method for the online estimation of multivariable process gains directly from row data, and creating the basis for a monitoring tool capable of detecting significant gain changes, particularly when working simultaneously with multivariable control applications like the combined LP (or QP) and MPC. The analysis presented here is still far from the final goal but highlights questions and dificulties that need to be answered and overcome to obtain the reliable estimations.

This paper is organized as follows: after the short introduction exposing the motivations and goals of this project, the subspace identification method is described in Section 2. The gain matrix estimator is defined in Section 3 base on the state space model matrices and Section 4 comments about monitoring alternatives for multivariable cases; however, the simpler Shewhart chart strategy was adopted for a closer inspection of the individual estimations. In Section 5, some simulation results are presented and the conclusions are given in Section 6, together with comments about future work associated to this subject.

2. SUBSPACE IDENTIFICATION (SID)

Early in the nineties, a new identification method for dynamic systems received the attention of many scholars and practitioners. The subspace identification method (SID) has the appealing feature of allowing the direct use of row data with scarce preprocessing needs and the ability of being applicable to multivariable process systems. Several analysis and applications were reported since then, to name a few: Van Overschee and De Moor (1996); Favoreel et al. (2000) and Katayama (2005).

Most subspace approaches fall into the unification theorem proposed by Van Overschee and De Moor (1996), being the followings the three better known: **N4SID** (Van Overschee and De Moor, 1994); **CVA** (Larimore, 1990) and **MOESP** (Verhaegen and Dewilde, 1992). These algorithms can be viewed as singular values decompositions of a weighted matrix. They provide reliable state-space models of multi-variable LTI systems directly from input-output data and, do not require iterative optimization procedures; this basically means that there are no problems of local minima, convergence, or initialization.

A main advantage of the subspace approaches is that preliminary or previous parameterization - a

complex task when dealing with MIMO systems - is not necessary before the identification process. Another advantage is the computational efficiency and robustness given by linear algebra tools like QR and SVD decompositions.

In this paper, our goal heads mainly to the online identification of steady-state gains of MIMO dynamic systems using a standard subspace approach (Overschee and De Moor, 1996) where specific matrices are obtained from projections of subspaces generated by the input and output data. These projections are useful to eliminate noisy components.

2.1. State space model

The method considers the following stochastic state-space model in an innovation form:

$$x(k+1) = Ax(k) + Bu(k) + Ke(k) \quad (1)$$

$$y(k) = Cx(k) + Du(k) + e(k) \quad (2)$$

where $x(k) \in \mathbb{R}^n$ stands for a *n*-dimensional state, $u(k) \in \mathbb{R}^m$ represents the *m* inputs to the system, $y(k) \in \mathbb{R}^l$ is the *l*-dimensional output, *K* is the steady state Kalman gain and, $e(k) \in \mathbb{R}^l$ is an unknown innovation with covariance matrix $R = E\{e(k)e(k)'\}$.

The problem solved by the SID method can be described as follows: given a large enought data set $\{y_k, u_k\}$ from an unknown system, find the model order *n*, the model matrices *A*, *B*, *C* and *D* for a state-space representation similar to (1) and (2), and estimate *K*.

2.2. Estimation of model matrices

An iterative substitution of Eqns. (1) yields the expresión

$$Y_f = \Gamma_i X_f + H_i^d U_f + H_i^s E_f \quad (3)$$

where the subscripts *f* and *p* denote future and past horizons respectively. The matrices $Y_f \in \mathbb{R}^{li \times j}$, $U_f \in \mathbb{R}^{mi \times j}$ and $E_f \in \mathbb{R}^{li \times j}$ are the output, input and noise block Hankel matrices respectively. H_i^d and H_i^s are low triangular Toeplitz matrices composed from the impulse responses of deterministic and stochastic subsystems respectively.

Notice that the system information is mainly in the first term, which includes the extended observability matrix and the state sequence X_f. Assuming the noise E_f is independent from the past input U_p, the past output Y_p and the future input U_f, then the following relationship can be obtained (see Appendix A for nomenclature usage)

$$Y_f /_{U_f} W_p = \Gamma_i . X_f \quad (4)$$

According with Van Overschee and De Moor (1996), this result tells the space column of Γ_i is the same as the space column of $Y_f /_{U_f} W_p$, which can be estimated from input-output data. Then, applying singular value decomposition the left side can be written as

$$Y_f /_{U_f} W_p = USV^T = U S^{1/2} S^{1/2} V^T \quad (5)$$

from where the extended observability matrix Γ_i can estimated by

$$\hat{\Gamma}_i = U S^{1/2} \quad (6)$$

Once this matrix is obtained, the model matrices in (1) and (2) can estimated in the following order: (*C*, *A*) and (*B*, *D*).

The matrix *C* is obtained directly from the first row block of Γ_i, i.e.,

$$C = \Gamma_i (0 : l-1, 0 : n-1). \quad (7)$$

The matrix *A* is then obtained using the invariant displacement property of Γ_i, this is:

$$\overline{\Gamma}_i = \underline{\Gamma}_i A, \quad (8)$$

where $\overline{\Gamma}_i \in \mathbb{R}^{l(i-1) \times n}$ and $\underline{\Gamma}_i \in \mathbb{R}^{l(i-1) \times n}$ are matrices obtained from the last $l(i\text{-}1)$ and the first $l(i\text{-}1)$ row blocks of Γ_i respectively. Hence, matrix *A* can be estimated by the mean squared solution as follows:

$$A = \underline{\Gamma}_i^{\dagger} \overline{\Gamma}_i \quad (9)$$

where $\underline{\Gamma}_i^{\dagger} = \left(\underline{\Gamma}_i' \underline{\Gamma}_i\right)^{-1} \underline{\Gamma}_i'$ is a pseudo inverse matrix.

Now, left and right multiplying Eqn. (2) by $\Gamma_i^{\perp}$ and $U_f^{\dagger}$ respectively

$$\begin{aligned} \Gamma_i^{\perp} Y_f U_f^{\dagger} = \Gamma_i^{\perp} \Gamma_i X_f U_f^{\dagger} + \\ \Gamma_i^{\perp} H_i^d U_f U_f^{\dagger} + \Gamma_i^{\perp} H_i^s E_f U_f^{\dagger} \end{aligned} \quad (10)$$

Notice $\Gamma_i^{\perp} \in \mathbb{R}^{(li-n) \times li}$ is a complete range matrix satisfying $\Gamma_i^{\perp} . \Gamma_i = 0$ and $U_f U_f^{\dagger} = I$. Hence, assuming negligible noise, the above expression simplifies to

$$\underbrace{\overbrace{\Gamma_i^{\perp} Y_f U_f^{\dagger}}^{Data}}_{\in \mathbb{R}^{(li-n) \times mi}} = \overbrace{\Gamma_i^{\perp}}^{Data} H_i^d \quad (11)$$

This is an over determinate system of linear equations where matrices *B* and *D* are the unknowns and that we can rewrite as follows:

$$\left(M_1\ M_2 \ldots M_i\right) = \left(L_1\ L_2 \ldots L_i\right) \begin{pmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{pmatrix}$$

and rearranging

$$\underbrace{\begin{pmatrix} M_1 \\ M_2 \\ \vdots \\ M_i \end{pmatrix}}_{\in \mathbb{R}^{i(li-n) \times m}} = \underbrace{\begin{pmatrix} L_1 & L_2 & \cdots & L_{i-1} & L_i \\ L_2 & L_3 & \ldots & L_i & 0 \\ L_3 & L_4 & \ldots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ L_i & 0 & 0 & \ldots & 0 \end{pmatrix}}_{\in \mathbb{R}^{i(li-n) \times li}} \underbrace{\begin{pmatrix} I_l & 0 \\ 0 & \underline{\Gamma}_i \end{pmatrix}}_{\in \mathbb{R}^{li \times (l+n)}} \begin{pmatrix} D \\ B \end{pmatrix} \quad (12)$$

where $M_k \in \mathbb{R}^{(li-n) \times m}$ and $L_k \in \mathbb{R}^{(li-n) \times l}$. This system can then be solved with a least squared approach.

Once the model matrices *A*, *B*, *C* and *D* in equations (1) and (2) are estimated, the process model gain matrix can be computed by using the estimator

$$\hat{G}(1) = \hat{C}[I - \hat{A}]^{-1}\hat{B} + \hat{D}\,. \qquad (13)$$

3. MONITORING ALGORITHMS

Modern data acquisition allowed process control systems to become multivariable and provided the technological base to develop monitoring applications capable of a simultaneous surveillance of several related characteristics. This motivated the challenge for extending several single variable statistical methods to applications in multivariable systems. Today there are at least three multivariable control statistics that rise as extensions of previous single-variable versions: the statistical distance or Hotelling's T^2 proposed by Hotelling (1947), multivariate acumulated sum or MCUSUM proposed by Woodall and Ncube (1985) and Crosier (1988), and the multivariate exponetially weighted moving average MEWMA proposed by Lowry et al. (1992). All of them have received attention from the industry and the academy; thus, many authors and references should be mentioned for barely covering the later contributions. In this context, it is worth to recall that a main difference between the T^2 statistic and MCUSUM or MEWMA is that the first one defines a sequence of hypothesis tests based just on the last multivariate observation while the others use a collection of past data of the process employing a different forgetting mechanism.

Despite the fact that important algorithms like those mentioned above are available, the application example exposed in the following section emphasizes the inspection of plain gain estimators of a known multivariable linear system, with the purpose of highlighting main difficulties appearing during the block estimation through the subspace approach method.

In general, monitoring systems are based on statistics defined such to estimate main parameters of the model representing the system to be watched. If the distribution density function of this estimator is known, then, a Shewhart-type control chart can be easily set by defining an interval with a given confidence level. Appendix B presents a short review of concepts providing fundaments to the well known two-side control chart of the Shewhart type. In this framework and considering the final motivation of this work, the MPC model gains would be taken as plausible parameter values (θ_0) to set the hypotesis tests implicit in plain Shewhart control charts for monitoring significant changes.

4. SIMULATION RESULTS

The gain estimation method proposed here is tested by numerical simulation where the process plant is represented by the 5x3 LTI system given in Table 1. This arbitrary example is defined with more outputs than inputs becouse this is a frequent characteristic of actual process systems where MPC is applied.

$\frac{-0.8}{15s^2+3s+1}$	$\frac{4.7}{10s^2+7.8s+1}$	$\frac{-2.0}{9.14s+1}$
$\frac{1.4}{8s^2+5s+1}$	$\frac{0.8}{10s^2+3s+1}$	0.0
$\frac{1.5}{7.5s+1}$	$\frac{-0.3}{10s^2+4s+1}$	$\frac{0.9}{15s^2+10s+1}$
$\frac{4.0}{7.8s+1}$	0.0	$\frac{2.5}{5s+1}$
$\frac{1.9}{6s^2+4s+1}$	$\frac{5.0}{16s^2+5s+1}$	$\frac{-2.8}{9.8s^2+10s+1}$

Table 1: Multivariable linear system used for testing the proposed gain estimation method.

Like most identification procedures, the experience here starts by introducing a PRBS signal in the manipulated input variables (Gaikwad and Rivera, 1996) as shown if Figure 1. In order to approach the conditions of typical signals coming from actual process systems, Gaussian noise is added to the output variables with such intensity that the noise-to-signal ratio reaches 0.1 approximately (Huang and Kadali, 2008). The exciting random signals are conditioned to comply with the persistent excitation requirements and starting with a different seed for each input to avoid simultaneous similar sequences, i.e. to avoid correlations between inputs. The adopted sampling time is 2 min., but the estimations are calculated every 400 min only, each one using the last 400 observations of the input and output vectors. Hence, these are "moving" estimations calculated with 200 "new" observations and 200 "old" observations. This procedure was adopted to have a gain-matrix estimation every 6.6 hours without resigning a conveniently large number of observations.

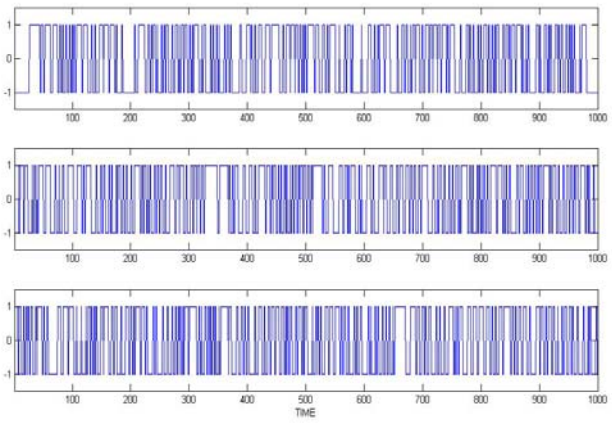

Figure 1: PRBS signals used in the input variables.

Two sequences of data were simulated. The first one assumes the system is stable and operating without receiving disturbances or gain changes; this set of data serves to desing the control limits for the individual

gains according with the analysis in Apendix B. Thus, for every gain $\theta_{ij} \equiv G_{ij}$, i = 1 to l; j = 1 to m, the limits are defined by

$$LS_{\theta_{ij}} = \bar{\theta}_{ij} + d_{(\alpha/2)}\hat{\sigma}_{\hat{\theta}_{ij}} = \bar{\theta}_{ij} + 3\hat{\sigma}_{\hat{\theta}_{ij}}$$

$$LI_{\theta_{ij}} = \bar{\theta}_{ij} - d_{(1-\alpha/2)}\hat{\sigma}_{\hat{\theta}_{ij}} = \bar{\theta}_{ij} - 3\hat{\sigma}_{\hat{\theta}_{ij}}$$

where subscripts i and j represent the output and input respectively, and the "3-sigma" convention is adopted for simplicity. The central line for these control charts are calculated using N = 49 estimations as follow:

$$\bar{\theta}_{ij} = \frac{\sum_{k=1}^{N} \hat{\theta}_{ij}(k)}{N},$$

and the standard deviation of the estimator is calculated by

$$\hat{\sigma}_{\hat{\theta}_{ij}} = \sqrt{\frac{\sum_{k=1}^{N} \left(\hat{\theta}_{ij}(k) - \bar{\theta}_{ij}\right)^2}{N-1}}.$$

The second data set is used for verification and demonstration of the ability to detect slow gain changes. Figure 2 shows the gain matrix estimations along 49 intervals of 200 mins; most of them show quite good accuracy though with different dispersion patterns. For instance, both gains G_{12} and G_{13} receive a 5% ramp change from time 3000 to 8000 min, i.e., from 7.5 to 20 in terms of the estimation number used in the horizontal axis in Figs. 2 to 4. Besides, G_{31} have also a 5% ramp change from time 4000 to 5000 (or equivalently 10 to 12.5 estimations time).

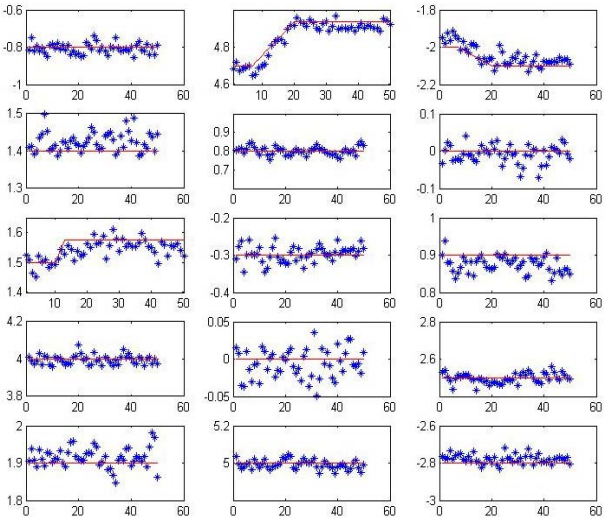

Figure 2: Responses of the gain matrix estimator vs. estimation number (one every 200 min) showing the effect of ramp changes in G_{12} G_{13} and G_{31} . Red lines indicate true gain values.

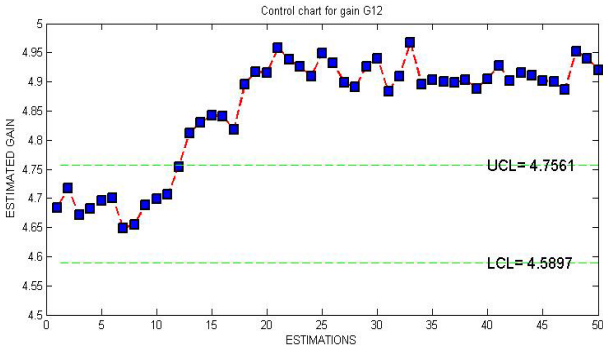

Figure 3: G_{12}-estimator response to a ramp change augmenting 5% the gain from 7.5 to 20 estimations. Green dash lines indicate 3-sigma control limits.

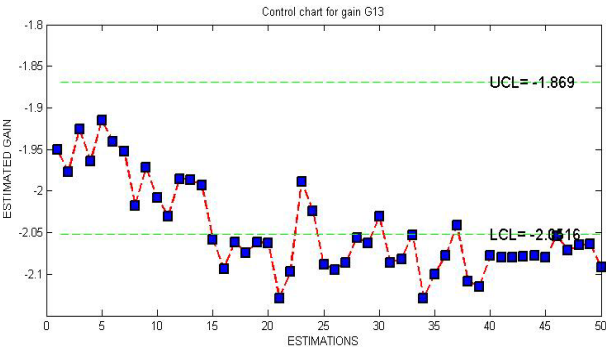

Figure 4: G_{13}-estimator response to a ramp change augmenting 5% the gain from 7.5 to 20 estimations. Green dash lines indicate 3-sigma control limits.

Though the specific changes are successfully detected – this is clearly observed in Figs. 3 to 5 – Figure 2 revels certain tendency for several individual estimators to present a bias. This exposes a problem to be analyzed in future works.

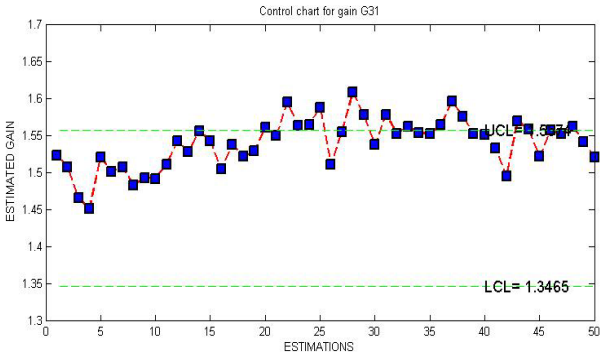

Figure 5: G_{31}-estimator response to a ramp change augmenting 5% the gain from 10 to 12.5 estimation time. Green dash lines indicate 3-sigma control limits.

5. CONCLUSIONS AND FUTURE WORK

The methodology presented here for online following of process gains shows promising results. However, several problems were highlighted by the presented analysis: i) slightly biased distribution functions with shapes doubtfully normal prevent from determining control limits associated to accurate detection rates or

false alarm frequencies; ii) several parameters used by the subspace identification method need to be optimized accordingly with the objectives; iii) the minimum necessary intensity of the persistent excitation when dealing with complex multivariable process systems is a pending duty to be face from a practical point of view; iv) the extension to closed loop identification appears as a mandatory alternative to be analyzed and developed.

APPENDIX A: Notation

The notation used in this paper follows that commonly used in the extensive literature available about subspace identification methods [Refs]. For instance, the block Hankel matrix of a single input signal is defined and written as

$$U_{0|2i-1} \triangleq \left(\begin{array}{ccccc} u_0 & u_1 & u_2 & ... & u_{j-1} \\ u_1 & u_2 & u_3 & ... & u_j \\ ... & ... & ... & ... & ... \\ u_{i-1} & u_i & u_{i+1} & ... & u_{i+j-2} \\ \hline u_i & u_{i+1} & u_{i+2} & & u_{i+j-1} \\ u_{i+1} & u_{i+2} & u_{i+3} & & u_{i+j} \\ & & & & \\ u_{2i-1} & u_{2i} & u_{2i+1} & & u_{2i+j-2} \end{array}\right) = \left(\frac{U_{0|i-1}}{U_{i|2i-1}}\right) = \left(\frac{U_p}{U_f}\right) = \left(\frac{U_{0|i}}{U_{i+1|2i-1}}\right) = \left(\frac{U_p^+}{U_f^-}\right)$$

where the indexes i and j are such that $i \geq n$ and $j \gg n$, and n is the assumed order of the system. Note that indexes p and f stand for "past" and "future" respectively; this is becouse each column of matrix U_p composes of i elements previous to the following i elements being part of U_f. This matrix structure is also extended to the block Hankel matrix for a single output variable, in this case using the notation $Y_{0|2i-1}, Y_p, Y_f$, Y_p^+ and Y_p^-. The above notation serves also to define the block Hankel matrix of past input and output data, i.e.,

$$W_p \triangleq \begin{pmatrix} U_p \\ Y_p \end{pmatrix}.$$

In a similar way, the state sequence

$$X_i = \begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-2} & x_{i+j-1} \end{pmatrix} \in \mathbb{R}^{n \times j},$$

is partitioned by setting $X_p = X_0$ and $X_f = X_i$.

The subspace identification algorithm used in this paper needs of other two important matrices: the observability matrix $\Gamma_i \in \mathbb{R}^{li \times n}$,

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{pmatrix}$$

and two low-triangular block Toeplitz, $H_i^d \in \mathbb{R}^{li \times mi}$ and $H_i^s \in \mathbb{R}^{li \times mi}$, which are written as follows:

$$H_i^d = \begin{pmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \cdots & D \end{pmatrix}$$

and

$$H_i^s = \begin{pmatrix} I & 0 & 0 & \cdots & 0 \\ CK & I & 0 & \cdots & 0 \\ CAK & CK & I & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ CA^{i-2}K & CA^{i-3}K & CA^{i-4}K & \cdots & I \end{pmatrix}$$

Assuming the pair $\{A, C\}$ is observable and $i \geq n$, then Γ_i is a full column range matrix, i.e., $Rank(\Gamma_i) = n$.

Finally, the notation A/B stands for the orthogonal proyection of the row space of A into the row space of B,

$$A/B \triangleq A\Pi_B = AB^T\left(BB^T\right)^\dagger B$$

Besides, $A/_C B$ denotes the oblique proyection of the row space of A on the direction of the row space of C into the row space of B,

$$A/_B C \triangleq \left[A/B^\perp\right]\left[C/B^\perp\right]^\dagger C$$

APPENDIX B: Confidence interval for a single estimator.

Assume θ is a parameter to be estimated using the estimator $\hat{\theta}$ whose random behavior is described by the probability density function $f(\hat{\theta})$. Then, the probability of $\hat{\theta}$ for being lower or grater than a lower and upper limit (UL and LL) respectively, can be defined as follows:

$$\alpha/2 = \int_{-\infty}^{LL} f(\hat{\theta})d\hat{\theta} = P(\hat{\theta} < LL)$$

$$1-\alpha/2 = \int_{-\infty}^{UL} f(\hat{\theta})d\hat{\theta} = P(\hat{\theta} \leq UL)$$

This is equivalent to say that the probability for $\hat{\theta}$ to fall inside the given interval is given by

$$P\left\{LL \leq \hat{\theta} \leq UL\right\} = 1-\alpha$$

For convenience, the estimator is normalized to have cero mean and unit variance, then rewriting the above expression we can arrive to

$$P\left\{-d_{(\alpha/2)} \leq \frac{\hat{\theta} - E\left\{\hat{\theta}\right\}}{\sigma_{\hat{\theta}}} \leq d_{(1-\alpha/2)}\right\} = 1-\alpha \qquad \text{(B1)}$$

where the quantities

$$d_{\alpha/2} = \frac{E\left\{\hat{\theta}\right\} - LL}{\sigma_{\hat{\theta}}} \qquad d_{1-\alpha/2} = \frac{UL - E\left\{\hat{\theta}\right\}}{\sigma_{\hat{\theta}}}$$

are the "$\alpha/2$ percent points" of the normalized distribution. Note that these quantities are completely defined by α and the normalized distribution function (with 0 mean and unit deviation). Let us analyze now the general case in which we deal with a biased estimator, i.e.,

$$E\left\{\hat{\theta}\right\} = \theta + b, \quad |b| \geq 0 . \tag{B2}$$

where *b* is the bias. Note that if for a given estimator $\hat{\theta}$, *b* = 0 can not be analytically demonstrated, the true population parameter θ can never be accurately estimated becouse the true bias *b* remains also uncertain. Then, in this case, all what we can do is to work in terms of an assumed, or expected, value θ_0, so we write

$$E\left\{\hat{\theta}\right\} = \theta_0 + b_0, \quad |b_0| \geq 0 . \tag{B3}$$

where all the uncertainty has been sent into the bias b_0. The above reasoning tells that through $\hat{\theta}$ we can only monitor the parameters characterizing the population of $\hat{\theta}$ values, and detect operating conditions producing significant changes in the observed values. This is equivalent to following parameter changes by using the bias estimation $\hat{b}_0 = \hat{\theta} - \theta_0$ referred to the specific value θ_0. In other words, two equivalent confidence intervals can be written, one directly in terms of the parameter estimator, and the other in terms of the bias estimator, as follows:

$$LL_{\hat{\theta}} = \hat{\theta} - d_{(1-\alpha/2)}\hat{\sigma}_{\hat{\theta}} \leq E\left\{\hat{\theta}\right\} \leq \hat{\theta} + d_{\alpha/2}\hat{\sigma}_{\hat{\theta}} = UL_{\hat{\theta}} \tag{B4}$$

$$LL_{\hat{b}} = \hat{b}_0 - d_{(1-\alpha/2)}\hat{\sigma}_{\hat{\theta}} \leq E\left\{\hat{b}_0\right\} \leq \hat{b}_0 + d_{\alpha/2}\hat{\sigma}_{\hat{\theta}} = UL_{\hat{b}}, \tag{B5}$$

both defined for a (1-α)% confidence level.

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