# COMPONENT FAULT DETECTION AND ISOLATION COMPARISON BETWEEN BOND GRAPH AND ALGEBRAIC APPROACH.

# Samir BENMOUSSA, Belkacem OULD BOUAMAMA, Rochdi MERZOUKI

LAGIS, FRE CNRS 3303, Polytech'Lille, Avenue Paul Langevin, 59655 Villeneuve d'Ascq, FRANCE Samir.benmoussa@polytech-lille.fr

# ABSTRACT

This paper makes a comparison of component fault detection and isolation between an algebraic approach and a bond graph one. The conditions of component fault detection and isolation are viewed in algebraic approach as an observation problem of the fault with respect to the input and the output. In Bond graph approach, these conditions are performed by analyzing the causal paths from faults to outputs using the notion of bicausality. It is shown that the use of bicausal bond graph helps to integrate many mathematical approaches particularly the algebraic one. The component fault detection and isolation performed from bond graph is much simpler as compared to the algebraic approaches in which an analytical model is needed and complex computations are performed to determine the diagnosability conditions. An illustrative example is given to show the efficiency and the simplicity of the bond graph approach compared to the algebraic one.

Keywords: Fault detection and isolation, Bond graph, Algebraic approach.

# 1. INTRODUCTION

Fault Detection and Isolation (FDI) has become an important tool in the ingredients of a modern automatic system. Its significance is based on enhancement in terms of safety, reliability, dependability and operating costs of the plant. And for that, FDI has been a widely exploited research topic in the recent years, and several methods have been developed among them ones based on the model of the system such as: parity equations (Gertler, 1997), observer model-based (Patton and Chen, 1997), and analytical redundancy relations (Staroswiecki and Comet-Varga, 2001).

Several FDI model-based approaches can be found in the literature. Among them, the algebraic approach (Fliess and Join, 2003; Fliess and Join, 2004; Cruz-Victoria, Martinez-Guerra and Rincon-Pasaye, 2008) consists of the ability of the detection, the identification, and the estimation of the fault variable. It is viewed as an observation problem of the latter with respect to the input and the output variables. These approaches are applied to linear and a class of nonlinear systems to detect sensor and actuator faults. Other approaches are based on structured and graphical models. They intend to create a graph that describes the mathematical model of the system. These approaches are based on a digraph G(S;A), where nodes S represent the state, the input, and the measurement output variables, and edges A are the interaction between these nodes. In (Commault, Dion, and Agha, 2008), the authors study the sensors location problem for internal faults in term of separator in the associated graph of the structured system. This separator gives the necessary and sufficient sensors to be added. Fault detection and isolation based on digraph has limits that it concerns only actuator and sensor faults.

The Bond Graph (BG) which is also a graph, can be an alternative for plant fault detection and isolation, since the nodes S represent not only state, input, and output variables but also the physical components of the system. It describes the power transfer between the passive and active components of multi-physical systems. It is the interface between the physical system and the mathematical model of the last.

Several problems have been solved structurally using this graphical approach, such as: observability and controllability (Sueur and Dauphin-Tanguy, 1991), system inversion (Ngwompo and Gawthrop, 1999), and FDI (El-Osta, Ould Bouamama and Sueur, 2006; Samantary and Ghosal, 2008). FDI BG model-based uses the analytical redundancy relations (ARR) which are generated from the BG model of the system using its causal and structural proprieties. A fault signature matrix (FSM) is defined to study the fault monitorability and isolability. This matrix depends mainly on the number of sensors, and the unknown variables to be eliminated, which is complex and difficult to apply for a nonlinear system.

This paper makes a comparison of component fault detection and isolation between an algebraic approach and a BG one, in which plant faults are modeled as additional modulated inputs to the system. The conditions of the detectability and isolability of plant faults are performed on the BG model using the notion of bicausality and disjoint causal path analysis. In this work, the detectability and the isolability of the fault are performed directly on the BG model without computations, which are usually complex. Since BG model is a graphical representation of the physical system, the concept of fault is much more physical than mathematical, contrary to the algebraic approach.

The paper is structured as follows: in Section 2 the problem statement. Conditions of detectability and isolability by algebraic approach are given in Section 3. And by BG approach in section 4. An illustrative example is introduced in Section 5. The paper ends with general synthesis and remarks.

# 2. PROBLEM STATEMENT

Let us consider a class of linear time invariant systems described by the following state space format as:

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u) \\ y(t) = Cx(t) \end{cases}$$
(1)

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the input vector,  $y(t) \in \mathbb{R}^p$  is the output vector, and *A*, *B*, *C* are matrices of appropriate dimensions.

The vector  $F \in \mathbb{R}^q$  such that  $q \leq p$ , represent internal components failure in the plant, are introduced in the state space format as:

$$\begin{cases} \dot{x}(t) = (A+F)x(t) + B(u) \\ y(t) = Cx(t) \end{cases}$$
(2)

Note that faults on inputs are not considered in this paper and we assume that all sensors are fault free. The system of Eq.(2) can be rewritten as:

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u) + MF(x,t) \\ y(t) = Cx(t) \end{cases}$$
(3)

where  $M \in \mathbb{R}^{n \times q}$  is a known matrix, and F is an unknown vector that needs to be detected.

In the next sections, the conditions of the detectability and isolability of the fault *F* are given.

# 3. FAULT DETECTION AND ISOLATION BASED ON AN ALGEBRAIC APPROACH

#### 3.1. Basics in algebraic approach

System  $\Lambda^{pert}$  is defined as a finitely generated free k[s]module, where k[s] is a commutative principal ideal domain of linear differential operators of the form  $\sum_{finite} c_v s^v, c_v \in k$ ; s is the usual symbol of derivation, and

k is the field of real or complex numbers.

In  $\Lambda^{pert}$  two finite subsets are distinguished, the fault variable *F* and the perturbation variable  $\pi$ , which do not 'interact', i.e.,  $span_{k[s]}(F) \cap span_{k[s]}(\pi) = \{0\}$ .

The nominal system is defined by:

$$\Lambda = \Lambda^{pert} / span_{k[s]}(\pi).$$

**Definition:** An input output system is a linear system,  $\Lambda^{pert}$  equipped with an input *u* and an output *y*, such that:

- The input of the linear system  $\Lambda^{pert}$  is a finite sequence  $u = (u_i), 1 \le i \le m$  of elements of  $\Lambda^{pert}$  such that  $\Lambda^{pert} / [u]_{k[s]}$  is torsion, the input u is assumed independent.
- The output of the linear system  $\Lambda^{pert}$  is a finite sequence  $y = (y_i), 1 \le i \le p$  of elements of  $\Lambda^{pert}$ .

**Assumptions**: the following propreties are assumed to be satisfied:

- $span_{k[s]}(u)span_{k[s]}(\pi) = \{0\},\$

This means that the control variable u does not interact with the perturbation and the fault variables.

# 3.2. Fault detection and isolation

The system described by Eq.(3) is an input output system  $\Lambda^{pert}$ .

**Theorem 1:** The system of Eq.(3) is observable (in the sense that the state is observable with respect to u and y), then it is diagnosable if, and only if, F is observable with respect to u, y, and x (Diop and Martinez-Guerra, 2001).

**Definition** (Algebraic Detectability): a fault F is said to be detectable if, it is observable over u and y (Fliess and Join, 2003).

**Definition** (Algebraic Isolability): any fault variable in F is said to be isolable if, and only if, there exists a system of parity equation (Fliess and Join, 2003)

$$M\begin{pmatrix}F_{1}\\\vdots\\F_{q}\end{pmatrix} = Q\begin{pmatrix}u_{1}\\\vdots\\u_{m}\end{pmatrix} + S\begin{pmatrix}y_{1}\\\vdots\\y_{p}\end{pmatrix}$$
(4)

where  $M \in k[s]^{q \times q}, Q \in k[s]^{q \times m}, S \in k[s]^{q \times p}, \det P \neq 0$ .

In other words, it is required that:

- The system must be observable: the states of the system can be expressed as a function of outputs and their derivatives,
- Each fault variable has to be written under a polynomial equation format  $F_i$  and finitely many time derivatives of u and y with coefficients in k[s].

$$\varphi(F_i, u, \dot{u}, ..., y, \dot{y}, ...) = 0$$
 (5)

# 4. FAULT DETECTION AND ISOLATION BASED ON BICAUSAL BOND GRAPH

# 4.1. Graph and Bond Graph

A graph theory approach is used to study and analyze structured systems which are independent of the system parameter values. This approach requires a low computation which allows dealing with large scale systems. The existing contributions related to the graph analysis proved that observability, controllability, inputoutput decoupling ... etc, can be simply deduced from the structural properties of the graph. There are different graphical methods used in the literature: Digraph (Dion, 2003), signed digraph (Maurya, Rengaswamy, and Venkatasubramanian, 2004), bipartite graph (Blanke, Kinnaert, Lunze, and Staroswiecki, 2003), and BG (Samantary, and Ghoshal, 2008).

**Definition**: The digraph, denoted by G(S;A), is deduced from state space equations. It is composed by a set of nodes (*S*),  $S = \{U \cup Y \cup X\}$  which corresponds to the system inputs, outputs, and states. The interactions between these nodes are represented by directed edges (*A*).

**Definition:** A graph G(S;A) is bipartite, if its vertices can be partitioned into two disjoint subsets Z (set of variables that defines the dynamic behavior of the system), and C (set of equations that defines the relations among the variable set),  $S = \{C \cup Z\}$ . The relations between these two subsets are represented by edges (A).

**Definition**: The BG which is also a graph G(S;A), is a unified graphical language for multi-physical domains. Unlike the others graphs mentioned above, the nodes *S* represent physical components, subsystems, and other basic elements called junctions. While the edges *A*, called power bonds represent the power exchanged between nodes.

In BG, the possible set of components  $S = \{R \cup C \cup I \cup TF \cup GY \cup Se \cup Sf \cup De \cup Df \cup J\}.$ 

The *R*- element represents passive energy dissipation phenomena, while *C*, and *I* model the passive energy storage elements. (*Se*), and (*Sf*) are the sources of effort and flow, respectively. Sensors are represented by flow (*Df*), and effort (*De*) detectors. Finally, *J* (which can be a 0 or a 1 junction), is used to connect the elements having the same effort (1-junction), or flow (0junction). The conservation of energy laws are obtained from the latter. *TF*, and *GY* are used to represent transformers and gyrators, respectively.

The difference between BG and the other graphical approaches is that the former is directly generated from the physical system, and not from state space equations. In addition, from the BG model state space equations can be generated manually using systematic way (Mukherjee, and Karmakar, 2000) or automatically from dedicated software such as *Symbol2000* (Ould-Bouamama, Medjaher, Samantaray, and Dauphin-

Tanguy, 2005) or 20-sim (Twentesim, 1996). Furthermore, system components are clearly represented in the BG model because of its graphical architecture.

#### 4.2. Multiplicative fault modeling on BG

The component fault is modeled using bond graph as an additional modulated input (MSe for 1- junction, MSf for 0-junction), placed at the same junction of the component element. Let consider as an illustrative example, a resistive (R) element in resistive causality with the following characteristic equation

$$e = R.f \tag{6}$$

If the element *R* is faulty, then an additional value  $R_f$  is added to the nominal value  $R_n$ , Fig.(1)-(a), so Eq.(6) can be rewritten as

$$e = R_n \cdot f + R_f \cdot f = e_n + e_d \tag{7}$$

where  $e_d$  is the effort brought by the fault, which is unknown time function, can be considered as an additional modulated input added to the 1-junction as given in Fig.1-(b).



Figure 1 : (a) The BG interpretation of Eq.(7); (b) Component fault modeling in BG.

# 4.3. Bicausal Bond Graph

The Bicausal BG is introduced to study control problems such as system inversion, state estimation, and unknown parameter estimation (Gawthrop, 2000). It overcomes the assignment statements that cannot be derived from the constraint equations of a so-called 'unicausal' bond graph model, Fig.2-(a), which implies two assignment statements :

$$e_1 \coloneqq e_2; \quad f_2 \coloneqq f_1 \tag{8}$$

the effort  $e_2$  and the flow  $f_1$  are used to determine the effort  $e_1$  and the flow  $f_2$  respectively, and they have an opposite direction. Contrary to 'unicausal' BG, bicausal one imposes the same direction to the effort and the flow, Fig.2-(b), which implies two assignment statements

$$e_2 \coloneqq e_1; \quad f_2 \coloneqq f_1 \tag{9}$$

Causal half strokes indicate the fixed or known variables of the bond, and determine the right hand side of the assignment statements form.



Figure 2 : (a) A bond with causality; (b) a bicausal bond.

Bicausal introduces also some additional BG elements (Ngwompo and Gawthrop, 1999), among which SS (Source-Sensor), AE (Amplifier of effort), and AF (Amplifier of flow). Table.1 gives causality assignment for Source-Sensor element.

Table 1: Source / Sensor causality assignment

'unicausal stroke'	SS element type	'Bicausal stroke'	SS element type
SS	Flow source/ Effort sensor	└── <b>&gt;</b> SS	Flow source/ Effort source
⊨	Effort source/ Flow sensor	r—→ ss	Flow sensor/ Effort sensor

The concept of bicausality allows fixing or imposing at the same time a variable and its conjugate as a bicausal bond thus decoupling the effort and flow causalities.

In the context of fault detection and isolation, imposing the output variable without modifying the energy structure (or constraint equations) of the system can be effectuated with an *SS* element having a flow source/effort source causality (Table 1). Then the output to be imposed plays the role of input variable of that *SS* element while its conjugate is set to a null value leading to null power propagation on that bond. Similarly, the fault variable to be isolated will be observed on another *SS* element with flow sensor/effort sensor causality.

# 4.4. Fault detection and isolation based on bicausal Bond Graph

The bicausal BG used to study fault detection and isolation is obtained by applying Sequential Causality Assignment Procedure for Inversion (SCAPI) algorithm as for system inversion (Ngwompo, Scavarda and Thomasset, 1996) on BG model in integral causality by replacing the input variable by the fault one, and assigning a derivative causality instead of an integral one.

Consider *n* as the number of the storage elements in integral causality, and *a* as the number of storage elements on the shortest path between the input and the output, which will take the derivative causality when the causality assignment algorithm for FDI is applied. n-a is the number of storage elements which are not on the shortest path, and which keep the same causality as they were in the BG in preferred integral causality.

# Proposition 1: (fault detectability)

If all storage elements take the derivative causality even ones that are not on the shortest path (n-a=0), so the fault variables modeled by modulated inputs are observable (detectable) with respect to the input u and the output y variables.

**Proof:** It was shown in (Sueur and Dauphin-Tanguy, 1991) that a system modeled by BG is observable when all the storages elements are derivative causality, so the inputs (sources or faults), and the states variables can be expressed in term of the output variable and their derivative.

On the Bicausal Bond graph, the fault isolability is done by analyzing the causal path from the SS element associated to the output variable to the SS element associated to the fault variable through the storage elements.

**Definition:** Two causal paths are said to be disjoint if, and only if, they do not share a common variable (Ngwompo and Gawthrop, 1999).

# Proposition 2: (fault isolability)

q faults are structurally isolable if, and only if, there are q disjoint causal paths linking the sensor to the fault through all storage elements C, and I that exist in the path.

**Proof:** Let consider *p* the number of sensors and *q* the number of component faults, such as  $q \le p$ . The *i*<sup>th</sup> causal path, where i = 1...q, links the component fault  $(F_i)$  which is represented in BG model by SS element to the  $y_i$  sensor, has *d* storage elements  $C_{l,i}$  and  $I_{l,i}$ , where l = 1...d. The covering causal path from the sensor (SS element) to the fault input (SS element) leads to the following oriented graph where  $x_{l,i}$  is the state variable, corresponds to the storage elements in BG model. From Fig.3, the algebraic equations Eq.(10) can be derived.



Figure 3 : Digraph representation of disjoint causal paths.

$$F_{i} = h_{i} \left( y_{i}, \dot{y}_{i}, \dots, y_{i}^{(d)} \right)$$
  

$$\vdots$$
  

$$F_{j} = h_{j} \left( y_{j}, \dot{y}_{j}, \dots, y_{j}^{(d')} \right)$$
(10)

The number of derivation d and d' corresponds to the number of the storage elements that belong to  $i^{th}$  and  $j^{th}$  causal path respectively.

It is clear that Eq.(10) satisfies Eq.(4), so the q component faults are isolable.

**Remark:** if one storage element remains in the integral causality when the assignment causality algorithm for FDI is applied on the BG model of the system, the q faults are not isolable even if there is a q disjoint causal paths.

# 5. ILLUSTRATIVE EXAMPLE

In this section, we will apply the presented methodology to detect and isolate component faults on the electromechanical system of the Fig.4.



Figure 4: The electromechanical system

The electromechanical system is composed of three principal parts: A DC motor, a gear, and a wheel part. The DC motor is a combination of electrical and mechanical domains. The gear is concerned with connecting the mechanical par with the load one (wheel + ground).

### 5.1. Case 1

In the first case, two faulty components R: Re and R:  $f_e$  are considered. The faults are modeled by a modulated source of effort, Fig.5. The available sensors for the current, and the wheel velocity, represented on the BG model in integral causality by Df:  $y_1$  and Df:  $y_2$ .



Figure 5 : The BG model of the faulty system of case 1 in integral causality.

#### 5.1.1. Algebraic isolability:

The state space form of the system can be derived from the BG model of the system in integral causality, it is given by the Eq.(11)

The faults  $F_1$  and  $F_2$  are detectable and isolable if, and only if, they satisfy the parity equation Eq.(4). As presented in the Eq.(12), the latter is verified, so the faults  $F_1$  and  $F_2$  are detectable and isolable.

#### 5.1.2. Bicausal BG detectability:

The system given in bicausal BG of the Fig.6 is diagnosable since all I and C elements are in derivative causality which means that the fault F can be expressed in terms of the input u and the output y. We can add that all states are observable with respect to u and y and their derivatives.

$$\begin{vmatrix} \dot{x}_{1} = -\frac{Re}{L} x_{1} - \frac{k_{e}}{J_{e}} x_{2} + u_{1} + F_{1} \\ \dot{x}_{2} = \frac{k_{e}}{L} x_{1} - \frac{f_{e}}{J_{e}} x_{2} - Kx_{3} + F_{2} \\ \dot{x}_{3} = \frac{1}{J_{e}} x_{2} - \frac{N}{J_{s}} x_{4} \\ \dot{x}_{4} = NKx_{3} - \frac{f_{s}}{J_{s}} x_{4} + u_{2} \\ y_{1} = \frac{x_{1}}{L} \\ y_{2} = \frac{x_{4}}{J_{s}} \end{vmatrix}$$
(11)

$$\begin{cases} F_{1} = (sL + Re) y_{1} + \frac{k_{e}}{NK} (s^{2}J_{s} + sf_{s} + N^{2}K) y_{2} \\ -u_{1} - s \frac{k_{e}}{NK} u_{2} \\ F_{2} = -k_{e}y_{1} + \left[ s^{3} \frac{J_{e}J_{s}}{NK} + s^{2} \frac{1}{NK} (f_{s}J_{e} + f_{e}J_{s}) + s \left( NJ_{e} + \frac{f_{e}f_{s}}{NK} + \frac{J_{s}}{N} \right) + \left( Nf_{e} + \frac{f_{s}}{N} \right) \right] y_{2} \\ - \left[ s^{2} \frac{J_{e}}{NK} + s \frac{f_{e}}{NK} + \frac{1}{N} \right] u_{2} \end{cases}$$
(12)



Figure 6 : The bicausal BG model of the faulty system of the case 1.

## 5.1.3. Bicausal BG isolability:

The Bicausal BG isolability is performed by analyzing the the causal paths from the outputs to the faults. For the faults to be isolable, it is necessary that there are disjoint causal paths equal to number of faults. In other words, q component faults are isolable if, and only if, there are q disjoint causal paths from the output to the fault passing through the storage elements (I, C) that exist in the causal path.

From the Fig.6, there are only two disjoint causal paths from the SS:  $y_1$  to SS:  $F_1$  and, SS:  $y_2$  to SS:  $F_2$ 

$$SS: y1 \to I: L \to SS: F_1$$
  

$$SS: y2 \to I: J_s \to C: \frac{1}{K} \to I: J_e \to SS: F_2$$
(13)

then, the two faults  $F_1$  and  $F_2$  are isolable.

From the bicausal bond graph, the order of derivation is also obtained, it is equal to the number of the storage elements in the path that links the output to the fault: in the expression of the fault  $F_1$ , the output  $y_1$  is derived one time n = 1, while for the fault  $F_2$ , the output  $y_2$  is derived three times n = 3.

By following the previous conditions, one is able to state the same conclusion as the algebraic approach ( $F_1$  and  $F_2$  are isolable) without any calculations.

Now, we are going to show how from BG model, the same equations as Eq.(12) are obtained.

#### 5.1.4. Fault indicator determination:

The fault indicator is derived by writing causal relations between BG variables (constraints and characteristic equations). For fault  $F_i$ , it is given by:

From the first 1-junction, the following equation can be derived

$$e_{F1} = -e_1 + e_2 + e_3 + e_4 \tag{14}$$

The efforts  $e_2$ ,  $e_3$ , and  $e_4$  are expressed in function of the input, and the output by following the causal path between the former and the latter. The effort  $e_2$  is linked to the output *SS*:  $y_1$  by the causal path of the Eq.(15), and it is given by  $e_2 = R_e f_{y_1}$ 

$$f_3 \to f_2 \to e_2 \tag{15}$$

The same reasoning for the effort  $e_3$ , it is given by:

$$e_3 = L\frac{d}{dt}f_{y1}$$

The effort  $e_4$  is linked to the output SS:  $y_2$  (There is no path between  $e_4$  and SS:  $y_1$ ) by

$$f_{y2} \to f_{12} \to e_{12} \to e_{11} \to e_{10} \to e_9 \to f_8 \to f_7 \to f_5 \to e_4$$
(16)

So, the effort  $e_4$  is given by:

$$e_{4} = \frac{k_{e}}{NK} \left( J_{s} \frac{d^{2}}{dt^{2}} f_{y2} + f_{s} \frac{d}{dt} f_{y2} + N^{2}K f_{y2} - \frac{d}{dt} u_{2} \right)$$
(17)

The fault indicator  $e_{FI}$  takes the following expression:

$$e_{F1} = R_e f_{y1} + L \frac{d}{dt} f_{y1} + \frac{k_e}{NK} \left( J_s \frac{d^2}{dt^2} f_{y2} + f_s \frac{d}{dt} f_{y2} + N^2 K f_{y2} - u_1 - \frac{k_e}{NK} \frac{d}{dt} u_2 \right)$$
(18)

From the second 1- junction, the  $e_{F2}$  is given by:

$$e_{F2} = -e_5 + e_6 + e_7 + e_8 \tag{19}$$

Same reasoning as for fault  $F_1$ , the effort  $e_5$ ,  $e_6$ ,  $e_7$ , and  $e_8$  are given by following the causal path between the former and a given output. They are given by:

$$e_{5} = k_{e} f_{y_{1}}$$

$$e_{6} = \frac{fe}{KN} \left( Js \frac{d^{2}}{dt^{2}} f_{y_{2}} + fs \frac{d}{dt} f_{y_{2}} + N^{2} K f_{y_{2}} - \frac{d}{dt} u_{2} \right)$$

$$e_{7} = \frac{J_{e}}{KN} \left( Js \frac{d^{3}}{dt^{3}} f_{y_{2}} + fs \frac{d^{2}}{dt^{2}} f_{y_{2}} + N^{2} K \frac{d}{dt} f_{y_{2}} - \frac{d^{2}}{dt^{2}} u_{2} \right)$$

$$e_{8} = \frac{1}{N} \left( J_{s} \frac{d}{dt} f_{y_{2}} + f_{s} f_{y_{2}} - u_{2} \right)$$
(20)

Thus, the expression of the fault variable  $F_2$  is:

$$e_{F2} = -k_e f_{y1} + \left[ s^3 \frac{J_e J_s}{NK} + s^2 \frac{1}{NK} \left( f_s J_e + f_e J_s \right) \right]$$
$$+ s \left( NJ_e + \frac{f_e f_s}{NK} + \frac{J_s}{N} \right) + \left( Nf_e + \frac{f_s}{N} \right) f_{y2} \qquad (21)$$
$$- \left[ s^2 \frac{J_e}{NK} + s \frac{f_e}{NK} + \frac{1}{N} \right] MSe_2.$$

To get the same algebraic equations as Eq.(12),  $e_{F1}$ ,  $e_{F2}$ ,  $Se_1$ ,  $MSe_2$ ,  $f_{y1}$ , and  $f_{y2}$  are substituted by  $F_1$ ,  $F_2$ ,  $u_1$ ,  $u_2$ ,  $y_1$ , and  $y_2$  respectively.

# 5.2. Case 2

In this case, three faults are considered, these faults are on R:  $R_e$ , R:  $f_e$ , and R:  $f_s$ . The available sensors are the current, the rotor velocity, and the wheel velocity.



Figure 7 : The faulty system of the case 2.

#### 5.2.1. Algebraic detectability and isolability:

The three faults cannot be expressed with respect to input and the output as shown in Eq(22).

The system is not diagnosable since the state  $x_3$  is not observable with respect to the input and the output, and the three faults are neither detectable nor isolable.

$$\begin{cases} F_1 = (sL + R_e) y_1 + k_e y_2 - u_1 \\ F_2 = -k_e y_1 + (sJ_e + f_e) y_2 + Kx_3 \\ F_3 = -NKx_3 + (sJ_s + f_s) y_3 - u_2 \end{cases}$$
(22)

# 5.2.2. Bicausal BG detectability and isolability:

The BG model of the system with the faulty elements is given in integral causality in Fig.7, and in bicausal one in Fig.8



Figure 8 : The bicausal BG of the faulty system of case 2.

From Fig.8, one storage element remains in integral causality when the causality assignment algorithm for FDI is applied on the system in integral causality of Fig.7, so the system is not diagnosable since proposition 1 is not satisfied.

The faults are not also isolable even if there are three disjoint causal paths, which link the outputs to the faults.

# CONCLUSIONS

It is shown in this paper that the detection and isolation of plant faults based on bond graph model is simpler than an algebraic approach. The latter is due to the fact that it is based on structural and graphical analysis of the model. The two approaches have similar results expect that bond graph model needs no complex computation compared to an algebraic approach.

#### REFERENCES

- Gertler, J., 1997. Fault detection and isolation using parity relations. Control Engineering Practice, 5:653-661.
- Patton, R.J., and Chen, J., 1997. observer-based fault detection and isolation : robustness and applications. Control Engineering Practice, 5:671-682.
- Staroswiecki, M., and Comtet-Varga, G., 2001. Analytical redundancy relations for fault detection and isolation in algebraic dynamic systems. automatica, 37:687–699.
- Fliess, M., and Join, C., 2003. An algebraic approach to fault diagnosis for linear systems. In CESA.
- Fliess, M., Join, C., and SIRA-RAMIREZ, H., 2004. Robust residual generation for linear fault diagnosis: an algebraic setting with examples. International journal of control, 77:1223–1242.
- Cruz-Victoria, J.C., Martinez-Guerra, R., and Rincon-Pasaye, J.J., 2008. On nonlinear systems diagnosis using differential and algebraic methods. Journal of the Franklin Institute, 345:102–117.
- Commault, C., Dion, J.M., and Agha, S.Y., 2008. Structural analysis for the sensor location problem in fault detection and isolation. Automatica, 44:2074–2080.

- Sueur, C. and Dauphin-Tanguy, G., 1991. Bond-graph approach for structural analysis of mimo linear systems. Journal of the Franklin Institute, 328:55– 70.
- Ngwompo, R.F. and Gawthrop, P.J., 1999. Bond graph-based simulation of non-linear inverse systems using physical performance specifications. Journal of the Franklin Institute, 336:1225–1247.
- El-Osta, W., Ould Bouamama, B. and Sueur, C., 2006. Monitorability indexes and bond graphs for fault tolerance analysis. In Safeprocess IFAC, Bejing, Chine, 29-1 Sept.
- Samantary, A.K., and Ghoshal, S.K., 2008. Bicausal bond graph for supervision : from fault detection and isolation to fault accommodation. Journal of the Franklin Institute, 345:1–28.
- Diop, S. and Martinez-Guerra, R., 2001. On an algebraic and differential approach of nonlinear systems diagnosis. In proceedings of the 40<sup>th</sup> IEEE : Conference On Decision and Control, 585– 589, Orlando, USA.
- Dion, J. M.,2003. Generic properties and control of linear structured systems: a survey. Automatica, 39:1125–1144.
- Maurya, M. R., Rengaswamy, R., and Venkatasubramanian, V., 2004. Application of signed digraphs-based analysis for fault diagnosis Of chemical process flowsheets. Engineering Applications of Artificial Intelligence, 17(5):501– 518.
- Blanke, M., Kinnaert, M., Lunze, J., and Staroswiecki, M., 2003. Diagnosis and Fault-Tolerant Control. Springer.
- Samantary, A.K., and Ghoshal, S.K., 2008. Bicausal bond graph for supervision : from fault detection and isolation to fault accommodation. Journal of the Franklin Institute, 345:1–28.
- Mukherjee, A., and Karmakar, R., 2000. Modelling and Simulation of Engineering Systems through Bondgraphs. Narosa, 1<sup>st</sup> edition.
- Ould-Bouamama, B., Medjaher, K., Samantaray, A.K., and Dauphin-Tanguy, G., 2005. Model builder using functional and bond graph tools for FDI design. Control Engineering Practice (CEP) journal, 13(7):875-891.
- Twentesim, 1996. Users Manual of Twentesim (20sim). Comtollab Products Inc, Box 217 NL-7500AEEnschede, Nederland.
- Gawthrop, P.J., 2000. Estimating physical parameters of nonlinear systems using bond graph models. In SYSID.
- Ngwompo, R.F., Scavarda, S. and Thomasset, D., 1996. Inversion of linear time-invariant siso systems modelled by bond graph. Journal of the Franklin Institute, 336:157–174.