
Design of a trajectory generator for a hexapod walking robot

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ABSTRACT

We present in this paper an original investigation on a walker hexapod robot for which we define a locomotion strategy considering the walk appearance characterized by the succession in time of the synchronized events of legs putting down and raising. The legs movements' synchronization is determined by the temporal phase displacement between the instants of legs putting down for which we define, for each one, the ratio between the duration passed spent on the ground and on the duration devoted to the replacement overhead phase. For the appearance implementation, we take into consideration the space distribution of legs ends trajectories, the one with regard to the other, and with regard to the gravity centre of the platform which plays a determining role on the locomotion stability. A simulation is made to illustrate the proposed approach results.

Keywords: *hexapod robot, walking robot, trajectories generation, robot cinematic modelization, robot locomotion, robot walking simulation*

1. INTRODUCTION

Papers When you want to pull robot's legs, we must provide high level orders to indicate in which direction and how fast it should move. This is called task-oriented instructions. These high level instructions come from a supervisory system, which can be represented by a module loaded on the machine or by an operator who directs the robot remotely. As appropriate, the robot can be autonomous or tele - operated. Once the machine is aware of the orders that have been assigned, it must generate instructions for walking

working gait. To this end, it has a locomotion module, responsible for selecting the speed to travel on, set the parameters and calculate the corresponding instructions to be sent to the legs actuators engines.

The walking gaits generation can be open or closed loop. It depends on the sensory capabilities of the robot. In open-loop locomotion, it is necessary to know at least the status of the actuators (typically, position and speed). These are associated proprioceptive sensors that provide such information. On the contrary, during a closed -loop locomotion, the command may consider certain information reflecting the robot's interactions with its environment. (Mahfoudi and al. 2003), (Martin, 1998).

The strategy of locomotion defines the nature of the walking working gait and its implementation. A walking working gait is characterized by the time sequence of events to putting down and raising legs. The timing of these events differs according to walking pattern. To define the synchronization mode, we define the temporal phase shifts between times to put down different legs. Furthermore, it must be determined for each leg the ratio of time spent on the ground and the time devoted to the replacement of air phase. Thus, the definition of a walking working gait requires control of these parameters. However, during the implementation of the gait, other parameters are taken into account. These depend on the structure of the robot. They concern the spatial distribution of trajectories of the ends of the legs relative to each other and from the gravity center of the platform. The position of gravity center with respect to the trajectories of legs plays a decisive role for locomotion stability. The definition of a walking motion reference is important for designing a walking robot and the study of control laws. These laws are often written to follow the imposed trajectories.

For one leg of a walking robot, the generation of a walking motion is to calculate the instructions reference: position, velocity and acceleration, which are a function of time and ensure the passage of the foot of the leg by an imposed trajectory, defined by mode of travel of the robot. (Hugel, 1999), (Randal, 1999).

As the robot moves in the operational space, the generation of movement of the leg is described in this space, this implies the use of reverse geometric method (Khalil et al., 1999), (Paul, 1981) to transform each point of this trajectory in corresponding articular coordinates.

The principle of the generation of motion in operational space is shown in Figure 1 (the exponents i, f and d designate respectively the initial, final and desired position).

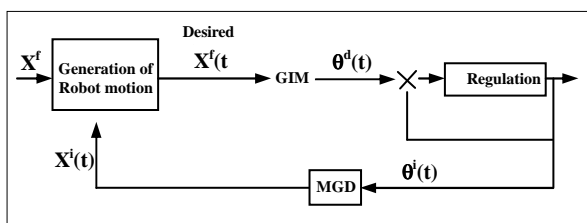


Figure 1: Generation of motion in the operational space

2. THE WALKING OF HEXAPOD

The type of walking seen in some insects is the alternated tripod where we observe at every moment, legs alternate in transfer and support phase, three by three (Ridderström, 2003) (Pfeiffer et al., 1995)

The hexapod walks using three legs at once, so a complete cycle consists of two phases (Figure 2): In the first phase, the robot is supported by legs (1,2 and 3); and the platform moves over a distance λ_0 . The legs (4,5 and 6) move into the air on a cycloidal path between times t_1 and t_2 .

During the second phase, the cycle of phase "1" is repeated except that it is the legs (4, 5, and 6) that are carrier and legs (1, 2 and 3) are in the air. Phase "2" occurs between times t_2 and t_3 (Figure 2).

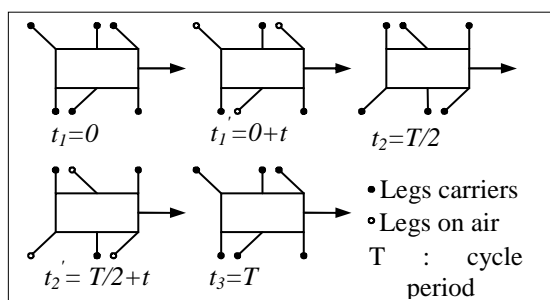


Figure 2: Composition of the hexapod motion during a cycle

2.1. Study of leg motion

During the walking, a step motion of the leg is done in two phases:

- Support Phase: the leg is in contact with the ground, it must bear the body and propel the robot forward;
- Transfer Phase: the leg is lifted and moved in the air passing from one step to the next.

2.1.1. Trajectory of lifted legs

The motion of a lifted leg affects the platforms in unstable manner. The choice of a trajectory of leg in the air must take into account the desired characteristics for the lift and pose moments, i.e. flexibility of motion and minimizing collisions with the ground, carrying out circuits through imposed time by the executed gait, adapting to changes in ground elevation.

A trajectory that respects these conditions is that of a cycloid (Villard, 1993), (Quinn et al., 2003). A cycloid is the trajectory in a plane of a point of a circle which rolls without slipping on a line on this plane as shown in Figure 3. Thus, for our case and as defined earlier a cycloid is the trajectory of the lifted leg in the absolute reference. Equations of x and z coordinates of the cycloid as a function of the rolling angle δ are:

$$\overrightarrow{OM} \begin{cases} x = a(\delta - \sin \delta) \\ z = a(1 - \cos \delta) \end{cases} \quad (1)$$

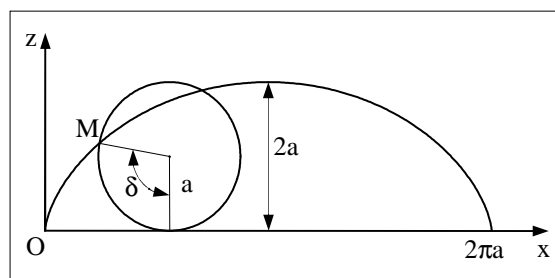


Figure 3: Leg Trajectory in a cycloid form

3. Procedure for generating hexapod motion

To apply the changes to relative motions between the center of gravity of the robot body and feet legs, we define the following parameters (Mahfoudi et al., March and May 2006) as shown in Figure 4:

- L_1, l_2, l_3, a, b : geometric parameters of respectively the leg and the platform
- R_0 : Landmark linked to the center of gravity. Its location is calculated with respect to the benchmark base R using the homogeneous transformation

- λ_0 : displacement Value of center of gravity for a sample of walking
- $P = [P_1, P_2, P_3]^T$: Vector coordinates of the leg foot with respect to the reference R
- $PP = [PP_1, PP_2, PP_3]^T$: Vector coordinates of the foot of the leg with respect to the benchmark R_0
- PP_0 : starting point of the cycloid with respect to the reference R.
- PP_n : end point of the cycloid, with respect to the reference R.
- λ_{00} : stride of the cycloid measured in the same plane of the cycloid
- N: a period of gait cycle
- S: a simple index that splits the interval n

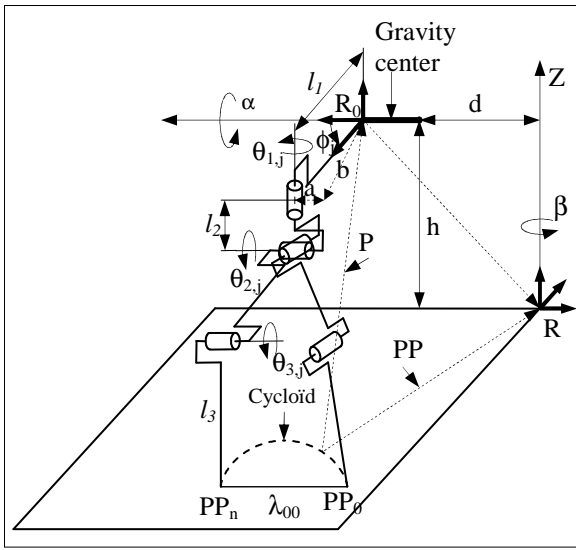


Figure 4: Generation Scheme of a walking leg

3.1. Approach of the platform motion generation

To generate a walking step of the hexapod, we must ensure coordination of the motion of the legs feet with the one of the center of gravity of the robot body. The procedure is as follows:

- Define the geometric parameters of the leg and the platform: l_1, l_2 and l_3 , (length of leg components), a, b (half-width and half length of the platform, respectively)

Initialize a location of the legs with respect to the center of gravity of the platform

$$\begin{aligned} P_{01} &= [0, l_2 + a, -h, 1]^T \\ P_{02} &= [-b, -l_2 - a, -h, 1]^T \\ P_{03} &= [b, -l_2 - a, -h, 1]^T \end{aligned} \quad (1)$$

$$P_{04} = [0, -l_2 - a, -h, 1]^T$$

$$P_{05} = [b, l_2 + a, -h, 1]^T$$

$$P_{06} = [-b, l_2 + a, -h, 1]^T$$

3.1.1 Equation of motion of the platform center of gravity.

We determine the equation of motion of the center of gravity of the robot (of origin R_0) in the reference R by the following parameters $X, Y, Z, \beta, h, \theta_0, d$ and α . This is actually the homogeneous transformation ${}^R T_0$ that permits to locate R_0 with respect to R. we define:

- $X = f(t)$: where $f(t)$ is a function of time
- $Y = g(x)$: The vertical axis of center of gravity may be a function of time and X
- $Z = h$: height of center of gravity. It can be fixed or variable with time, X and Y
- $\theta_0 = \arctg(\dot{y})$: Rotation around the axis z_0
- $d = \sqrt{x^2 + y^2}$: Distance covered by the center of gravity
- $\beta = \arctg(y/x)$: Angle of overall rotation around the z axis of the reference
- α : angle of rotation around the axis x_0 of the platform. It can be fixed or variable with respect to time

3.1.2. Motion of carrying legs

During the t_i and t_{i-1} moments the platform receives a combined motion of a rotation $\theta_{0i} - \theta_{0i-1}$ around the z_0 axis and a translation of the center of gravity λ_0 . Assuming that the carrying legs are (1, 2 and 3) then the points of support $P_j = [P_{1,j}, P_{2,j}, P_{3,j}]^T$ will push back with respect to the center of gravity (Figures 5, 6).

$$\text{Rot}(\theta_{0i} - \theta_{0i-1}) = \begin{pmatrix} \cos(\theta_{0i} - \theta_{0i-1}) & -\sin(\theta_{0i} - \theta_{0i-1}) & 0 & 0 \\ \sin(\theta_{0i} - \theta_{0i-1}) & \cos(\theta_{0i} - \theta_{0i-1}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

$$\lambda_0 = \sqrt{(X_i - X_{i-1})^2 + (Y_i - Y_{i-1})^2} \quad (3)$$

The corresponding homogeneous translation matrix is:

$$\text{Trans}(\lambda_0) = \begin{pmatrix} 1 & 0 & 0 & -\lambda_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

The new coordinates of carrying legs (1, 2, and 3) with respect to the center of gravity are:

$$P_j = \text{Rot}(\theta_{oi} - \theta_{oi-1}) \text{Trans}(\lambda_o) P_{Oj} \quad (j = 1, 2, 3) \quad (5)$$

3.1.3. The motion of legs in the air phase

The legs (4, 5, and 6) are in the air phase. They each carry a cycloid that has parameters $PP_{0,j}$, $PP_{n,j}$ and $\lambda_{00}(j)$ defining respectively the starting, the ending and the stride of each leg. These parameters are calculated in advance.

For legs that carry motion on the air phase along a cycloid, we must calculate the intermediate points belonging to the cycloid. This calculation is made with respect to the R reference as follows (Figures 5 and 6):

- **Calculation of the takeoff point of the leg :**

$$\text{Rot}(\beta_o) = \begin{pmatrix} \cos \beta_o & -\sin \beta_o & 0 & 0 \\ \sin \beta_o & \cos \beta_o & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

$$\text{Rot}(\beta_o - \theta_{00}) = \begin{pmatrix} \cos(\beta_o - \theta_{00}) & -\sin(\beta_o - \theta_{00}) & 0 & 0 \\ \sin(\beta_o - \theta_{00}) & \cos(\beta_o - \theta_{00}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

$$\text{Trans}(d_o) = \begin{pmatrix} 1 & 0 & 0 & d_o \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8)$$

$$\text{Trans}(\lambda_o) = \begin{pmatrix} 1 & 0 & 0 & h_o \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

The initial positions of the feet coordinates of legs (4, 5, and 6) with respect to R are:

$$PP_{0,j} = \text{Rot}(\beta_o) \text{Trans}(h_o) \text{Trans}(d_o) \text{Rot}(\beta_o - \theta_{00}) P_{Oj} \quad (10)$$

- **Calculation of the landing point of the leg:**

Although we are at the starting point, one expects to predict the landing point of the leg to determine the equation of the cycloid that must follow the foot of the

leg. For this, we assume that we are at the point $n/2$; we then obtain PP_{nj} the same way as PP_{0j} .

$$PP_{nj} = \text{Rot}(\beta_{n/2}) \text{Trans}(h_{n/2}) \text{Trans}(d_{n/2}) \text{Rot}(\beta_{n/2} - \theta_{0n/2}) P_{Oj} \quad (11)$$

$j = (4, 5, 6)$

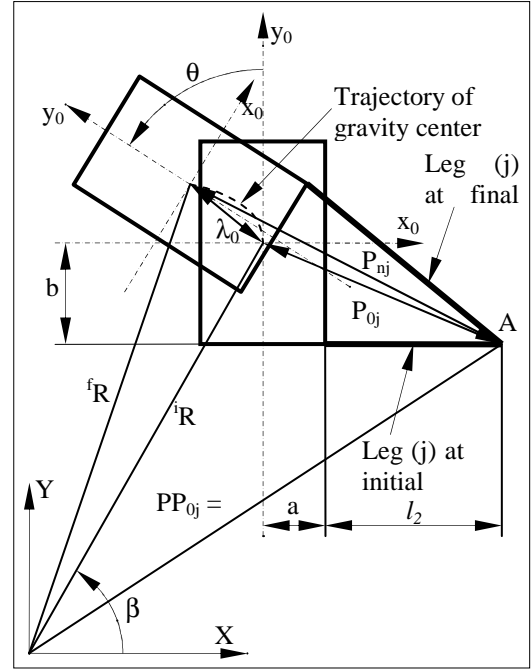


Figure 5 : Carrying leg

- **Calculation of travelled distances $\lambda_{00}(j)$ by the feet of legs:**

$$\lambda_{00}(4) = \sqrt{(PP_{n4x} - PP_{04x})^2 + (PP_{n4y} - PP_{04y})^2} \quad (12)$$

$$\lambda_{00}(5) = \sqrt{(PP_{n5x} - PP_{05x})^2 + (PP_{n5y} - PP_{05y})^2} \quad (13)$$

$$\lambda_{00}(6) = \sqrt{(PP_{n6x} - PP_{06x})^2 + (PP_{n6y} - PP_{06y})^2} \quad (14)$$

- **Calculation of intermediate points (i) constituting the cycloid:**

To find the equation of the cycloid corresponding to our case, we introduce the following parameters:

$$\begin{cases} a_{cy} = \frac{\lambda_{00}(j)}{2\pi} \\ \zeta = \frac{1}{2}, \quad t = s - \frac{n}{2}, \quad T = n \\ \delta = \frac{2\pi}{(1-\zeta)T} t = \frac{4\pi}{n} t \end{cases} \quad (15)$$

With

a_{cy} : radius of the circle

t : time variable of transfer phase
 T : period of a walking step

A limitation of this choice is the trajectory height $2a_{cy}$ which is related to the travelled distance. To obtain a usable path, we propose a cycloid corrected time. Using a factor k , we can choose the height of the trajectory. The equation of the cycloid in the travel plane for each leg "j" is:

$$\begin{cases} X_{\lambda}(i) = \frac{\lambda_{00}(j)}{2\pi} \left\{ \frac{4\pi}{n} t - \sin\left(\frac{4\pi}{n} t\right) \right\} \\ Z_{\lambda}(i) = k \frac{\lambda_{00}(j)}{2\pi} \left\{ 1 - \cos\left(\frac{4\pi}{n} t\right) \right\} \\ Y_{\lambda}(i) = 0 \end{cases} \quad (16)$$

$k \leq 1$: correction factor of the height of the cycloid, i varies from 0 to $n/2$

The foot velocity at the ideal moment for both leg lifting and setting down is equal to zero. The events of setting down and lifting can then occur without collision or serious interruption of motion of the leg.

The speed components expressions \dot{X} and \dot{Z} are:

$$\begin{cases} \dot{X}_{\lambda}(i) = \frac{2\lambda_{00}(j)}{n} \left\{ 1 - \cos\left(\frac{4\pi}{n} t\right) \right\} \\ \dot{Z}_{\lambda}(i) = k \frac{2\lambda_{00}(j)}{n} \sin\left(\frac{4\pi}{n} t\right) \\ \dot{Y}_{\lambda}(t) = 0 \end{cases} \quad (17)$$

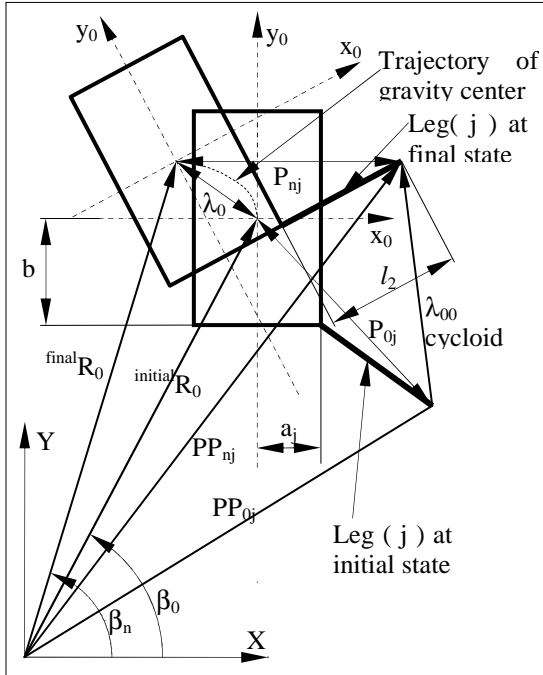


Figure 6 : Leg in air phase

Like any motion must be referenced to the reference base R , then the equation of the cycloid in this benchmark is given by:

$$\begin{cases} X_{PP}(i) = (X_{PPn} - X_{PP0}) \frac{X_{\lambda}(i)}{\lambda_{00}(j)} + X_{PP0} \\ Y_{PP}(i) = (Y_{PPn} - Y_{PP0}) \frac{X_{\lambda}(i)}{\lambda_{00}(j)} + Y_{PP0} \\ Z_{PP}(i) = Z_{\lambda}(i) \end{cases} \quad (18)$$

Its speed in R is given by:

$$\begin{cases} \dot{X}_{PP}(i) = (X_{PPn} - X_{PP0}) \frac{\dot{X}_{\lambda}(i)}{\lambda_{00}(j)} + \dot{X}_{PP0} \\ \dot{Y}_{PP}(i) = (Y_{PPn} - Y_{PP0}) \frac{\dot{X}_{\lambda}(i)}{\lambda_{00}(j)} + \dot{Y}_{PP0} \\ \dot{Z}_{PP}(i) = \dot{Z}_{\lambda}(i) \end{cases} \quad (19)$$

Finally, we must rewrite all vectors PP_j , $j = (4, 5, \text{ and } 6)$ in the benchmark R_0 :

$$P_j = \text{Trans}(-h_i) \text{Trans}(-d_i) \text{Rot}(-\beta_i - \theta_{0i}) \text{Rot}(-\beta_i) PP_j \quad (20)$$

- Calculation of new initial positions:

At the end of each calculation step we must reset the new positions of the feet of the legs with respect to the benchmark R_0 as follows:

$$P_{0j} = P_j \quad j = (1, 2, \dots, 6)$$

Transition from phase "one" to phase "two" corresponding to $i = n / 2$ in the gait cycle

We separate the two phases of a walking step so that the transfer phase begins when ends one of the support. This separation is achieved through the division of the interval n into two halves and using a test variable s which is compared with the value $n / 2$

We then define the motion performed during each phase. When s exceeds the value $n/2$, starts the second phase which consists to swap the legs (1,2,3) in air phase while (4,5,6) will be carriers. Finally the walking cycle ends when s reaches the value of n .

- Calculation of articular coordinates

we use the geometric inverse method (GIM) to calculate the articular coordinates $\theta_{1,j}$, $\theta_{2,j}$ and $\theta_{3,j}$ and for $j = (1, 2, \dots, 6)$ as a function of operational coordinates PP_j of legs feet with respect to the reference R . Finally, we draw at each step calculation the corresponding configuration of the hexapod.

3.2. Simulation of hexapod motions

To implement this procedure a set of subroutines linked by a main program was programmed with Matlab software to generate walking gaits for hexapod. A graphical representation of the hexapod is thus possible.

The following figures show the movement of the hexapod during a cycle.

Figure 7 shows a leg tip simulation for a walking cycle

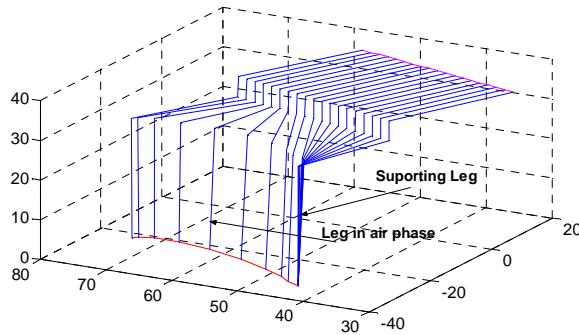


Figure 7: Simulation of a walking step

Figure 8 shows in the operational space and in degree unit the profile of the three suitable articular coordinates and also the desired coordinates of the end of the leg (in cm). Figure 9 shows the hexapod moving in a straight line on a horizontal plane

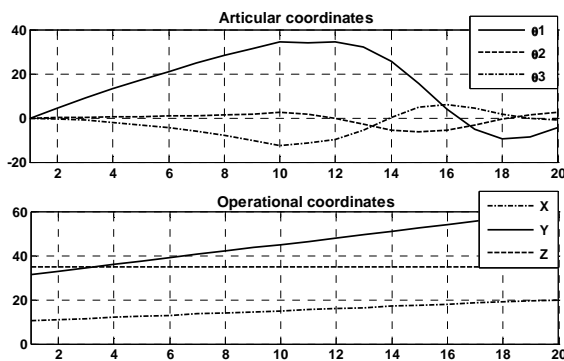


Figure 8: Pattern Evolution of articular and operational coordinates

Figure 9 shows the hexapod walking in accordance with a trajectory of equation $Y = 3X$ and making 80 cycles (for not to clutter up the diagram, we have represented 1 cycle of 15). Figures 10 and 11 represent the detailed movement of the hexapod for a walk cycle. We have imposed to the platform of the hexapod a sinusoidal trajectory in a vertical plane that intersects with the horizontal plane according to the equation $Y = 3X$.

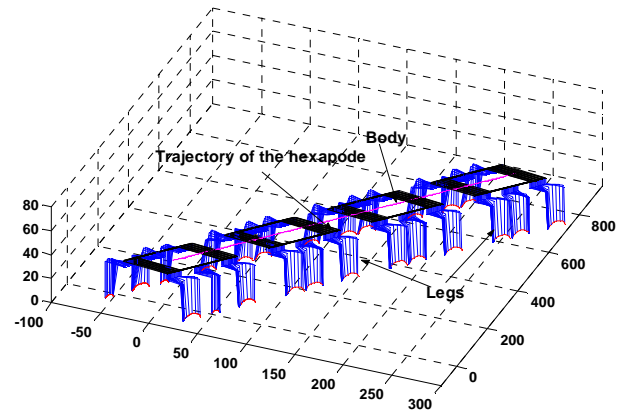


Figure 9: Hexapod motion performing several cycles

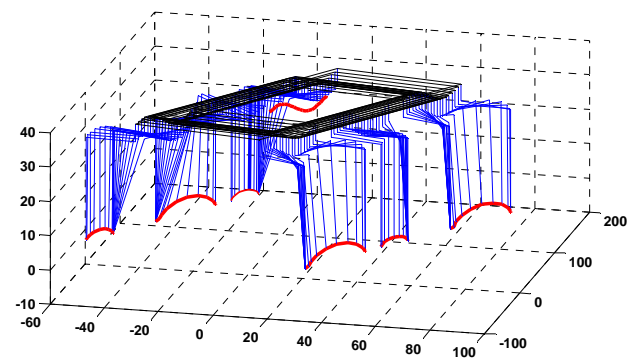


Figure 10: The hexapod walking following a sinusoidal vertical trajectory during a cycle

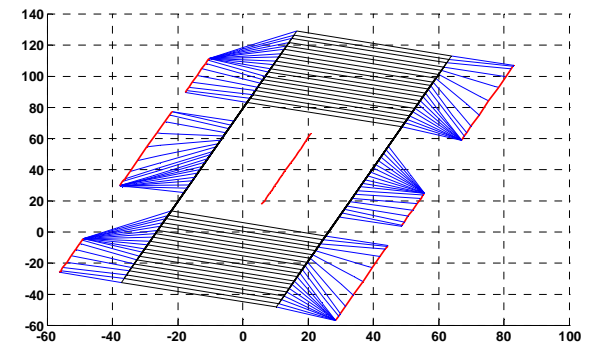


Figure 11: Top view of the hexapod motion

Figures 12 and 13 show respectively the articular coordinates and articular velocities. The hexapod is walking on a straight line of equation $Y = 3X$, on a horizontal plane for two cycles.

Figures 14 show the operational coordinates of the tips of legs expressed in the benchmark associated to the platform. The hexapod is walking on a straight line of equation $Y = 3X$ on a horizontal plane for two cycles.

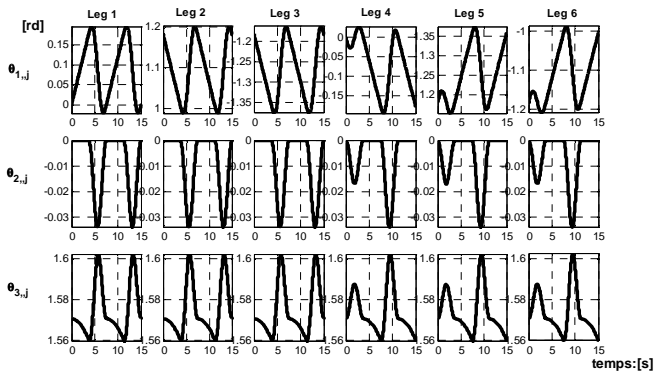


Figure 13: Articular velocities of the six legs of the hexapod

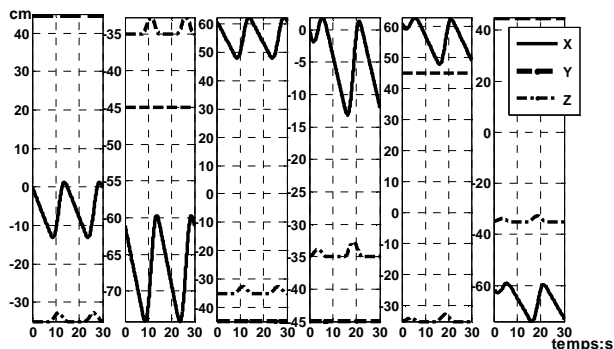


Figure 14: Operational tips Coordinates of the hexapod six legs

4. Experimentation

Therefore, a First experimentation was conducted on our experimental walking machine figure (15). Our experimental hexapod uses two hardware cards. The First one, based on a Microchip 2 micro-controller is dedicated for low level motors control. While the second one, which is a PC104 card under real time OS, is dedicated for computed torque control and real time force distribution computing. Finally, a PC is used for trajectory planning and real time monitoring.

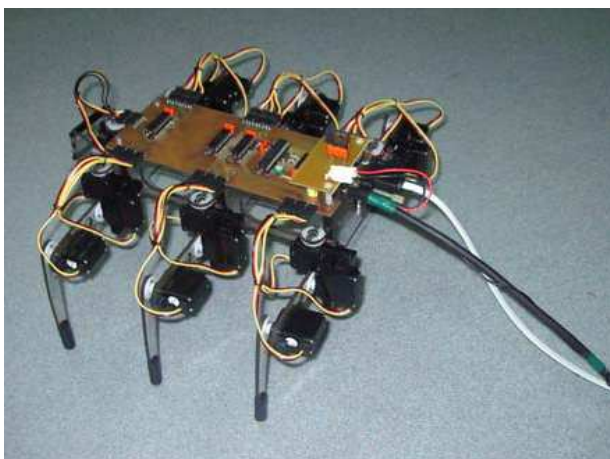


Figure 15: Experimental hexapod

5. Conclusion

The main objective of this work is to implement a quite general walking generator. The trajectory generator uses the alternating tripod gait. The platform can move following any time of law, but according to the constraint of attainable field legs carriers. Thus the robot simulates walking on flat field following trajectories with a quite large radius of curvature. The trajectory generator conceived in such manner permit to read back at any time values of the articular coordinates, velocities and accelerations, all the center of gravity kinematic of the platform, as well as coordinates of the tips of the legs with their corresponding velocities and accelerations in the workshop benchmark.

2.1. Length of the Paper

The International Program Committee will accept two types of camera ready papers: extended paper (10 pages, 2 columns); regular paper (6 pages, 2 columns) both for regular and special sessions.

2.2. Margins

The left and the right margins must be 1,9 cm on each page, and the gutter between the two columns of text must be 1,25 cm in width. The top and the bottom margins must be 2,54 cm.

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