ANALYSIS OF CONTROLLED SEMI-MARKOV QUEUEING MODELS

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ABSTRACT
The present paper is devoted to the research of the controlled semi-markov queueing model M/M/n*/N*. Research of income functional is carried out at control of several parameters. The control is based on the system structure. In the article the income functional is constructed on the trajectories of the controlled semi-markov process. The main problem is to search an optimal control strategy for the given model. The given research algorithm also can be used for various models at control with both discrete and continuous model parameters. Algorithmization of the problem is presented.

Keywords: optimization, semi-markov controlled process, control strategy, income functional

1. INTRODUCTION

The main part of research works of queueing theory showed that the model structure is considered invariable, and the model characteristics are investigated at the fixed initial parameters and functions. The interarrival time distribution, the service time distribution, the number of servers, the system capacity are considered to be fixed. In the present paper we change the model structure and investigate the system model using variables of controls.

The purpose of the queueing model theory is to present recommendations on maintenance for high efficiency system functioning.

Notice that the functioning increase can be reached using control parameters. There can be several control parameters in the model. There may be the interarrival time distribution, the service time distribution, the number of servers, the system capacity and others. We are interested in several control parameters, that means the controls set expansion.

One of the main results of the controlled semi-markov processes theory is strategy determination that gives the maximum value for the functional. The main theory of semi-markov processes can be found in the following works (Bellman R.A. 1957, Jewell W. S., 1967).

It is important to note that we can obtain new results using investigation for not only stationary characteristics (stationary queue length, loss probability, etc.), but using structure model changing.

Stochastic character of the interarrival time and the service time generates a stochastic process in the queueing model. For the controlled stochastic process the problem of a choice of optimum control strategy is given. In the present paper a controlled semi-markov process is used. On the trajectories of the controlled semi-markov process the income functional is constructed.

The result of the functional research is a choice of optimal control strategies for the given model.
Similar works linked with the control in queueing systems (Banik A.D. 2009, Swishchuk A. 1999, Kashtanov V.A. 2010), one of the advantages of the given work is simultaneous control using several parameters of system.

To calculate parameters of the system functioning quality and to choose optimum control, the following algorithm is used:

- Construct semi-markov process,
- Determine structure of control,
- Enter constants describing incomes and costs,
- Calculate additional characteristics,
- Calculate income functional and search for optimum control.

2. PROBLEM STATEMENT

The queueing model can be represented using Kendall’s notation M/M/n*/N*.

For the given model:

- the interarrival time distribution is a Poisson distribution with parameter λ,
- the service time distribution is an exponential distribution with parameter μ (service intensity),
- the number of servers channels is a parameter of control, n* is a maximum possible value,
- the maximum number of places in the queue (system capacity) is also a parameter of control. N* is a maximum possible value.

The present paper is devoted to the research of the controlled semi-markov queueing model using model structure control.

Describe the system in details and enter some assumptions:

- Maximum possible number of servers is n, each of which can serve simultaneously only one requirement. The number of servers is a control parameter and can change from 0 up to n.
- Maximum possible number of places in the queue is N. It is a parameter of control and can change from 0 up to N.
- Markov moments are the moments of the service termination and the arrival epoch. The control strategy is to be chosen at the markov moments only.
- Each state of the system can be described using pair of parameters (i, j), where
- i - the numbers which are served, $0 \le i \le n$.
- j - the numbers which are in the queue, $0 \le j \le N$.
- It is not rational to increase the number of empty/free places in the queue more, than by one at every markov moment. Therefore in the store never will be more than one empty seat in the queue.
- Instead of addition of an extra seat being in state (0,0) it is possible to add extra servers (if it is more profitable, expense for the free working server is less than an expenses for an additional places in the queue.
- We assume, that the queueing model is directed on service of the greatest possible number of requirements, hence, we assume, that it is more profitable to send the requirement to server (if such opportunity is represented). Thus, when from the store the requirement sends on service, the number of places in the queue decreases for one unit.
- If one of the servers has finished service and there are requirements in the queue model passes from set (i, j) to set (i, j-1).
- When one of channels has finished service and in the store there are no requirements, the system passes from set (i, 0) in set (i-1,0).
- Note, that the control is carried out at markov moments. The transition probability depends on the made decision (strategy).

3. RESEARCH ALGORITHM

To calculate parameters of the system functioning quality and to choose optimum control, the algorithm investigated by Kashtanov is used.

For construction of the controlled semi-markov process it is necessary to define the parameters: markov moments, the states set, the set of controls, a semi-markov kernel. The next step is to determine:

- transition probabilities of the embedded markov chain,
- conditional mathematical expectations of the saved up income provided that process is in state (i,j) and through time t will pass in state (i',j'),
- mathematical expectations of the saved up income for the full period when process is in state (i,j).
- Then, a parameter value of the functioning quality (the income functional) is calculated.

Parameter of functioning quality is the income functional. The income functional is defined as follows:

$$S = \lim_{t\to\infty}\frac{S_i(t)}{t} = \frac{\sum_{i\in E} s_i\pi_i}{\sum_{i\in E} m_i\pi_i} \qquad (1)$$

where

s_i - mathematical expectation of the saved up income for the full period when process is in state i,

m_i - mathematical expectation of the saved up income for the full period when process is in state i,

π_i - stationary probabilities of the embedded markov chain.

4. CONSTRUCTION OF THE CONTROLLED SEMI-MARKOV PROCESS

Semi-markov process $\xi(t)$ is described with a 2-dimensional markov chain (ξ_n,θ_n), $n \ge 0, \xi_n \in E,\ \theta_n \in R^+ = [0,\infty)$.

The markov chain (ξ_n,θ_n) is defined with transition probabilities (semi-markov kernel):

$$\begin{aligned}&P\{\xi_{n+1} = j, \theta_{n+1} < t | \xi_n = i, \theta_n = \tau\} = \\ &P\{\xi_{n+1} = j, \theta_{n+1} < t | \xi_n = i\} = Q_{ij}(t)\end{aligned} \qquad (2)$$

where $n \ge 1, i, j \in E,\ t \in [0,\infty)$ and using initial distribution

$p_i = P\{\xi_0 = i\} \ge 0,\ i \in E,\ \sum_{i\in E} p_i = 1,$

assume $P\{\theta_0 = 0\} = 1$.

A 3-dimensional markov chain (controlled semi-markov process) is described as follows:

(ξ_n,θ_n,u_n), $n \ge 0, \xi_n \in E,\ \theta_n \in R^+ = [0,\infty)$, $u_n \in U$.

Where $E = \{1,2,...,N\}$ - the final states' set, θ_n - a component is identified as time duration. On the set $R^+ = [0,\infty)$ σ-algebra is stated. U -is a control set with σ-algebra A of a subset of this set.

Markov chain (ξ_n,θ_n,u_n) is defined using transition probabilities:

$P\{\xi_{n+1}=j,\theta_{n+1}<t,u_{n+1}\in B|\xi_n=i,\theta_n=\tau,u_n=u\}$,
$i,j\in E,t,\tau\in R^+,B\in A,u\in U$

Assume that:

$$P\{\xi_{n+1}=j,\theta_{n+1}<t,u_{n+1}\in B|\xi_n=i,\theta_n=\tau,u_n=u\}= \\ =P\{\xi_{n+1}=j,\theta_{n+1}<t,u_{n+1}\in B|\xi_n=i\} \quad (3)$$

and

$$p_i=P\{\xi_0=i\}\geq 0,\ i\in E,\ \sum_{i\in E}p_i=1,\ P\{\theta_0=0\}=1.$$

The probability $Q_{ij}(m,t,B)=\{\xi_{n+1}=j,\theta_{n+1}<t,u_{n+1}\in B|\xi_n=i\}$ generates on the set (U_i,A_i) probabilities measures:

$$G_i(B)=P\{u_{n+1}\in B/\xi_n=i\}=\sum_{i\in E}Q_{ij}(m,B),i\in E,B\in A_i.$$

Then $Q_{ij}(m,t,B)\leq G_i(B)$, so:

$$Q_{ij}(m,t,B)=\int_B Q_{ij}(m,t,u)G_i(du)$$

where $Q_{ij}(m,t,u)=P\{\xi_{n+1}=j,\theta_{n+1}<t|\xi_n=i,u_{n+1}=u\}$

$$Q_{ij}(m,t,B)= \\ \int_B P\{\xi_{n+1}=j,\theta_{n+1}<t|\xi_n=i,u_{n+1}=u\}G_i(du) \quad (4)$$

Each system state can be described by pair of parameters (i, j), where

i - the number of requirements which are on service $0\leq i\leq n$.

j - the number of requirements which are on places in the queue $0\leq j\leq N.$

Define the control set. We can control choosing the number of servers and the number of additional/extra places in the queue.

The control set of the servers quantity is $U=\{0,1,.....,u\}$, where:

0 - we do not add servers,

1 - we make decision to add 1 extra server.

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u - we add u additional servers, where $1\leq u\leq n-i$.

The control set of the extra places is $V=\{0,1\}$, where:

v=0 – we do not add places,

v=1 – we add one place for expectation in the queue.

Note, that at the markov moments in each state the desicion {u,v} can be made, that depends on the concrete system state.

In state (0,0) there are no requirements either on service, or in the queue in the system.

In the given state it is possible to make a decision on addition of an extra place (or inclusion of the server).

Hence, in state (0,0) following controls U = {0,1}, V = {0,1} are possible. It is meaningful to add only one seat in the queue or to include one server/device.

In state (i,0), $0<i\leq n$ the following controls are possible U={0,1},V={0,1}.

In state (0,j), $0<j\leq N$, there are no requirements on service, there are j requirements in the queue. So in state (0,j) the controls U={0,u},V={0,1} are possible. And $0\leq u\leq\min(j,n)$.

In state (i,j), $0<i<n,0<j<N$, there are i requirements on service, there are j requirements in the queue. In state (i,j) the controls U={0,u},V={0,1} are possible. And $0\leq u\leq\min(j,n-i)$.

In state (i,N), $i<n$, are i requirements on service, there are N requirements in the queue (maximum possible). So in (i,N) the controls U={0,u},V={0} are possible. And $0\leq u\leq\min(N,n-i)$.

In state (n,j), $0<j<N$, there are n requirements on service (maximum possible), there are j requirements in the queue. The controls U={0},V={0,1} are possible.

In state (n,N) there are n requirements on service (maximum possible), there are N requirements in the queue (maximum possible). So in state (n,N) the controls U={0},V={0} are possible.

Construct semi-markov kernel for the given system.

The interarrival time distribution is a Poisson distribution with parameter λ, $F(t)=1-e^{-\lambda t}$.

The service time distribution is an exponential distribution with parameter μ (service intensity), $G(t)=1-e^{-\mu t}$.

Semi-markov kernel $Q_{(i,j)(i',j')}(t,u,v)$ is a probability of that the semi-markov process will pass in state (i',j') and the time before this transition will not surpass t provided that process is in state (i,j) and in this state a decision from the set of controls (u,v) is made.

In state (0,0) $U_{(0,0)}=\{0,1\}$, $V_{(0,0)}=\{0,1\}$.

$$Q_{(0,0)(0,1)}(t,0,1)=Q_{(0,0)(1,0)}(t,1,0)=Q_{(0,0)(0,0)}(t,0,0)= \\ F(t)=1-e^{-\lambda t}$$

In state (i,0), $0<i\leq n$ $U_{(i,0)}=\{0\}$, $V_{(i,0)}=\{0,1\}$.

$$Q_{(i,0)(i-1,0)}(t,0,1)=Q_{(i,0)(i-1,0)}(t,0,0)=\frac{i\mu}{\lambda+i\mu}(1-e^{-t(\lambda+i\mu)})$$

$$Q_{(i,0)(i,1)}(t,0,1) = Q_{(i,0)(i,0)}(t,0,0) = \frac{\lambda}{\lambda + i\mu}(1 - e^{-t(\lambda + i\mu)}).$$

In state (0,j) $U_{(i,0)} = \{\overline{0, \min(j,n)}\}$, $V_{(i,0)} = \{0,1\}$

If $u = j, j \le n$ then:

$$Q_{(0,j)(j,1)}(t,j,1) = Q_{(0,j)(j,0)}(t,j,0) = \frac{\lambda}{\lambda + j\mu}(1 - e^{-t(\lambda + j\mu)})$$

$$Q_{(0,j)(j-1,0)}(t,j,0) = Q_{(0,j)(j-1,0)}(t,j,1) = \frac{j\mu}{\lambda + j\mu}(1 - e^{-t(\lambda + j\mu)})$$

Else:

$$Q_{(0,j)(u,j-u+1)}(t,u,1) = Q_{(0,j)(u,j-u)}(t,u,0) = \frac{\lambda}{\lambda + u\mu}(1 - e^{-t(\lambda + u\mu)})$$

$$Q_{(0,j)(u,j-u-1)}(t,u,0) = Q_{(0,j)(u,j-u-1)}(t,u,1) = \frac{u\mu}{\lambda + u\mu}(1 - e^{-t(\lambda + u\mu)})$$

In state (i,j), $0 < i < n, 0 < j < N$, $U_{(i,j)} = \{\overline{0, \min(j, n-i)}\}$, $V_{(i,0)} = \{0,1\}$, $u \in U_{(i,j)}$.

If $u = j, j \le n - i$ then:

$$Q_{(i,j)(i,1)}(t,j,1) = Q_{(i,j)(i+j,0)}(t,j,0) = \frac{\lambda}{\lambda + (i+j)\mu}(1 - e^{-t(\lambda + (i+j)\mu)})$$

$$Q_{(i,j)(i+j-1,0)}(t,j,0) = Q_{(i,j)(i+j-1,0)}(t,j,1) = \frac{(i+j)\mu}{\lambda + (i+j)\mu}(1 - e^{-t(\lambda + (i+j)\mu)})$$

Else:

$$Q_{(i,j)(i,j-u+1)}(t,u,1) = Q_{(i,j)(i+u,j-u)}(t,u,0) = \frac{\lambda}{\lambda + (i+u)\mu}(1 - e^{-t(\lambda + (u+a)\mu)})$$

$$Q_{(i,j)(i+u,j-u-1)}(t,u,0) = Q_{(i,j)(i+u,j-u-1)}(t,u,1) = \frac{(i+u)\mu}{\lambda + (i+u)\mu}(1 - e^{-t(\lambda + (i+u)\mu)})$$

In state (i,N), $i < n$, $U_{(i,N)} = \{\overline{0, \min(N, n-i)}\}$, $V_{(i,N)} = \{0,1\}$, $u \in U_{(i,N)}$.

And if $V_{(i,N)} = \{1\}$, then $U_{(i,N)} = \{\overline{1, \min(N, n-i)}\}$.

If $u = N, N \le n - i$ then:

$$Q_{(i,N)(i,1)}(t,N,1) = Q_{(i,N)(i+N,0)}(t,N,0) = \frac{\lambda}{\lambda + (i+N)\mu}(1 - e^{-t(\lambda + (i+N)\mu)})$$

$$Q_{(i,N)(i+N-1,0)}(t,N,1) = Q_{(i,N)(i+N-1,0)}(t,N,0) = \frac{(i+N)\mu}{\lambda + (i+N)\mu}(1 - e^{-t(\lambda + (i+N)\mu)})$$

Else:

$$Q_{(i,N)(i,N-u+1)}(t,u,1) = Q_{(i,N)(i+u,N-u)}(t,u,0) = \frac{\lambda}{\lambda + (i+u)\mu}(1 - e^{-t(\lambda + (u+a)\mu)})$$

$$Q_{(i,N)(i+u,N-u-1)}(t,u,0) = Q_{(i,j)(i+u,N-u-1)}(t,u,1) = \frac{(i+u)\mu}{\lambda + (i+u)\mu}(1 - e^{-t(\lambda + (i+u)\mu)})$$

In state (n,j), $0 < j < N$, $U_{(n,j)} = \{0\}$, $V_{(n,j)} = \{0,1\}$.

$$Q_{(n,j)(n,j)}(t,0,0) = Q_{(n,j)(n,j+1)}(t,0,1) = \frac{\lambda}{\lambda + n\mu}(1 - e^{-t(\lambda + n\mu)})$$

$$Q_{(n,j)(n,j-1)}(t,0,0) = Q_{(n,j)(n,j-1)}(t,0,1) = \frac{n\mu}{\lambda + n\mu}(1 - e^{-t(\lambda + n\mu)})$$

In state (n,N), $U_{(n,N)} = \{0\}$, $V_{(n,j)} = \{0\}$.

$$Q_{(n,N)(n,N)}(t,0,0) = \frac{\lambda}{\lambda + n\mu}(1 - e^{-t(\lambda + n\mu)})$$

$$Q_{(n,N)(n,N-1)}(t,0,0) = \frac{n\mu}{\lambda + n\mu}(1 - e^{-t(\lambda + n\mu)})$$

Notice, that cases of states (0, j), (n, j), (i, N), (n, N) are described by formulas for a case (i, j), considering the controls sets for each concrete state.

5. ADDITIONAL CALCULATIONS

For construction of a conditional mathematical expectation of the saved up income $R_{(i,j)(i',j')}(t,u,v)$ provided that process is in state (i,j) and through time t will pass in state (i',j') and in this state a decision from the set of controls (u,v) is made, we shall enter the constants describing incomes and costs:

c_1 - income received per service of one number;

$-c_2$ - payment per time unit of server work device during the service time of the customer ;

$-c_3$ - payment per time unit in case the number is in the queue;

$-c_4$ - payment per time unit in case extra place in the queue is not used;

$-c_5$ - payment per one lost number;

$-c_6$ - payment per time unit in case server is working, but not used (without service).

Then for all possible states $R_{(i,j)(i',j')}(t,u,v)$ can be described.

Conditional mathematical expectation of the saved up income $R_{(i,j)(i',j')}(t,u,v)$ provided that process is in state $(0,0)$ can be described:

$$R_{(0,0)(0,1)}(t,0,1) = R_{(0,0)(0,0)}(t,0,1) = -c_4 t$$
$$R_{(0,0)(1,0)}(t,1,0) = R_{(0,0)(0,0)}(t,1,0) = -c_6 t$$
$$R_{(0,0)(0,0)}(t,0,0) = -c_5$$

In state (i,j):

$$R_{(i,j)(i+u,j-u)}(t,u,0) = -(c_2(i+u)+c_3(j-u))t - c_5$$
$$R_{(i,j)(i+u,j-u+1)}(t,u,1) = -(c_2(i+u)+c_3(j-u)+c_4)t$$
$$R_{(i,j)(i+u,j-u-1)}(t,u,0) = -(c_2(i+u)+c_3(j-u))t + c_1$$
$$R_{(i,j)(i+u,j-u-1)}(t,u,1) = -(c_2(i+u)+c_3(j-u)+c_4)t + c_1$$

If $u = j, j \le n-i$ then:

$$R_{(i,j)(i+j,0)}(t,j,0) = -c_2(i+j)t - c_5$$
$$R_{(i,j)(i+j,1)}(t,j,1) = -(c_2(i+j)+c_4)t$$
$$R_{(i,j)(i+j-1,0)}(t,j,0) = -c_2(i+j)t + c_1$$
$$R_{(i,j)(i+j-1,0)}(t,j,1) = -(c_2(i+j)+c_4)t + c_1$$

We can define conditional mathematical expectation of the saved up income for all possible system states (see semi-markov kernel).

Calculate $m_{(i,j)}$ mathematical expectations of the saved up income for the full period when process is in state (i,j).

$$m_{(i,j)} = \int_0^\infty \left[1 - \sum_{(i',j')\in E} Q_{(i,j)(i',j')}(t)\right] dt \tag{5}$$

$$Q_{(i,j)(i',j')}(t) = \sum_{u,v} Q_{(i,j)(i',j')}(t,\{u,v\}) p_{u,v}^{[i,j]} \tag{6}$$

$$m_{(i,j)} = \int_0^\infty \left[1 - \sum_{(i',j')\in E} \sum_{u,v} Q_{(i,j)(i',j')}(t,\{u,v\}) p_{u,v}^{[i,j]}\right] dt \tag{7}$$

Where $p_{u,v}^{[i,j]}$ is a probability that in state (i,j) we choose strategy (u,v) from the given control sets.

For example, in state (0,0), U={0,1},V={0,1}.

$$m_{(0,0)} = \int_0^\infty \left[1 - \sum_{(i',j')\in E} \sum_{u,v} Q_{(0,0)(i',j')}(t,\{u,v\}) p_{u,v}^{[0,0]}\right] dt$$

$$m_{(0,0)}(0,0) = m_{(0,0)}(0,1) = m_{(0,0)}(1,0) = \frac{1}{\lambda}$$

In state (i,j), $0 < i < n, 0 < j < N$, U={0,u},V={0,1}, $0 \le u \le \min(j, n-i)$.

If $u = j, j \le n-i$ then:

$$m_{(i,j)}(j,0) = m_{(i,j)}(j,1) = \frac{1}{\lambda + (i+j)\mu}$$

Else:

$$m_{(i,j)}(u,0) = m_{(i,j)}(u,1) = \frac{1}{\lambda + (i+u)\mu}$$

Notice, that cases of states (0, j), (n, j), (i, N), (n, N) are described by formulas for a case (i, j), considering the controls sets for each concrete state.

Provide additional calculations. Calculate variables $s_{(i,j)}$. Variables $s_{(i,j)}$ are mathematical expectations of the saved up income for the full period when process is in state (i,j).

$$s_{(i,j)}(t,\{u,v\}) = \sum_{(i,j)\in E} \int_o^t R_{(i,j)(i',j')}(x,\{u,v\}) dQ_{(i,j)(i',j')}(x,\{u,v\}) + \sum_{(i,j)\in E} \int_t^\infty R_{(i,j)(i'j')}(x,t,\{u,v\}) dQ_{(i,j)(i',j')}(x,\{u,v\}) \tag{8}$$

$$s_{(i,j)}(t) = \sum_{u,v} s_{(i,j)}(t,\{u,v\}) p_{u,v}^{[i,j]} \tag{9}$$

Consider $s_{(i,j)} = \lim\limits_{t\longrightarrow\infty} s_{(i,j)}(t)$.

$$s_{(i,j)} = \sum_{u,v} \sum_{(i',j')\in E} \int_0^\infty R_{(i,j)(i',j')}(x,\{u,v\}) dQ_{(i,j)(i',j')}(x,\{u,v\}) p_{u,v}^{[i,j]} \tag{10}$$

Taking into consideration degenerated distributions (see next page) we have:

$$s_{(i,j)} = \sum_{(i',j')\in E} \int_0^\infty R_{(i,j)(i',j')}(x,\{u^{[i,j]},v^{[i,j]}\}) dQ_{(i,j)(i',j')}(x,\{u^{[i,j]},v^{[i,j]}\}) \tag{11}$$

For example, in state (i,N):

$$s_{(i,N)}(u,1) = \frac{-(c_2(i+u)+c_3(j-u)+c_4)+c_1(i+u)\mu}{\lambda+(i+u)\mu}$$

$$s_{(i,N)}(u,0) = \frac{-(c_2(i+u)+c_3(N-u))-c_5\lambda+c_1(i+u)\mu}{\lambda+(i+u)\mu}$$

Notice, that cases of states (0, j), (n, j), (i, N), (n, N) are described by formulas for a case (i, j), considering the controls sets for each concrete state.

Stationary probabilities of the embedded Markov chain are defined from the equations:

$$\begin{cases} \pi_l = \sum_{k \in E} \pi_k p_{kl} \\ \sum_{k \in E} \pi_k = 1 \end{cases} , \qquad (12)$$

where $p_{ij} = \lim_{t \longrightarrow \infty} Q_{ij}(t, \{u, v\})$

Taking into consideration that model state is described using pair of parameters, the system can be defined:

$$\pi_{(i',j')} = \sum_{(i,j) \in E} \pi_{(i,j)} p_{(i,j)(i',j')}, \ \sum_{(i',j') \in E} \pi_{(i',j')} = 1 \qquad (13)$$

where

$$p_{(i,j)(i',j')} = \lim_{t \longrightarrow \infty} Q_{(i,j)(i',j')}(t, \{u, v\}) .$$

Use theorem for controlled semi-markov processes for the income functional calculation (Kashtanov and Medvedev 2002) and find S (1).

Note, that calculations are to be automated because of complexity.

Using the theorem about the accumulation functional fractional-linearity concerning the distributions defining structure of the accumulation (see Appendix), and the theorem about a maximum of fractional-linear functional, we use the basic conclusion: it is possible to search for optimum strategy in the set of determined strategies of control (Kashtanov and Medvedev 2002). So all results of the calculations are obtained substituting degenerated distributions.

We receive the functional, depending on the variables which are responsible for control $S((v_{00},..v_{ij}..,v_{nN}),u_{00},..u_{ij}...,u_{nN})$.

Further it is necessary to define a maximum of the functional S depending on u_{ij}, v_{ij} (see set of controls). Then we receive the solve of the problem. That is the strategy $((v^*_{00},....,v^*_{nN}),u^*_{00},....,u^*_{nN})$, at which the maximal income is received. Hence, when the process is in state (i,j) we take the decision $\{u^*_{ij},v^*_{ij}\} \in U$. The choice of the decision defines the most effective system work.

6. CALCULATION EXAMPLE

The calculation results have shown the opportunities of control expansion, in other words using several parameters of control leads to more effective model functioning. For example for model M/G*/1/2* we can use discrete and continuos controls (Kashtanov and Kondrashova 2010).

There are three states in the system.

State {0} – there are no numbers in the system;

State {1} – one number is being served, there are no numbers in the queue;

State {2} – there are one number is on service, there is one number in the queue.

In state 1 we can add 0,1,2 additional places in the queue. In state 2 we can add 0 or 1 additional places in the queue. We receive, that the quantity of every possible strategy combinations for choice v equals six. u –service duration in state, $u \in [0,\infty)$ can be chosen in each state (control parameter).

In state i we choose the service time and we make the decision on creation of additional/extra places in the queue (except state i=0). Hence, we use two parameters of control from the given set of controls: {u, v}.

Write down the strategy pairs for states 1 and 2 : (0,0); (0,1); (1,0); (1,1); (2,0); (2,1). Last strategy pair concerns to a classical case of the problem when all places are used and we control only with the service time. And the control of service duration is also used (τ_1,τ_2 – service duration in 1 and 2 state, that take maximum value for income functional at fixed v).

Table 1: Calculation example

Strategy	Maximum value S	Value τ_1	Value τ_2
(0,0)	8	0	-
(0,1)	8	0	-
(1,0)	2	0	-
(1,1)	2	0	-
(2,0)	0,24418416	1,05	0
(2,1)	1,16934403	1,367	0,674
(0,0)	1	0	-
(0,1)	1	0	-
(1,0)	-0,743454243	0,879	-
(1,1)	-0,743454243	0,879	-
(2,0)	1,54211326	1,387	0
(2,1)	2,28817567	1,482	0,448

7. CONCLUSION

In the paper the research of the controlled semi-markov queueing model M/M/n*/N* is carried out. The control is based on the system structure. In the paper the construction of the controlled semi-markov process and the construction of the income functional on its trajectories are used.

The formula for the income functional calculation for the given system is received. The calculation example for the model is demonstrated. Algorithmization of the problem is presented.

The calculation results have shown the opportunities of control expansion, in other words using several parameters of control leads to more effective system functioning.

APPENDIX A

Theorem. If there is a controlled semi-markov process, for which the embedded Markov chain is ergodic, at *least, one of the distributions* $Q_{ij}(t)$ *is non-trellised and the average income* s_i *is final at any* $i \in E$ *, then:*

$$S = \lim_{t\to\infty}\frac{S_i(t)}{t} = \frac{\sum_{i\in E} s_i\pi_i}{\sum_{i\in E} m_i\pi_i}, \qquad (14)$$

where π_i *are stationary probabilities of conditions' distribution of the embedded Markov chain.*

APPENDIX B

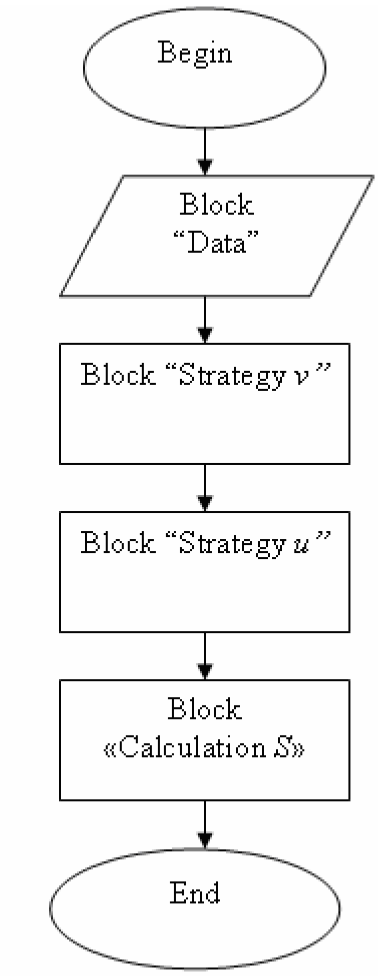

Figure 1: Block-diagram of the automated decision for M/M/n*/N*

Block "Data": input n, N, λ, μ, cost constants c_1, c_2, c_3, c_4 , c_5 , c_6.

Block "Strategy *v"*: generating of strategy massive for v for every possible combinations.

Block "Strategy *u"*: generating of strategy massive for u for every possible combinations.

Block "Calculation S": additional calculations for necessary characteristics for each strategy combination (cycle). Choice of optimum functional value and optimum strategy.

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