TRAFIC CONTROL AND QUEUES MANAGEMENT FOR A SINGLE INTERSECTION

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ABSTRACT
It is by now well known that, the traffic signal control is the most effective measure for reducing traffic congestion in an intersection. Traffic congestion arises when arrival rate exceeds capacity in the intersection. The most undesirable symptom of this kind of instability is an unbounded accumulation of vehicles in the system. Our contribution in this paper is to show that, for a single intersection, congestion can be prevented by using a state feedback controller having an appropriate positive structure. It is shown that two main objectives are achieved with the proposed controller: firstly, when traffic flow is not in excess, the queuing of automobiles cannot exceed intersection capacity. Secondly, in order to fit physically reasonable signalization, the controller respects the boundary conditions. Example is worked out to illustrate the results.

Keywords: Traffic lights control, Single intersection, State feedback control.

1. INTRODUCTION

As the number of vehicles and the need for transportation grow, cities around the world face serious road traffic congestion problems. In general there exist different methods to tackle the traffic congestion problem. The most effective measures in the battle against traffic congestion seem to be a better control of traffic through traffic management. Traffic light control can be used to manage the traffic flow in urban environments by providing a smooth circulation of the traffic. The purpose of traffic lights is to provide safe and efficient interaction of vehicles within the intersection.

In the past 40 years, great effort has been made in the area of signal timing optimization techniques (Gazis 1974, Green 1968, Kaltenbach and Koivo 1974, Dans and Gazis 1976, Michalopoulos and Stephanopolos 1978, Wey 2000, Barisone, Giglio, Minciardi and Poggi 2002, Abu-Lebdeh and Benekohal 2000, Girianna and Benekohal 2000). Most of these control strategies are based on a preprogrammed periodic cyclic rhythm. These approaches are easy to implement but may be not efficient and flexible because they do not take into account traffic changes.

Michalopoulos and Stephanopolos (Michalopoulos and Stephanopolos 1978) were probably the first to propose a two-stage timing method. The method attempts to find an optimal switch-over point during the over-saturated period to interchange the timing of the approaches. Their model is a continuous type and does not address the problem of cycle length optimizing. Recently, Tang-Hsien and Lin (Tang-Hsien and Lin 2000), have studied a discrete delay type model, involving *bang-bang* like control, to improve Michalopoulos and Stephanopolos model. The *bang-bang* control operates alternatively and sequentially with a given only minimal and maximal green time boundaries. However, there is a limited evolution of this control strategy because the *bang-bang* control does not consider the deviation of a system from its nominal behavior caused by deviation of system components and parameters from their nominal performance characteristics. Another notable work in this area is the one presented by Schutter and Moor (Schutter and Moor 1998) in which the authors have shown how an optimal traffic light switching scheme for an over-saturated intersection of two streets can be determined. In general, this leads to a minimization problem over solutions set of an extended linear complementarity problem. However, it is well-known that the linear complementarity problem is NP-hard. Hence, as claimed by authors, this approach is not feasible if the number of switching cycles is large.

In conclusion, all these works are based mainly on an open loop and rolling horizon techniques. However, due to the substantial changes in the traffic characteristics at different times of the day, control strategies should be such that the signal takes into account the effects of the abrupt change in any period. In fact, the feedback control method may be the best means to achieve this aim. This is the objective of the paper.

Our contribution in this paper is to show that congestion can be prevented by using a state feedback controller having an appropriate positive structure. More precisely, we propose a state feedback control scheme able to achieve the objective of congestion avoidance and to satisfy a queue length of vehicles that does not exceed the intersection capacity.

2. SYSTEM DESCRIPTION AND MODELING

We consider here an isolated two-phase intersection with controllable traffic lights on each corner (Figure 1). It is supposed that there are two traffic flows of vehicles to be served (movements 1 and 2). To be able to present the model, a certain number of definitions are necessary.

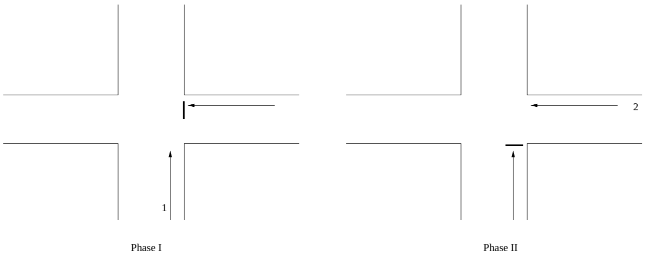

Fig. 1. Four-leg intersection with two-phase signal control.

Lost time L: time when an intersection is not effectively used by any approach. Lost time occurs during a part of the change interval and at the beginning of each green due to start-up delay. It is defined by $L = l_1 + l_2$, where l_1 represents the startup lost time and l_2 the clearance lost time.

Effective green time g_e: time actually available for movement. It excludes start-up delay that occurs at the beginning of the phase as well as any lost time that might occur near the end of the phase. It is defined by

$$g_e = g + y - L,$$

where g is the green time interval, y represents the amber interval. The effective green time g_{e_2} on phase II is determined from the effective green times g_{e_1} on phase I by the relation

$$g_{e_1} + g_{e_2} = c. \tag{1}$$

where c is the cycle length. Notice that it makes no sense to speak of negative g_{e_i}. Therefore, equation (1) leads to constraints on the pair of control variables (g_{e_i}, c), that is

$$0 \leq g_{e_i} \leq c; \quad c > 0.$$

In other words, the effective green time can be at most equal to the length of the cycle. However, certain boundary points may not fit physically reasonable signalization settings. In particular, $g_{e_1} = 0$ corresponds to no effective green time for phase I, $g_{e_1} = c$ corresponds to no effective green time for phase II. Therefore, it is important to select tow boundary values g_{min} and g_{max} so that

$$g_{min} \leq g_{e_i} \leq g_{max} \tag{2}$$

The values of g_{min} and g_{max} must be well selected. For instance, too short effective green lights are impractical and too long effective green lights are unacceptable to the stopped drivers of the other approach. Hence, any g_{e_i} satisfying this constraint leads to a viable signalization control strategy.

Saturation flow rate s: the discharge process of the vehicles in the queue is controlled by reaction times and desired acceleration rates of drivers as well as acceleration rates of vehicles ahead. At the beginning of the green interval, the discharge rate at stop lane starts to increase. As the queuing vehicles have reached a constant speed at stop line, the discharge rate has reached its maximum, called the saturation flow rate. More precisely, the saturation flow rate is defined as the maximum number of vehicles being able to use the corridor without interruption during the effective green time g_e. The saturation flow rate may vary from cycle to cycle, but an average value can be used for given conditions.

Let us now write down a discrete model that describes the evolution of the queue lengths. This will then yield the equations that give the relation of the queue lengths between cycle k and $k+1$. Let us denote $I = \{1, 2\}$ and define

x_i : queue length of movement $i \in I$,
q_i : input flow rate of movement $i \in I$,
s_i : saturated flow rate of movement $i \in I$,
g_{e_i} : effective green time of phase $i \in I$

where in general $q_i < s_i$. The fundamental idea of the described traffic model design technique is based on the traffic flow conservation principle. It means that the queue at time $k+1$ is equal to the sum of the previous queue at time k and the number of arrivals $E_i(k)$ minus the number of departures $D_i(k)$ from the lane i.

The number of arrivals $E_i(k)$ is given by the input flow rate q_i of movement i and the cycle length c. The quantity $E_i(k)$ can be written as $E_i(k) = q_i\, c(k)$. The maximal number of passing vehicles $D_i(k)$ is given by the saturation flow s_i and the effective green time g_{e_i}. The quantity $D_i(k)$ can be written as $D_i(k) = s_i\, g_{e_i}(k)$.

Now, let $x_i(k+1)$ be a queue length of approach i when the cycle k is terminated, then the relation of the queue lengths between cycle k and $k+1$ can be represented by the following equation:

$$x_i(k+1) = x_i(k) + E_i(k) - D_i(k) \tag{3}$$

Hence, equation (3) becomes:

$$x_1(k+1) = x_1(k) + q_1 c - s_1 g_{e_1}(k)$$
$$x_2(k+1) = x_2(k) + q_2 c - s_2 g_{e_2}(k)$$

Since $g_{e_1} + g_{e_2} = c$, it follows

$$x_1(k+1) = x_1(k) + (q_1 - s_1)c + s_1 g_{e_2}(k)$$
$$x_2(k+1) = x_2(k) + q_2 c - s_2 g_{e_2}(k)$$

The above equations can be equivalently restated as a state space expression

$$x(k+1) = x(k) + Bu(k) + L \tag{4}$$

where $x(k)$ is the state vector, $u(k) = g_{e_2}(k)$ is the control variable, and

$$B = \begin{pmatrix} s_1 \\ -s_2 \end{pmatrix} = \begin{pmatrix} b_1 > 0 \\ b_2 < 0 \end{pmatrix};\ L = \begin{pmatrix} (q_1 - s_1)c \\ q_2 c \end{pmatrix} = \begin{pmatrix} \ell_1 < 0 \\ \ell_2 > 0 \end{pmatrix}$$

Notice that, it makes no sense to speak of negative queue. Hence, if x_0 describes queue at time $k = 0$ then $x(k) \geq 0$ for all $k \in \mathbb{N}$. Therefore, the system (4) has physical meaning only if x belongs to the region of admissible states $\Omega_x = \{x \in \mathbb{R}^n / x \geq 0\}$.

3. CONTROL PROBLEM FORMULATION

The control problem is formulated as follows. We consider the system (4) with a maximal capacity of each lane x_1^* and x_2^*. Then, congestion may occur in the system if the queue length of each lane exceeds its capacity. The most undesirable symptom of this kind of instability is an unbounded accumulation of vehicles in the system.

The control objective is then to find a state feedback controller that respects the constraint (2) and is able to satisfy

$$0 \le x(k) \le x^*, \quad \forall k \ge 0, \quad \forall\, 0 \le x_0 \le x^* \tag{5}$$

Motivated by (Rami, Tadeo, and Benzaouia 2007), the following proposition gives an answer to this problem.

Proposition 1: Consider system (4) *with parameters* $x^* > 0$, $g_{\min} > 0$ *and* $g_{\max} > 0$. *Suppose that* $1 + \frac{b_1 b_2}{x_i^*} \ge 0$. *Then, if there exists a positive scalar* α *with* $\alpha \ge \max(\frac{-\ell_1}{b_1}; \frac{-\ell_2}{b_2})$ *such that*

$$B(b_1 + b_2 + \alpha) + L \le 0 \tag{6}$$

$$b_1 + \alpha \le g_{\max} \tag{7}$$

$$b_2 + \alpha \ge g_{\min} \tag{8}$$

then there exists a positive control $u(k) = Q^T x(k) + \alpha$ *such that* $0 \le x(k) \le x^*$ *for all* $0 \le x_0 \le x^*$ *and* $g_{\min} \le u(k) \le g_{\max}$.

Proof: Define the vector $Q^T = [\frac{b_2}{x_1^*}, \frac{b_1}{x_2^*}]$ and observe that since $1 + \frac{b_1 b_2}{x_i^*} \ge 0$, it follows $I + BQ^T \ge 0$. Furthermore, $\alpha \ge \max(\frac{-\ell_1}{b_1}; \frac{-\ell_2}{b_2})$ implies $B\alpha + L \ge 0$. Now, under the state feedback control $u = Q^T x + \alpha$ the closed-loop system becomes

$$x(k+1) = (I + BQ^T)x(k) + B\alpha + L \tag{9}$$

Since $I + BQ^T \ge 0$ and $B\alpha + L \ge 0$, it is easy to show that $x(k) \ge 0$ for all $x_0 \ge 0$. With this in mind, observe that $b_1 + b_2 = Q^T x^*$, hence (6) implies

$$B(b_1 + b_2 + \alpha) + L = BQ^T x^* + B\alpha + L < 0. \tag{10}$$

This and the fact that $I + BQ^T \ge 0$ and $x_0 \le x^*$ imply

$$\begin{aligned} x(1) &= (I + BQ^T)x_0 + B\alpha + L \\ &\le (I + BQ^T)x^* + B\alpha + L \\ &= x^* + BQ^T x^* + B\alpha + L \le x^* \end{aligned}$$

it follows

$$\begin{aligned} x(2) &= (I + BQ^T)x(1) + B\alpha + L \\ &\le (I + BQ^T)x^* + B\alpha + L \\ &= x^* + BQ^T x^* + B\alpha + L \le x^* \end{aligned}$$

By induction it can be shown that

$$\begin{aligned} x(k) &= (I + BQ^T)x(k-1) + B\alpha + L \\ &\le (I + BQ^T)x^* + B\alpha + L \\ &= x^* + BQ^T x^* + B\alpha + L \le x^* \end{aligned}$$

Hence, the first objective (5) of the control is achieved with the proposed controller. Now, observe that

$$u(k) = Q^T x(k) + \alpha = \frac{b_2}{x_1^*} x_1 + \frac{b_1}{x_2^*} x_2 + \alpha$$

Since $0 \le x_1 \le x_1^*$, $0 \le x_2 \le x_2^*$, $b_1 > 0$ and $b_2 < 0$ then

$$b_2 + \alpha \le u(k) \le b_1 + \alpha$$

Hence, (7) and (8) imply

$$g_{\min} \le u(k) \le g_{\max}$$

which completes the proof. ■

It follows immediately from Proposition 1 that, for an arbitrarily selected positive $\alpha > \max(\frac{-\ell_1}{b_1}; \frac{-\ell_2}{b_2})$ that satisfies constraints (6), (7) and (8), the feedback controls defined by $u(k) = Q^T x(k) + \alpha$ exhaust the class of control laws u which maintain the queue length of each lane less than its capacity. Hence, Proposition 1 gives explicit and non-unique solution of the congestion control.

Remark 1: As we can see, the control law $u(k) = Q^T x(k) + \alpha$ is positive and its input data are the estimated values of the queues level during each cycle k. Hence, the control policy evaluates the green light for each traffic cycle following the calculation process of vehicles stored in the intersection. The real-time availability of $x(k)$ can be measured by a traffic detector installed at the corresponding lanes. This situation allows us to use effectively the available information on the system to perform closed loop control of the traffic signals in real time.

The performance of the proposed control law is illustrated in the simulation studies of two-phase intersection. Figure 2 represents the evolution of the queue lengths x_1 and x_2 stored in the system within 6000 cycles. One can immediately observe that the application of feedback control $u(k) = Q^T x(k) + \alpha$ prevents the queue lengths reaching the lanes capacities and hence avoids the congestion. This simulation is carried out by holding account the following data: $s_1 = 0.2veh/s; s_2 = 0.2veh/s; q_1 = 0.1veh/s; q_2 = 0.1veh/s; c = 160s; u_{max} = 120s; u_{min} = 30s; x_1^* = 50veh; x_2^* = 45veh$. According to these parameters, we have $1 + \frac{b_1 b_2}{x_1^*} = 0.9992 \ge 0$, $1 + \frac{b_1 b_2}{x_1^*} = 0.9991 \ge 0$, $Q^T = (-0.0041, 0.0044)$ and the scalar α is found to be 80.

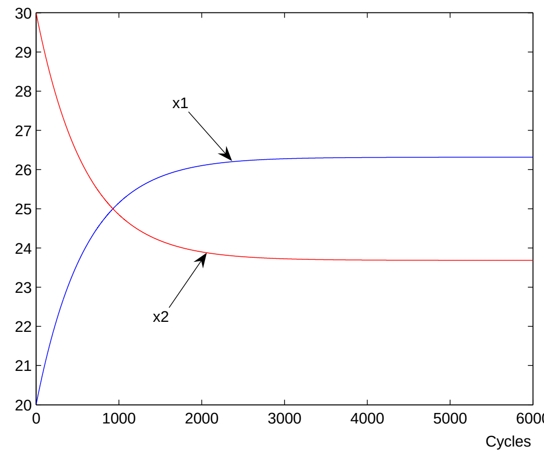

Fig. 2. Queue lengths evolution system

In the following section, we address the case where the cycle length is time varying and it is considered as a second control variable. The importance of this comes from the fact that it gives us a degree of freedom and a flexibility for the control strategy, because its variation will take into account the effects of the abrupt change in real traffic situation.

4. CASE WHERE THE CYCLE LENGTH IS TIME VARYING

In this case equation (3) becomes:

$$x_1(k+1) = x_1(k) + (q_1 - s_1)c(k) + s_1 g_{e_2}(k)$$
$$x_2(k+1) = x_2(k) + q_2 c(k) - s_2 g_{e_2}(k)$$

The above equations can be equivalently restated as a state space expression

$$x(k+1) = x(k) + Bu(k) \tag{11}$$

where $x(k)$ is the state vector, $u(k) = (c(k), g_{e_2}(k))^T$ is the control variable, and

$$B = \begin{pmatrix} q_1 - s_1 & s_1 \\ q_2 & -s_2 \end{pmatrix}$$

Furthermore, the system is to be analyzed under the following constraint

$$\begin{pmatrix} c_{\min} \\ g_{\min} \end{pmatrix} \le \begin{pmatrix} c(k) \\ g_{e_2}(k) \end{pmatrix} \le \begin{pmatrix} c_{\max} \\ g_{\max} \end{pmatrix}$$

which is equivalent to

$$0 < u_{\min} \le u \le u_{\max}. \tag{12}$$

As in the preceding section, the control objective is then to find a state feedback controller that respects the constraint (12) and is able to satisfy

$$0 \le x(k) \le x^*, \quad \forall k \ge 0, \quad \forall\, 0 \le x_0 \le x^*$$

This control problem is the same as the one described in section II except that the control variable in this case is depending on effective green time $g_{e_2}(k)$ and the cycle length $c(k)$. This situation leads us towards a square matrix B. Thus, the feedback control law derived in section II does not work in this context because the control is not a scalar. To overcome this inherent difficulty the following lemma will be useful.

Lemma 1: Consider a linear system of the form:

$$x(k+1) = x(k) + Bv(k) + L \tag{13}$$

with $x^* > 0$ and $v^* > 0$, if there exist three positive vectors $y^1, y^2, \alpha \in \mathbb{R}^2$ such that

$$B(y_i^1 + y_i^2 + \alpha) + L \le 0 \tag{14}$$
$$B\alpha + L \ge 0 \tag{15}$$
$$y^1 + y^2 + \alpha \le v^* \tag{16}$$
$$1 + b_i \frac{y^i}{x_i^*} \ge 0 \tag{17}$$
$$b_i \frac{y^j}{x_j^*} \ge 0, \text{for } i \ne j \tag{18}$$

where b_i is the i row of the matrix B, then there exists a positive control $v(k) = Qx(k) + \alpha$ such that $0 \le x(k) \le x^*$ and $0 \le v(k) \le v^*$ for all $0 \le x_0 \le x^*$.

Proof: Define the matrix $Q = [\frac{1}{x_1^*}y^1, \frac{1}{x_2^*}y^2]$ and observe that (17) and (18) imply $I + BQ \ge 0$. Now, under the state feedback control $v = Qx + \alpha$ the closed-loop system becomes

$$x(k+1) = (I + BQ)x(k) + B\alpha + L \tag{19}$$

Since $I + BQ \ge 0$ and $B\alpha + L \ge 0$ in view of (15), it is easy to show that $x_k \ge 0$ for all $x_0 \ge 0$. With this in mind, observe that $y^1 + y^2 = Qx^*$, hence (14) and (19) imply

$$B(y_1 + y_2 + \alpha) + L = BQx^* + B\alpha + L \le 0. \tag{20}$$

This and the fact that $I + BQ \ge 0$ and $x_0 \le x^*$ imply

$$\begin{aligned} x(1) &= (I + BQ)x_0 + B\alpha + L \\ &\le (I + BQ)x^* + B\alpha + L \\ &= x^* + BQx^* + B\alpha + L \\ &\le x^* \end{aligned}$$

it follows

$$\begin{aligned} x(2) &= (I + BQ)x(1) + B\alpha + L \\ &\le (I + BQ)x^* + B\alpha + L \\ &= x^* + BQx^* + B\alpha + L \\ &\le x^* \end{aligned}$$

By induction it can be shown that

$$\begin{aligned} x(k) &= (I+BQ)x(k-1) + B\alpha + L \\ &\le (I+BQ)x^* + B\alpha + L \\ &= x^* + BQx^* + B\alpha + L \\ &\le x^* \end{aligned}$$

Hence, the first objective of the control is achieved with the proposed controller. Now, since $0 \le x(k) \le x^*$ and $Q > 0$, then $0 \le u(k) = Qx(k) + \alpha \le Qx^* + \alpha = y^1 + y^2 + \alpha \le v^*$ in view of (16), which completes the proof. ■

Now, the congestion control problem of system (11) can be solved by putting $u = v + u_{\min}$. In this case the constraints (12) yield $0 \le v \le u_{\max} - u_{\min} = v^*$ and system (11) becomes

$$x(k+1) = x(k) + Bv(k) + L \tag{21}$$

where $L = Bu_{\min}$, which is the same as equation (13). Hence, if all conditions of Lemma 1 are verified then the control $u = v + u_{\min} = Qx + \alpha + u_{\min}$ respects the constraint (12) and achieves the following objective:

$$0 \le x(k) \le x^*, \quad \forall k \ge 0, \quad \forall\, 0 \le x_0 \le x^*.$$

The performance of the proposed control law is illustrated in the simulation studies of two-phase intersection. Figure 3 is obtained with the following data: $s_1 = 0.6veh/s; s_2 = 0.4veh/s; q_1 = 0.3veh/s; q_2 = 0.2veh/s; c_{max} = 160s; c_{min} = 120s; g_{max} = 96s; g_{min} = 30s; x_1^* = 80veh; x_2^* = 70veh$. According to these parameters, we have found $y^1 = (18.2789, 5.0991)^T$, $y^2 = (7.6539, 7.8673)^T$, $\alpha = (6.5315, 33.2657)^T$.

As in the preceding section, one can conclude that the application of feedback control $u(k) = Q^T x(k) + \alpha + u_{\min}$ prevents the queue lengths reaching the lanes capacities and hence avoids the congestion.

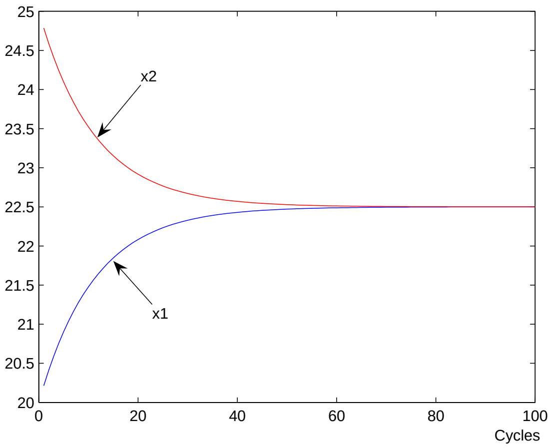

Fig. 3. Queue lengths evolution system

5. CONCLUSION

In this paper, it was shown that congestion in a single intersection can be prevented by using a state feedback controller having an appropriate positive structure. We have shown that two main objectives are achieved with the proposed controller: firstly, when traffic flow is not in excess, the queuing of automobiles cannot exceed intersection capacity. Secondly, in order to fit physically reasonable signalization, the controller respects the boundary conditions.

Upon application of the control strategy described in this paper, the process model is used on-line. Great computational effort is therefore desirable in this context. However, the rapid development of the numerical performance of computer hardware makes computationally extensive control concepts, such as on-line application, more and more feasible and also reasonable in terms of cost.

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