AN INFORMATION FRACTAL FOR SUPPLY NETWORK INVENTORY OPTIMISATION

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ABSTRACT
This paper develops a new conceptual framework for an information fractal to optimise inventory across the supply network by identifying the optimum safety stock, inventory policy and cycle stock with the lowest logistics cost as well as out of stock prevention.
The proposed framework consists of two levels: top and bottom level fractals. Fractals in the bottom level analyse demand, optimise safety stock and recommend an inventory policy. Then transmit output to the top level fractal to investigate the effect of different replenishment frequencies to determine the optimum cycle stock for each fractal in the bottom level by integrating the inventory holding costs and transportation costs to minimise the logistics cost.
The proposed framework provides a systematic method through which practitioners are able to decide upon the demand analysis, safety and cycle stock optimisation.

Keywords: Fractal supply network, supply network modelling, and inventory optimization.

1. INTRODUCTION
In today's competitive world, increased competitiveness in the global business environment and improvements in manufacturing technology mean that traditional production management methods that have failed to improve the integrity of their processes have lost their effectiveness; companies need to create systematic integration in all production processes from supplier to the final consumer. Supply chain management as an integrated approach has the ability to meet these requirements to manage the flow of raw materials and final products, information and funds. Supply chain integration allows companies and their suppliers to act together, leading to performance improvement through the chain (Kannan & Tan, 2002). The major responsibilities within the industrial units are planning and inventory control. Despite the costs associated with inventory holding, having an inventory is inevitable for supply chain members because inventory shortages can lead to irrecoverable losses including stopped production, loss of sales opportunities, damage to the reputation of the organisation and so on. Inventory control strategies in the supply chain management are classified as either centralised inventory control (Gross, 1963; Zheng & Zipkin, 1990; Marklund, 2002) or decentralised inventory control (Andersson & Marklund, 2000; Jemai & Karaesmen, 2007; Hall & Zhong, 2002). In terms of centralised inventory control, decisions in the supply chain can be made by a centralised decision maker who has access to all the necessary information to improve system performance; this situation is possible when the whole supply chain is under the control of a centralised decision maker who has a high level of coordination and communication with other members in the supply chain. None of the members (e.g. supplier or retailer) can control the entire supply chain and each of them has their own goals and priorities to optimise their individual performance. Therefore, each member controls and manages their inventory position and places orders to their resources based on their own priorities; in such cases, the inventory control strategy is categorised as decentralised. In this study, an information system based on fractal features is developed which is a combination of both centralised and decentralised inventory control. Each member in the supply chain has a responsibility to analyse the demand of its downstream members, determine its safety stock, inventory reorder point and inventory policy, and share with the information centre in the chain. This in turn must determine the optimum cycle stock for each member to minimise the logistics costs in the supply chain by integrating both inventory holding costs and transportation costs.
Among all areas of potential improvement in supply chain management, information sharing is of greatest interest. When a company uses information from other companies in the supply chain, the negative effects of uncertainty in the modern business environment such as high inventory levels, wrong demand forecasts and defective orders can be reduced. To have the greatest improvement in organisational performance and increase their competitive advantage, firms can take advantage of information technology to develop information sharing and knowledge capabilities throughout the whole supply chain (Wagner & Buko, 2005). It has been noticed that there was few reported research articles tried to show the benefit of information

sharing in supply chain inventory management although most of the models that were introduced were relatively simple and developed in a two- or three-stage supply chain. Gavirneni, Kapuscinski, & Tayur (1999) investigated and analysed the benefits of information sharing in a two-echelon supply chain by considering one supplier and one retailer with several levels of information sharing, including when there is no demand for information flow to the supplier except historical data, when the supplier has information regarding the type of inventory control policy and demand distribution of the retailer and, in the third level, when the supplier has full access to the retailer's daily inventory position. Lau, Huang, & Mak (2004) analysed the effect of information sharing on inventory replenishment in three-stage supply chains with one manufacturer, distribution centres and retailers. They investigated four types of information sharing among nodes, including order information sharing among nodes, demand, safety factors and inventory information sharing from retailers to their distribution centres, sharing retailers' order information with manufacturers from distribution and order information sharing from retailers to distribution centres and from distribution centres to manufacturers. Lee, So, & Tang (2000) developed a simple two-stage supply chain with manufacturer and retailer and indicated how the manufacturer can achieve benefits from information sharing by decreasing the inventory and saving costs directly.

In this case, a conceptual information fractal framework is developed by considering multiple retailers, distribution hubs, manufacturers, supplier facilities and information chain centres which are also considered as fractals separately. Each fractal has its own structure but with the same inputs and outputs, the ability to choose and use appropriate methods to optimise itself and divide large problems into small ones, and perform a goal-formation process to generate their own goals by coordinating processes with the participating fractals, modifying goals if necessary. Finally, each fractal has the ability to adapt to the dynamically changing environment.

2. THE PROPOSED FRAMEWORK FOR THE INFORMATION FRACTAL SUPPLY NETWORK (IFSN)

Figure 1 displays the new proposed framework of an IFSN with two levels including an information fractal chain centre as a top level fractal and an information fractal supplier's facility, information fractal manufacturer, information fractal distribution hub and information fractal retailer as bottom level fractals. In this paper, the information fractal structure for each fractal consists of five functional models including observer, analyser, resolver, organiser and reporter as a basic fractal unit (BFU) (Ryu, Moon, Oh, & Jung, 2013).

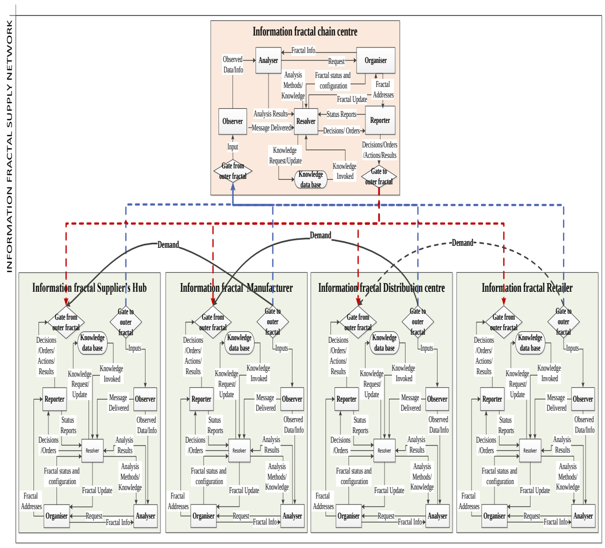

Figure 1: The proposed framework for an Information Fractal Supply Network (IFSN)

In the bottom level fractal, observers as an input gate of each fractal must monitor, trace and receive data and messages (e.g. demand) from outer fractals (e.g. retailer, distribution hub, manufacturer) and the environment (e.g. customer). Observers in the sourcing fractals trace and receive the demand from destination fractals, transmit the demand data to analysers and notify resolvers receiving the demand.

Analysers use an appropriate method to analyse current demand based on a set of demand statistics to determine demand class and then transmit it to resolvers. The demand class enables resolvers to recognise different types of demand and allocate an appropriate method to calculate safety stock. Resolvers determine the expected safety stock, recommending inventory policy and inventory policy parameters as part of the safety stock optimisation. Organisers in all the fractals, including top and bottom level fractals, observe, control and manage the fractal structure to adapt to the continuous change in the environment. Reporters as an output gate have a responsibility to report fractal outputs to outer fractals. In the bottom level fractal, reporters report resolvers' decisions regarding expected safety stock, inventory policy and associated parameters to the fractals in the top level.

In the top level fractal, observers trace and receive decisions which are made by each fractal in the bottom level (e.g. safety stock, inventory policy and so on.), transmit them to analysers and notify resolvers. Analysers investigate and analyse the different amounts of cycle stock on both transportation costs and inventory holding costs based on replenishment frequencies for each fractal in the bottom level. Resolvers integrate inventory holding costs and transportation costs based on analysers' reports to achieve an optimum amount of cycle stock with the lowest logistics cost for each non-production fractal and also determine the optimum production frequency for the production fractals. In the top level fractal, reporters report resolvers' decisions regarding optimum cycle stock, production and replenishment frequencies to the fractals in the bottom level. This paper concentrates on two main functions, analyser and resolver, to optimise both safety stock and cycle stock in the supply network.

2.1. Bottom level fractal

It is important to determine how much inventory must be held against the variability in both demand and lead times. Therefore, understanding the demand variability is essential to calculate safety stock. Analysers in the bottom level fractal use an appropriate method to analyse demand based on a set of demand statistics. During the demand analysis process, demand is aggregated, outliers are recognised and a set of demand statistics is provided. Analysers use demand statistics and demand classification threshold values to determine the demand classification (e.g. Slow, Lumpy, Erratic and Smooth).

Analysers perform the following steps to analyse current demand:

- Step 1: Determine aggregate demand for the specified aggregation period which can be based on daily, weekly and monthly demand.
- Step 2: Provide a set of demand statistics to classify the demand.
- Step 3: Classify demand based on demand statistics which are provided in step 2.

To set up a demand class, analysers use set demand classification thresholds that affect how demand is classified and how analysers determine the appropriate approach for safety stock calculation. Demand classification thresholds include demand frequency, intermittency and dispersion which determine by non-zero demand count (M_{NZ}), inter-demand interval mean (p) and squared coefficient of variation of non- zero demand (CV^2_{NZ}), respectively. Outlier, variability and clumpiness are specified by non-zero demand standard division (σ_{NZ}). Demand classification threshold values are determined based on the firm's conditions (see Figure 2).

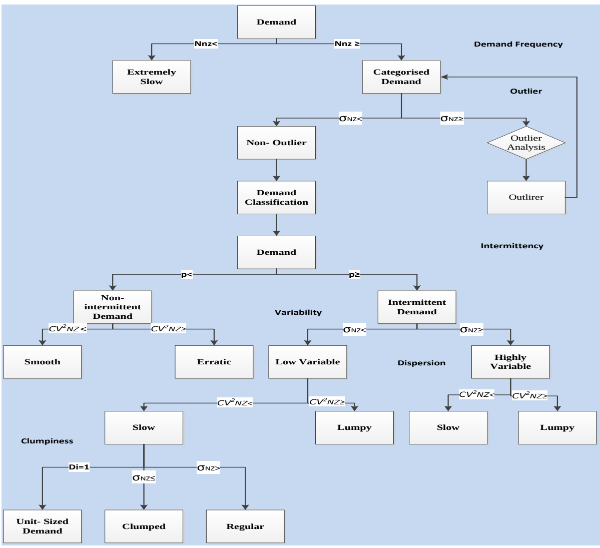

Figure 2: Demand classification diagram

An extremely slow class will occur when the demand count is lower than the demand count adjusted in the demand classification thresholds. This class has a large inter-demand interval mean.

Analysers recognise outliers based on non-zero demand standard division and non-zero demand mean values during the demand classification process:

- If (σ_{NZ}) is less than the default number in the demand classification threshold, analysers ignore the outlier recognising process and continue to demand classification.
- If (σ_{NZ}) is greater or equal to the default number in the demand classification threshold, the outlier recognising process is started. Analysers consider the aggregation period with the largest demand size and determine it as an outlier if it is greater or equal to (σ_{NZ}) in the demand classification threshold *(μ_{NZ}) from the rest of the demand.

There are two options for analysers for handling the outliers:

- Outliers are considered in the demand statistics where they were recognised.
- Replace outliers with the demand mean of the rest of the demands which are smaller than the outlier and recalculate the non-zero demand standard deviation and return to the first step of the process.

Intermittency specifies how frequently demand occurs, based on the average time between adjacent demands.

- If the average time between the demands is lower than the intermittency threshold, it is known as non-intermittent demand. It means that demand happens regularly with few exceptions during the demand period. If (CV^2_{NZ}) is greater than the default number in the threshold, this demand is classified as erratic and if (CV^2_{NZ}) is less, the demand is classified as smooth.
- If the average time between the demands is greater than the intermittency threshold, it is known as intermittent demand. It means that there is irregularity of when demand happens during the demand period. Intermittent demand can be considered as a low or high variable, and is slow or lumpy. Low variable demand has a lower (σ_{NZ}) in comparison to highly variable demand, and slow demand has a lower (CV^2_{NZ}) in comparison to lumpy demand.

Clumpiness shows how demand points are close to each other and has a reasonably fixed demand with variability close to zero. The demand size for unit-sized demand is always one, and there is no variability for this demand class.

Once analysers have finished the demand analysis, resolvers start to specify the required safety stock by considering demand and lead-time variability. Resolvers use a target service level to calculate optimum safety stock. Service level is a measure to indicate a fractal's ability to provide products to downstream fractals.

There are different types of service level which are used in industry including type 1 (probability of not stocking out), type 2 (fill rate) and type 3 (ready rate). In this research paper, service level type 1 is used. Resolvers in the bottom level fractal determine the safety stock level, inventory policy and reorder point as part of the safety stock optimisation.
There are three models to calculate safety stock and reorder point which may happen during the demand period (Heizer & Render, 2014):
The following notation is adopted:

- SS =Safety stock
- σ_{dLT} = Standard division of demand during the lead time
- σ_d= Standard deviation of demand per day
- LT=Lead time
- Z= Service level
- ROP= Reorder point
- μ_{dLT}= Demand mean during the lead time
- μ_d= Average daily demand
- d_D= Daily demand
- σ_{LT}= Standard deviation of lead time in days
- μ_{LT}= Average lead time

2.1.1. Demand is variable and lead time is constant

$$SS=Z\times\sigma_{dLT} \tag{1}$$

where:

$$\sigma_{dLT}=\sigma_d\times\sqrt{LT} \tag{2}$$

and

$$ROP=\mu_{dLT}+Z\sigma_{dLT} \tag{3}$$

where:

$$\mu_{dLT}=\mu_d\times LT \tag{4}$$

2.1.2. Lead time is variable and demand is constant

$$SS=Z\times d_D\times\sigma_{LT} \tag{5}$$

and

$$ROP=(d_D\times\mu_{LT})+Z\times\sigma_{LT} \tag{6}$$

2.1.3. Both lead time and demand are variable

$$SS=Z\times\sigma_{dLT} \tag{7}$$

where:

$$\sigma_{dLT}=\sqrt{(\mu_{LT}\times\sigma_d^2)+(\mu_d)^2\times\sigma_{LT}^2} \tag{8}$$

and

$$ROP=(\mu_d\times\mu_{LT})+Z\times\sigma_{LT} \tag{9}$$

As part of the safety stock optimisation, resolvers define the demand series and lead time demand distribution parameters; they specify a lead time demand distribution and determine an inventory policy. Resolvers use demand class and lead time demand distribution which is determined based on the lead time demand distribution parameters (lead time demand and lead time demand standard deviation) in order to recommend inventory policies (see Table 1).

Table1: Inventory policy recommendation based on demand class lead time demand distribution

Demand Class Details	Lead-Time Demand Distribution	Policy
Extremely Slow	None	Make-to-Order
Smooth	Normal	R,Q
Erratic	Mixture of Distributions	s, S
Slow-Low Variable	Poisson/Mixture of Distributions	Base Stock
Slow-Highly Variable	Poisson/Mixture of Distributions	s, S
Lumpy	Negative Binomial	T,S

2.2. Top level fractal

As part of the cycle stock optimisation in the supply network (Saad and Bahadori, 2015), analysers of the fractals in the top level have to measure the replenishment cycle stock of both finished products and components, inventory holding costs and transportation costs by investigating different days between replenishment during the demand period. Therefore, mathematical equations governing the problem of cycle stock replenishment, inventory holding costs and transportation costs are presented in the following sections:

- To calculate replenishment cycle stock in a supply network, analysers consider the days between replenishment; period time and the flow quantity per period from source fractal to destination fractal, which is the sum of the total demand and safety stock (see equation 10 and 11).

$$RCS=DBR\times\left(\frac{q_{i\to r}}{2T}\right) \tag{10}$$

where:

- RCS= replenishment cycle stock
- DBR = days between replenishment
- q = flow quantity per period
- i= the index for source fractal
- r= the index for destination fractal

- T= period time

where:

$$q_{i\rightarrow r}=TD + SS \quad (11)$$

where:

- TD= total demand.

➢ The inventory holding cost of components in each fractal in the upstream stage can be calculated using total components inventory which is the sum of the safety stock, replenishment cycle stock and the in-transit component inventory where the in-transit component inventory comprises components that are on order but have not arrived, component value, time period and inventory carrying cost (see equations 12, 13 and 14). The inventory holding cost in each fractal in the downstream stage can be calculated using total finished products which is the sum of the safety stock, cycle stock and in-transit finished products inventory where the in-transit component inventory comprises finished products that are on order but have not arrived, product value, time period and inventory carrying cost (see equations 15, 16 and 17).

$$IHC_{(C)}=T_{(CI)}\times C_{(v)}\times \frac{T}{365}\times I_{(cc)} \quad (12)$$

where:

- $IHC_{(C)}$ = inventory holding cost of components
- $T_{(CI)}$ = total components inventory
- $I_{(cc)}$ =Inventory carrying cost

where:

$$T_{(CI)}=SS+RCS+IT_{(CI)} \quad (13)$$

where:

- $IT_{(CI)}$ = in-transit component inventory

$$IT_{(CI)}=\frac{q_{i\rightarrow r}\times t}{T} \quad (14)$$

where:

- t= Transportation time

$$IHC_{(Pr)}=T_{(Pr)}\times Pr_{(v)}\times \frac{T}{365}\times I_{(cc)} \quad (15)$$

where:

- $IHC_{(Pr)}$ = inventory holding cost of finished products
- $T_{(Pr)}$ = total finished products inventory

where:

$$T_{(pr)}=SS+RCS+IT_{(PrI)} \quad (16)$$

where:

- $IT_{(PrI)}$ = in-transit finished products inventory

where:

$$IT_{(PrI)}=\frac{q_{i\rightarrow r}\times t}{T} \quad (17)$$

➢ To calculate transportation cost, analysers determine the number of shipments during the demand period between the source fractal and destination fractal by dividing the flow quantity per period from source fractal to destination fractal to the replenishment quantity (see equations 18 and 19).

$$NOS = \frac{q_{i\rightarrow r}}{RQ} \quad (18)$$

where:

- NOS = numbers of shipment
- RQ= replenishment quantity

where:

$$RQ = DBR \times \mu_d \quad (19)$$

As one of the fractal units, analysers use the number of shipments to specify total travel distance from source fractal to destination fractal (see equation 20).

$$T_{td}= td \times NOS \quad (20)$$

where:

- T_{td} = total travel distance
- td=travel distance

Finally, transportation costs from source fractal to destination fractal are calculated using equation 21:

$$T_{(c)\, i\rightarrow r}= T_{td} \times A_{(C)} \quad (21)$$

where:

- $T_{(c)\, i\rightarrow r}$ = transportation cost from source fractal to destination fractal
- A(c) = average transportation cost per mile.

Since different numbers of days between replenishments were investigated among fractals by analysers, resolvers integrate both inventory holding costs and transportation costs to choose the best match and find the optimum amount of cycle stock to achieve lower total logistics cost among fractals. Moreover, resolvers determine the optimum production frequencies for the production fractals based on logistics cost optimisation results.

3. CONCLUSION

In this paper, a new proposed framework for the information fractal with two levels named top and bottom level fractals was proposed to manage and optimise inventory in the supply network. Fractals In the bottom level traced observed and analysed its downstream fractal demand and determined optimum safety stock and inventory policy which in turn shared with fractal information centres in the top level fractal. Based on these information, information fractal chain centres of the top level fractal achieved the lowest total logistics cost among fractals of the bottom level fractal by integrating both inventory holding costs and transportation costs and determined and shared optimum cycle stock for each fractal. It is expected that one of the benefits of the proposed framework is the increase of both collaboration and integration through the supply network. Moreover, it will provide a systematic method through which practitioners should be able to decide upon the demand analysis, optimisation of both safety stock and cycle stock.

Examining the proposed framework to explore its benefits was reported for future work through which it will be applied on real supply network utilising simulation software, mathematical programming and full experimental design techniques to consider all the combinations with a full statistical analysis in order to have a comprehensive set of results, which may lead to possible generalisation. This work has been commenced and will be reported in different research paper in near very future.

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