ABSTRACT
Optimization problem for planning the reconfiguration of a production-inventory-transportation network in a supply chain is studied. A dynamic multi-commodity planning model is presented to support the reconfiguration of a production-inventory-transportation network in a supply chain under conditions of uncertainty and structure dynamics.

Keywords: production-inventory-transportation network, structure dynamics, reconfiguration, supply chain, optimal control.

1. INTRODUCTION
Production-inventory-transportation network (PITN) planning is a referenced research problem that is vital in many supply chains in order to successfully meet the customer needs while improving the performance efficiency. Given a location structure from facility planning, customer demand from forecasting, and order quantities from inventory management and sourcing planning, the aggregate transportation volumes between suppliers and customers need to be determined for a middle-range period of time (e.g., one month) so that the total costs (e.g., transportation and inventory) and service level are improved (Amiri, 2006; Chopra & Meindl, 2012; Hinojosa, Kalcsics, Nickel, Puerto, & Velten, 2008; Mula, Péidro, Díaz-Madronero, & Vicens, 2010; Tayur, Ganeshan, & Magazine, 1999). The problem is typically constrained by limited capacity of nodes and arcs in the network, given service level, reverse flows, product quality, etc. (Akkermann, Farahani, & Grunow, 2010; Chen, 2010; Costantino, Dotoli, Falagario, Fanti, & Mangini, 2012; Liang, 2008; Mula et al., 2010).

PITN is a form of business organisation, which contains the following specific features: network and enterprise collaboration are organised project-oriented, supply chains in the PITN are formed customer-oriented without predetermined long-term suppliers structures and product programmes, each network participant specialises on certain functional competencies, network participants are independent and act autonomously and self-goal-oriented, for each project a coordinator exists, who is responsible for the project success (customer side) and coordinating network participant activities (supply side). The PITN must be configured according to the project goals and reconfigured in dynamics according to the current execution environment. In practice, PITN redesign decisions are centered on adapting and rationalizing the supply chains in response to permanent changes of network itself and its environment (Harrison et al., 2005). Efficient supply chain reconfiguration is a critical source of competitive advantage given that as much as 80% of total product cost may be fixed by these decisions (Harrison et al., 2005). About 75% of operative costs in value chains are caused by supply chains (Wannenwetsch, 2005). That is why we consider the problem of PITN dynamical reconfiguration as a critical point in the PITN research. Subject of this contribution is to elaborate a methodological basis of reconfiguration for the PITN in the settings of adaptation. The focus lies on the PITN, which are characterized by high structure dynamics due to flexible customer-oriented networking of core competencies.

The goal of this study is to develop a DN planning and analysis model and to apply it for a concrete case-study and based on an original methodical approach called structure dynamics control (SDC) (Ivanov et al. 2010).

Modern SCM (Supply Chain Management) is a complex system consisting of various objects, including plants in the structure to distribute, distribution centres with the capabilities of physical transformation products. Key elements of the supply chain are often presented (Ivanov, Sokolov, 2010) as the diagram shown in Fig. 1.

Fig. 1. The main elements of the supply chain.
following operations: receiving the raw materials and storage of raw materials, production of finished products (processing of raw materials), storage of finished products, transportation of raw materials and finished products, delivery of finished products to the consumer. It should be noted that PITN nodes can perform some or all of the above operations.

Logical interconnection of process operations performed in PITN nodes is shown in Fig. 2.

2. PROBLEM STATEMENT AND MODEL

As an example, let’s consider the production-inventory-transportation network of SCM.

The triangles in Fig. 3 characterize the warehouse of the network nodes for raw materials (gray triangles), for finished products (light triangles). The rectangles in Fig. 3 characterize the manufacturing process in the nodes. The supplier delivers various types of raw materials through the nodes 1 and 6. Node 4 is the central node of distribution of finished products. Node 5 is the regional centre of distribution of finished products. Raw materials can be processed in nodes 1 and 6, and in intermediate terminals 2, 3 and 7. In order to take into account possible problems with channel 4-5, 7 terminal outsourcing is used as an alternative route for supplies to distribution centre 5. In addition, you can move finished products, in small quantities, from terminal 2 into centre 5 directly.

We assume that as a result of the destructive effects of the different nature, the structure and parameters of PITN can change, but, at that, they can be constant in some time intervals (Ivanov et al., 2013; Pavlov, 2013). Version of such structural reconfiguration is shown in Fig. 4.

Meaningful statement of a task for PITN planning has the following features. If the volumes of raw materials exceed PITN capabilities of processing, transport, storage, then unclaimed raw material is sent back to the warehouse outside of PITN. In addition, if the volume of finished products exceed PITN capabilities of transportation, warehousing and customer demand, the unclaimed finished products are disposed. All of this requires additional costs. It is also necessary to consider the constant transportation and storage costs, variable costs for the transportation, storage, processing of products. As a whole, the problem consists of finding a plan for the production, storage, transportation of supplied products with the purpose of meeting consumer demand with minimal total cost.

To formalize the proposed problem, we introduce the following functions describing the peculiarities of the supply, production, storage, transportation, and delivery of raw materials and finished products:

\[ V_{\rho}^i(t) \] - volume of the warehouse of raw materials of \( \rho \) type in each \( A_i \) node in the PITN;

\[ V_{\tau}^i(t) \] - volume of a warehouse of finished products of \( \tau \) type in each \( A_i \) node in the PITN (or \( V_{\tau}^m(t) \) - volume of multipurpose warehouse of raw materials and finished products in each \( A_i \) node of PITN);

\[ \gamma_{\rho}(t) \] - intensity of deliveries of raw material of \( \rho \) type in \( A_i \) node of PITN;

\[ \gamma_{\tau}(t) \] - intensity of output of finished products of \( \tau \) type in \( A_i \) node of PITN;

\[ k_{\rho \tau} \] - the ratio of raw material consumption of \( \rho \) type (the ratio of consumed raw material volume to volume of the resulting finished products) for output of the finished product unit of \( \tau \) type in \( A_i \) node of PITN;
\( \omega_{\rho t}^c(t) \) - intensity of transportation of raw materials of \( \rho \) type between nodes of PITN;
\( \omega_{\rho t}^p(t) \) - intensity of transportation of the finished product of \( \tau \) type between nodes of PITN;
\( \phi_{\tau}(t) \) - intensity of delivery of finished products of \( \tau \) type from \( A_i \) node of PITN to the consumer.

From substantial statement of the PITN planning problem, it follows that the dynamics of changing the quantity of raw materials and finished products passing through \( A_i \) node of PITN can be described by the following relations

\[
\dot{X}_{\rho t}^c(t) = \dot{X}_{\rho t}^c(t) + \dot{Y}_{\rho t}^c(t) + \dot{Z}_{\rho t}^c(t), \quad i \in N = \{1, 2, \ldots, n\}, \quad \rho \in \mathbb{P},
\]
\[
\dot{X}_{\rho t}^p(t) = \dot{X}_{\rho t}^p(t) + \dot{Y}_{\rho t}^p(t) + \dot{Z}_{\rho t}^p(t), \quad i \in N = \{1, 2, \ldots, n\}, \tau \in \Theta.
\]

In these expressions (1)-(2), designated via \( \dot{X}_{\rho t}^c(t), \dot{X}_{\rho t}^p(t) \) is, respectively, intensity of deliveries of raw materials and finished products in \( A_i \) node of PITN, \( \dot{Y}_{\rho t}^c(t), \dot{Y}_{\rho t}^p(t) \) - intensity of return of unclaimed raw materials and disposal of finished products in \( A_i \) node of PITN.

In accordance with the designation introduced in a meaningful statement of the problem, intensity of the supply of raw materials into \( A_i \) node can be described by the following relation (3):

\[
\dot{X}_{\rho t}^c(t) = \psi_{\rho t}^c(t) + \sum_{j=1}^{n} e_{\rho j}^c(t) \cdot \omega_{\rho j}^c(t) \cdot u_{\rho j}^c(t), \quad i \in N, \rho \in \mathbb{P},
\]

where \( u_{\rho j}^c(t) \in \{0,1\} \) - management of transportation of raw materials on PITN. If \( u_{\rho j}^c(t) = 1 \), then raw material of \( \rho \) type is transported from \( A_j \) node to \( A_i \) node, and if \( u_{\rho j}^c(t) = 0 \), then the raw materials are not transported.

The intensity of the raw material reduction in \( A_i \) node of PITN can be described by the relation (4)

\[
\dot{X}_{\rho t}^c(t) = \sum_{t \in \Theta} k_{\rho t}^c \cdot \gamma_{\rho t}^c(t) \cdot \theta_{\rho t}^c(t) + \sum_{j=1}^{n} e_{\rho j}^c(t) \cdot \omega_{\rho j}^c(t) \cdot u_{\rho j}^c(t), \quad i \in N, \rho \in \mathbb{P},
\]

where \( \theta_{\rho t}^c(t) \in \{0,1\} \) - management of finished products output in \( A_i \) node. Here, if \( \theta_{\rho t}^c(t) = 1 \), then the costs of raw materials of \( \rho \) type for the finished products of \( \tau \) type in \( A_i \) node of PITN, otherwise \( \theta_{\rho t}^c(t) = 0 \).

Accordingly, the intensity of supply and the finished products reduction can be described by relations (5) - (6):

\[
\dot{X}_{\rho t}^c(t) = \psi_{\rho t}^c(t) \cdot \theta_{\rho t}^c(t) + \sum_{j=1}^{n} e_{\rho j}^c(t) \cdot \omega_{\rho j}^c(t) \cdot u_{\rho j}^c(t), \quad i \in N, \tau \in \Theta,
\]

where \( u_{\rho j}^c(t) \in \{0,1\} \) - management for finished products transportation on PITN, \( \theta_{\rho t}^c(t) \in \{0,1\} \) - management of finished products delivery to the consumer from \( A_i \) node of PITN.

Relations (1)-(6) describe the dynamics of production, transportation, delivery, storage, and additional operations with raw materials and finished products on PITN. It should be noted that \( \psi_{\rho t}^c(t), \psi_{\rho t}^p(t), \omega_{\rho j}^c(t), \omega_{\rho j}^p(t), \phi_{\tau}(t) \) are prescribed functions, \( e_{\rho j}^c(t) \) describes structural reconfiguration of PITN on the appropriate script in the management interval \( T = (t_0, t_f] \), while the functions \( y_{\rho t}^c(t), y_{\rho t}^p(t), z_{\rho t}^c(t), z_{\rho t}^p(t), u_{\rho j}^c(t), u_{\rho j}^p(t) \) are unknown quantities, and management \( u_{\rho j}^c(t), u_{\rho j}^p(t), \theta_{\rho j}^c(t), \theta_{\rho j}^p(t) \) uniquely define the operations of return of unclaimed raw materials \( z_{\rho t}^c(t) \) and disposal of finished products \( z_{\rho t}^p(t) \), dynamics of accumulation and transportation of raw materials \( y_{\rho t}^c(t) \) and finished products \( y_{\rho t}^p(t) \) from the warehouse of \( A_i \) node in the PITN.

Therefore, the functions \( y_{\rho t}^c(t), y_{\rho t}^p(t), z_{\rho t}^c(t), z_{\rho t}^p(t) \) can be considered as the dynamic system state.

In these conditions, the PITN planning model will include the following main elements. Model of the PITN planning process is prescribed by relations (1)-(6). Restrictions for the possible managements and states can be described as follows

\[
0 \leq y_{\rho t}^c(t) \leq V_{\rho t}^c, \quad 0 \leq y_{\rho t}^p(t) \leq V_{\rho t}^p, \quad \forall i \in N, t \in (t_0, t_f], \rho \in \mathbb{P}, \tau \in \Theta,
\]

\[
0 \leq \sum_{t \in \Theta} y_{\rho t}^c(t) + \sum_{t \in \Theta} y_{\rho t}^p(t) \leq V_{\rho i}^m, \quad \forall i \in N, t \in (t_0, t_f],
\]
\[
\begin{align*}
\forall i, j \in N, t \in (t_0, t_f), \rho \in P, \tau \in \Theta, \\
J_{ij}^c(t), u_{ij}^p(t) \in [0,1], \theta_{ij}^p(t), \gamma_{ij}^p(t) \in [0,1] \\
z_{ij}^c(t) \geq 0, z_{ij}^p(t) \geq 0, \dot{z}_{ij}^c(t) \geq 0, \\
\dot{z}_{ij}^p(t) \geq 0 \forall i \in N, t \in (t_0, t_f), \rho \in P, \tau \in \Theta.
\end{align*}
\]

The boundary conditions look as follows:
\[
\begin{align*}
y_{ij}^c(t_0) &= y_{ij}^p(t_0) = z_{ij}^c(t_0) = z_{ij}^p(t_0) = 0 \\
y_{ij}^c(t_f) \geq 0, y_{ij}^p(t_f) \geq 0, z_{ij}^c(t_f) \geq 0, \\
z_{ij}^p(t_f) \geq 0 \forall i \in N, \rho \in P, \tau \in \Theta.
\end{align*}
\]

Admissible managements \(u_{ij}^c(t), u_{ij}^p(t), \theta_{ij}^p(t), \gamma_{ij}^p(t)\), that satisfy equations (1)-(6), describing change of the state along the trajectory \((7)-(9)\) and boundary conditions \((10)\), can be evaluated according to various quality indexes for PITN functioning.

Such indexes can be:
\[
\begin{align*}
J_1 &= \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} v_{ij}^c(t) \cdot z_{ij}^c(t) dt + \\
&+ \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} v_{ij}^p(t) \cdot z_{ij}^p(t) dt \\
&\text{the unclaimed raw materials and disposal of finished products;} \\
J_2 &= \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} h_{ij}^c(t) \cdot \phi_{ij}^c(t) \cdot \gamma_{ij}^c(t) dt - \text{cost of the finished products delivered to the consumer;} \\
J_3 &= \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} g_{ij}^c(t) \cdot \gamma_{ij}^c(t) \cdot \epsilon_{ij}^c(t) dt + \\
&+ \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} \sum_{j_1}^n \delta_{ij}^c(t) \cdot \omega_{ij}^c(t) \cdot \epsilon_{ij}^c(t) dt + \\
&+ \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} d_{ij}^c(t) \cdot \phi_{ij}^c(t) \cdot \epsilon_{ij}^c(t) dt + \\
&+ \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} \sum_{j_1}^n f_{ij}^c(t) \cdot V_{ij}^c(t) \cdot \epsilon_{ij}^c(t) dt \\
&\text{fixed manufacturing, transport and storage costs;}
\end{align*}
\]

\[
J_4 = \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} \sum_{j_1}^n r_{ij}^c(t) \cdot \omega_{ij}^c(t) \cdot u_{ij}^c(t) dt + \\
+ \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} \sum_{j_1}^n \delta_{ij}^c(t) \cdot \gamma_{ij}^c(t) \cdot \theta_{ij}^p(t) dt + \\
+ \int_{t_0}^{t_f} \sum_{\rho_0 \in P} \sum_{\tau} \sum_{j_1}^n \delta_{ij}^c(t) \cdot \gamma_{ij}^c(t) \cdot \gamma_{ij}^c(t) dt
\]

- the costs of production, transportation, delivery, and storage of products.

3. EXPERIMENTAL RESULTS

The presented model falls into the class of optimal program control (OPC) models. The structure dynamics (i.e., the execution scenarios according to different structural states), inventory dynamics, and transitions between the intervals can be modeled in the proposed dynamic OPC model. However, the detail degree of the OPC model is too high for aggregate planning. Besides, in this model the right parts of the differential equations undergo discontinuity at the beginning of interaction zones. Piecewise functions \(e_{ij}^c(t)\) are contained in the right parts of Eqs \((3)-(6)\) in order to describe the structure dynamics. The considered problems can be regarded as control problems with intermediate conditions. Another feature is the form of time-spatial, technical, and technological non-linear conditions that are mainly considered in control constraints and boundary conditions. This is why the application of known direct methods for solving the above-stated OPC problem becomes complicated.

However, the structure and the parameters of PITN undergo changes at discrete time points \((t_0, t_1, \ldots, t_f)\). These points divide the planning interval \((t_0, t_f] \) into sub-intervals \(L\), \(T = (t_0, t_1, t_2 \ldots, t_{k-1}, t_k \ldots, t_{L-1}, t_L = t_f]\). The PITN structure does not vary at each \(k\)-sub-interval \(T_{ik} = (t_{k-1}, t_k]\). The assumption on the intervals of structural constancy allows transit from the dynamic to static models (Ivanov et al., 2013). Analysis of the static problem showed that in spite of its high dimensionality it could be efficiently solved via special decomposition procedures and the method of successive improvement of plans for LP problems with two-sided constraints. For experiments with only the static model, computations can be performed in commercial optimization software that supports simplex method.

Let us consider the scenario defined in Fig. 5. The PITN is composed of two mega-hubs (nodes 1 and 6), a central distribution hub (node 4), two intermediate terminals (nodes 2 and 3), an outsourcing terminal (node 7), and a regional distribution centre as the strategic inventory holding point (node 5). The execution in each of the nodes and
transportation arcs is limited by maximal warehouse capacity, processing intensity, and transportation intensity respectively. The triangles refer to the warehouse capacity, and numbers on the arcs refer to maximal transportation intensity. The suppliers first deliver goods to the mega-hubs 1 and 6. Then, the goods will be processed in the central distribution hub 4. The goods from hub 1 are additionally processed at intermediate terminals 2 and 3. From hub 4, the goods are moved to the regional distribution centre 5, which has a certain demand in each of the periods (i.e., 100 units). In practice, a number of the regional distribution centres are in the PITN. Without loss of generality, we are reducing this formulation in this particular paper to only one centre. This is the centralized PITN structure. In order to take into account possible problems with the channel 4→5, an outsourcing terminal is used as an alternative way for deliveries to reach the distribution centre 5. Besides, it is possible to move small quantities (maximal 30 units) directly from terminal 2 to centre 5. Three intervals of the structural constancy are considered. The problem is to maximize the service level under the assumption of the demand of 300 units for the planned period of three months (i.e., 100 units each month) while minimizing the costs as composed of the storage, transportation, return, sourcing, and fixed costs. For simplification, processing costs and capacities are not considered in this example.

The same example has been calculated with greedy algorithm. A greedy algorithm is an algorithm that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding a global optimum. The corresponding plan is presented in Fig. 7.

![Fig. 5. Scenario of reconfiguration.](image)

In Fig. 6, results of optimal planning subject to the data from Fig. 5 are presented.

![Fig. 6. Distribution plan for scenario of reconfiguration.](image)

Table 1. Planning results

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Proposed model</th>
<th>Greedy algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of delivered goods</td>
<td>300.0</td>
<td>280.0</td>
</tr>
<tr>
<td>Volume of returned goods</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Volume of inventory total</td>
<td>120.0</td>
<td>150.0</td>
</tr>
<tr>
<td>Transportation costs</td>
<td>30.0</td>
<td>26.6</td>
</tr>
<tr>
<td>Inventory costs</td>
<td>8.4</td>
<td>10.5</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Sourcing costs</td>
<td>124.0</td>
<td>124.0</td>
</tr>
<tr>
<td>Total costs</td>
<td>252.4</td>
<td>251.1</td>
</tr>
<tr>
<td>Total revenue</td>
<td>300.0</td>
<td>280.0</td>
</tr>
<tr>
<td>Profit</td>
<td>47.6</td>
<td>28.9</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

The problem of optimization of production-inventory-transportation network (PITN) supply chain has been investigated. Some extensions of this model in future can be considered. First, it is possible to extend the developed model to operative detailed decisions on transportation planning. The possibility of addressing decision components of different time horizons and levels of detail arises from a combination of a static LP and dynamic OPC models. This combination can result in a hybrid multi-period distribution-transportation model that can be investigated in detail in future research. Second, in comparing the presented approach with the literature, dimensions of agility and flexibility as well as environmental performance can be integrated subject to the trade-off “efficiency-flexibility-resilience”. Different
data, e.g., fuzzy representation, can be considered. Additional restrictions, e.g., total budget with time value or other optimization objectives, e.g., minimizing delivery times, can be included in future analysis. On-line adaptation can be the next possible future research direction. Here different adaptation options (e.g., flow re-direction, capacity adjustment, and structure adjustment) and their costs can be compared. In addition, a comparison between investments in robustness vs. costs of adaptation can be made.

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