WHATIF: A DECISION SUPPORT SYSTEM FOR PLANNING AND MANAGEMENT OF RAIL FREIGHT NETWORKS

Andrea Emilio Rizzoli[(a)], Matteo Salani[(a)], Corrado Valeri [(a)], Janos Barta[(a)], Simone Croci-Torti [(b)], Aldo Croci [(b)] Angelo Barbone [(b)], Luca Maria Gambardella [(a)]

[(a)]IDSIA USI/SUPSI, Istituto Dalle Molle di Studi sull'Intelligenza Artificiale, CH 6928 Manno, Switzerland
[(b)]Hupac Intermodal SA, 6830 Chiasso,

[(a)]andrea@idsia.ch [(b)]scrocitorti@hupac.ch

ABSTRACT
We present a Decision Support System to assist operators of rail freight networks in planning their activities and daily operations.
The DSS is interfaced with the information system of the rail freight company in order to access the most complete and up-to-date information on the location of rolling stock over the rail network, and the maintenance status of each wagon, on the current state of train circulation.
Given static information on the characteristics of the rolling stock fleet, and given the scheduled connections among terminals, an efficient rolling stock circulation plan, i.e. the routes of train compositions periodically connecting two or more terminals, is computed. Thanks to dynamically updated information and on the above mentioned circulation plan, a set of efficient train dispatching assignments, that is, the assignment of incoming train in a terminal to outgoing train rotations, is also obtained and proposed to the operational manager.

Keywords: rail freight network, decision support system, train circulation optimization, train dispatching.

1. INTRODUCTION

Managing a rail freight network at the continental scale is a complex business, not only for the sheer combinatorial complexity due to the large numbers of terminal relations to be served, and the dynamic complexity due to the temporal dimension of transports, but also for the extremely tight competition by road transport, which offers great flexibility, higher timeliness, and a competitive cost structure.

Yet, rail-based transport is often seen as the "way to go" and Europe in particular has been supporting rail-based transport with a number of policies, and targeted research projects over the years (Héritier et al. 2001, Marco Polo, European Commission, 2012). This is because of the ecological advantages of rail-based transport: less pollution, less particulate matter, more energy efficiency, and also less noise.

Rail-based transport is unfortunately fraught with a number of problems, the most relevant ones are the lack of an efficient network of intermodal terminals over most of Europe to allow for a quick and efficient modal change, and the lack of capacity on some key links. In particular, the high congestion observed on the European road network is also present in the rail network, and such traffic intensity makes the network more fragile and more susceptible to disruptions. As a consequence, the reliability of rail freight transport, which often has to yield to passengers transport, is at stake.

Rail freight companies face a difficult situation: they need to reduce their cost structure in order to compete with road-based transport, which has the great advantage of not externalizing its environmental costs (CO_2 taxes on fuel are negligible, as well as road freight transport taxes which are present only in some countries such as Germany, Austria and Switzerland).

While reducing costs, rail freight companies need to match a demand that is expected to grow by 80% by 2050 (European Commission, 2011) but at the same time can be highly volatile, especially during the current economical crisis. Furthermore, companies need to gain access to a rail network which has not been adequately developed during the past 50 years, at least since the 60s, when focus during the European economic boom shifted to road-based transport.

The logical consequence is that rail freight companies need to increase their efficiency, and this can be achieved by optimising the amount of rolling stock by a better planning of its usage and maintenance, and optimising the number of trains serving relationships among terminals.

In order to support rail freight companies in their operations, IDSIA has teamed with Hupac Intermodal SA, one of the leading European companies, in order to develop a suite of methods, algorithms and efficient and user friendly implementations to support the managers at various management levels: from strategic planning to operational management. Strategic planning involves the rolling stock circulation problem (Alfieri et al., 2006, Fioole et al., 2006) while operational management concerns online train dispatching (D'Ariano and Pranzo, 2009, Krasemann, 2012, Corman et al. 2011).

The result is an innovative decision support system (DSS) named WHATIF, which we describe in this paper.

In the next sections we first introduce the formalization of the problems we wanted to solve. We then show some examples of its use. Finally we discuss the first feedback obtained during the development phase, and we prospect future developments.

2. THE FORMALIZATION OF THE PROBLEM

The approach to the problem was inspired by the two level decision support system by Soncini et al. (1990), which decomposes a system management problem into two hierarchically structured problems: first a management problem is solved offline on a longer horizon, using a feedforward scheme; the solution of the above problem is then used as the reference value (set-point) for the solution of an online operational management problem, which is solved on a shorter time horizon. As shown in Figure 1, WHATIF DSS is structured in the following elements:

- WHATIF Composer (WCO): a software application that computes optimized train circulation plan. WCO can be used for strategic planning regarding the opportunity of opening new or closing existing relationships and by modifiying the schedule or the timetable. WCO solves the rolling stock circulation problem as shown in Section 2.1.
- WHATIF Planner (WPL): a software application for the operational dispatching of trains from terminal to terminal. WPL, also using the WHATIF simulator briefly described below, solves the train dispatching problem, as shown in Section 2.2.
- WHATIF Simulator: given the train dispatching decisions, the impacts and future evolution of the network state are projected in the near future.

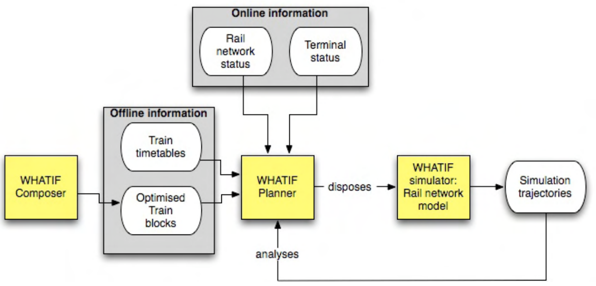

Figure 1: The logical structure of the WHATIF DSS and its modules.

In our knowledge, this is the first time such an integrated approach has been adopted in the context of rail freight planning and management. Previous developments of DSSs to the rail domain were mostly focused either on train rescheduling at the rail network operational management level or on terminal management issues (e.g. yard management). On the other hand, WHATIF DSS is directly aimed at and focuses on the business model of the intermodal transport company.

2.1. The Rolling Stock Circulation Problem

In the context of the rail freight network problem, the offline management problem is solved to compute the optimized train circulation, using as inputs the total availability of rolling stock and the timetables which specify the availability of a connection between two or more terminals during a specified time period. Currently, timetables are purchased by the freight operator from network operators such as DB in Germany, RFI in Italy, SBB in Switzerland. We refer the reader to Caprara et al. (2002), Peeters (2003) and Cacchiani (2007) for the mathematical modeling and solution procedures to the planning of optimal timetables. Timetables are purchased according to forecasts of transport demand. Unfortunately there are many constraints on the availability of timetables, especially passenger transport and the limited capacity for infrastructure, and therefore they cannot be considered a decisional variable for the rail freight company. Therefore, we consider the timetable as given.

The timetable is specified as a set of services S between one or more terminals. We denote by o_s^k and d_s^k the k-th origin and destination of service $s \in S$, respectively. With each service s are associated departure time a_s^k and arrival time b_s^k at origin and destination terminals, respectively. Departure and arrival times include the time necessary to load and unload freight and to perform the necessary security checks.

The rolling stock is classified into wagon types, denoted by the set H, having specific characteristics, i.e., the ability of carrying different cargo types (different classes of freight, from containers to semi-trailers) and specific length. In order to harmonize the circulation in the network and to reduce the operational costs of frequent shunting operations, freight transportation companies classify services according to recurrent demand patterns referred as families of composition or train units. Each pattern $p \in P$ is defined by the number of modules of each wagon type composing the pattern and denoted by q_p^h. The estimated traffic demand on service s is therefore converted into a specific demand for pattern q_s^p.

We represent the rolling stock circulation problem as a multi-commodity flow with additional constraints on a time-space network $G = (N, A)$. The network is made of nodes $n \in N$, which represent events (either departures or arrivals) at specific terminals and arcs $a \in A$ which represent either travel durations when connecting nodes at different terminals or waiting times when connecting nodes at the same terminal.

Connections must be feasible, i.e., $a_s^k \geq b_s^k \; \forall \; k \in K, \forall \; s \in S$.

The definition of sets, parameters and variables used to model the Rolling Stock Circulation (RSC) are provided in Tables (1)-(3).

Table 1: Sets for the RSC model.

Description	*Set*
Set of terminals (nodes) indexed by n	N
Set of services indexed by s	S
Set of available patterns indexed by p	P
Ordered set of all events on the time-space network, indexed by t	T
Set of nodes in the time-space network	$V(n,t)$
Set of inbound services to node (n,t)	$I(n,t)$
Set of outbound services from node (n,t)	$O(n,t)$

Table 2: Parameters for RSC model.

Description	*Parameter*
Demand of pattern p in service s	q_s^p
Number of available patterns p	N^p
The first event in the time line at terminal n	$minE_n$
The last event in the time line at terminal n	$maxE_n$

Table 3: Decision variables for RSC model.

Description	*Variable*
Number of patterns p on service s	x_s^p
Number of patterns p at terminal n immediately after time t	y_{n,t^+}^p
Number of patterns p at terminal n immediately before time t	y_{n,t^-}^p

The RSC model is defined as follows:

$$min \sum_{n \in N, p \in P} y_{n,minE_n^-}^p \tag{1}$$

$$s.t. \; x_s^p \geq q_s^p \; \forall s \in S, \forall p \in P \tag{2}$$

$$y_{n,t^-}^p + \sum_{s \in I(n,t)} x_s^p = y_{n,t^+}^p + \sum_{s \in O(n,t)} x_s^p \; \forall (n,t) \in V, p \in P \tag{3}$$

$$y_{n,minE_n^-}^p = y_{n,maxE_n^+}^p \; \forall n \in N, p \in P \tag{4}$$

$$x_s^p \in \mathbb{Z}^+ \; \forall s \in S, p \in P \tag{5}$$

$$y_{n,t}^p \geq 0 \; \forall (n,t) \in V, p \in P \tag{6}$$

Objective function (1) minimizes the number of necessary patterns to perform the schedule. If $\sum_{n \in N} y_{n,minE_n^-}^p \leq N^p \; \forall p \in P$, there exist a feasible and optimal rolling stock circulation plan. Otherwise, $N^p - \sum_{n \in N} y_{n,minE_n^-}^p$ represents the number of patterns p necessary to fulfill the schedule. In such case, the company has to rent additional wagons from third parties. Constraints (2) impose that each service must be formed by an appropriate amount of patterns. Constraints (3) are flow conservation constraints. Constraints (4) are optional. When imposed, they induce a cyclic rolling stock circulation plan. Finally, bounds on variables are defined by constraints (5)-(6). The model is solved using a general purpose MILP solver.

Currently, decision makers planning for rolling stock circulation at our industrial partner use a FIFO policy, i.e., at each terminal, each incoming train is associated with the earliest outgoing train. Under this policy, the rolling stock circulation can be solved in polynomial time by a dynamic programming algorithm.

2.2. The Dispatching Problem

At the operational level, the role of the train dispatchers is to maintain the train circulation plan. In each terminal the dispatcher sees a number of incoming trains, which need to be unloaded and then reassigned to one of the outgoing train compositions. A train composition is a set of wagons with specific characteristics with respect to the load they can carry. According to the specific terminal relationship, different compositions will be used.

Had everything been running perfectly, the same train compositions would circulate connecting the same terminals, according to the circulation plan.

This is unfortunately impossible due to unexpected disturbances on the rail network (e.g. breakdowns), strikes and breakdowns in terminals, and delays. The dispatcher needs therefore assistance in making the best decisions in order to stick as much as possible to the train circulation plan, which guarantees the most efficient use of the fleet. Whenever a train is rerouted on a different relationship, its composition might be less than ideal, and therefore a compromise needs to be made.

The profitability indicator measures the length of the incoming train with respect to the desired length of the outgoing train. Train length is an aggregated and widely used performance criterion in railway freight transportation. Mathematically, let $\sum_{p \in P} \mathrm{len}(q_{in}^p)$ be the total length of the patterns composing the incoming train in. Then the profitability indicator is defined as:

$$I_{pr} = \min\left\{1, \sum\nolimits_{p \in P} len(q_{in}^p)/len(q_{out}^p)\right\}$$

Two conformity indicators measure the compliance of wagon patterns circulating on the incoming train with respect to the desired wagon patterns of the outgoing train. We remark that some wagon types are able to accommodate the cargo type of other wagon types, even if they are not specifically conceived for it. Therefore, it is possible to define a compatibility indicator between two different patterns. Mathematically, we define the following conformity indicators:

$$I_{con} = \frac{\sum_{p \in P} \frac{\min\{q_{in}^p, q_{out}^p\}}{q_{out}^p}}{|P|}$$

For each pattern of the incoming train not accounted in the conformity indicator, we compute the compatibility with each pattern of the outgoing train and retain the maximum. We remove the pattern pair and iterate the process. We denote the compatibility computed as above by I_{com}.

The final performance indicator is used to estimate the ability of a given relationship to "absorb" the accumulated delay of the incoming train. This indicator, named the delay risk indicator I_{dr} is computed using a stochastic approach based a large set of historical data.

Essentially, we estimate the delay propagation as follows. Let r_0 be an arriving train with delay δ_0 and $\{r_1, \dots, r_m\}$ the train sequence that follows train r_0. The propagation of the delay δ_0 on the sequence $\{r_1, \dots, r_m\}$ can be modeled by a stochastic sequence $\{X_k\}_{k=0,\dots,m}$ where each X_k is a random variable describing the state of the delay at the arrival of train r_k. In our model delays are classified in 5 clusters $\{C_0, \dots, C_4\}$ usually adopted by the company: cluster C_0 means no delay, C_1 a delay within 1 hour, C_2 between 1 and 3 hours, C_3 between 3 and 6 hours and C_4 indicates a delay over 6 hours. Each random variable X_k can assume any of the 5 delay states $\{0, \dots, 4\}$ corresponding to the 5 delay clusters. In this context it is reasonable to assume that the delay state at the time instant $k+1$ depends only on the current delay state at time instant k and not on previous delay states. Therefore the stochastic sequence $\{X_k\}_{k=0,\dots,m}$ can be classified as a discrete-time Markov chain. The key idea of our approach is to extrapolate the transition probabilities $h_{ij}(k) = P[X_{k+1} = j \mid X_k = i]$ of the delay propagation Markov chain from the historical delay data set. In other terms we define the transition probabilities in such a way, that the Markov chains generated are consistent with the real delay propagation patterns observed in the past. In order to generate the transition matrices $H(k) = [h_{ij}(k)]$ we focused on the delay propagation pattern of each couple of consecutive trains $\{r_k, r_{k+1}\}$ in the chain. More precisely, we analyzed the delay data of 20'000 compositions circulated on the Hupac transportation network during a period of 6 months and we aggregated them per clusters and per couples of consecutive relations. Then for each couple of relations we calculated a transition matrix based on the historical data set. As an example we report in Table 4 the transition matrix of the couple of relations {BUS2-KOEL, KOEL-BUS2} based on 166 samples.

Table 4: Transition matrix of the couple of relations {BUS2-KOEL, KOEL-BUS2}

	C0	C1	C2	C3	C4
C0	0.39	0.43	0.10	0.04	0.04
C1	0.34	0.16	0.34	0.05	0.11
C2	0.36	0.35	0.18	0.04	0.07
C3	0.56	0.22	0.11	0.00	0.11
C4	0.32	0.09	0.27	0.14	0.18

For example, we read from Table 4 that 39% of the trains arrived on time on the relation BUS2-KOEL, came back on time to BUS2, while 43% of the trains on time the way there report a delay in cluster C_1 on the way back. The transition matrices of the couples of consecutive relations enable us to forecast the risk of delays after a given sequence of relations. In fact, given an initial delay distribution $d(0)$ and a sequence of transition matrices $\{H(k)\}_{k=0,\dots,m-1}$, the forecasted delay distribution $d(m)$ after the train sequence can be calculated on the basis of the Chapman-Kolmogorov equation as

$$d(m) = \left(\prod_{k=0}^{m-1} H(k)\right)^T \cdot d(0) .$$

As an example, let's consider a train with a delay of 80 minutes on the relation BUS2-KOEL, which is assigned to the rotation {KOEL-BUS2, BUS2-KOEL}. What kind of delay distribution might be expected after the rotation? By means of the transition matrices of the 2 couples of relations involved we obtain the output distribution $d(2) = [0.43, 0.21, 0.19, 0.05, 0.12]$. This result means that, according to the delay evolution observed in similar situations in the past, we can expect that the train will absorb the present delay with a probability of 43%, whereas a delay of cluster C_1 can be expected with a probability of 21%, and so on.

In order to compare the quality of the assignment of a train t_0 to different possible rotations it is necessary to reduce the information of the output distribution $d(m)$ to a numerical indicator that can be easily interpreted. The most straightforward indicator of the delay propagation is the expected value of the output distribution $d(m)$. In the previous example we obtain an expected value of 1.21, which means that in average we might expect an output delay of cluster C_1. For each assignment selected by the dispatcher WHATIF calculates a normalized delay risk indicator I_{dr} which depends on the sequence of trains $\{r_k\}$ and on the initial delay δ_0

In summary, we compute the above described performance indicators for every feasible pair of incoming and outgoing trains at a terminal and we formulate the dispatching problem as a maximum weight matching on a bipartite graph. Let $G = (V, E)$ be a bipartite graph, i.e. a graph in which there exists a partition X, Y such that $V = X \cup Y$, $X \cap Y = \emptyset$ and $E \subseteq X \times Y$. X represents the set of incoming trains at every station and Y represents the set of rotations originating at some station. An edge $e \in E$ exists between $x \in X$ and $y \in Y$, if it is feasible to cover rotation y using the composition running on train x. The feasibility of the assignment is determined by time-space constraints, i.e., the two services must be incident at the same terminal and the incoming train must be fully unload before the expected loading time of the outgoing train. Additional feasibility constraints are imposed by dispatchers in order to improve the performances of the assignment. In particular, referring to previously introduced performance indicators, lower thresholds on performance indicators are set, we refer to these thresholds as $\tilde{I}_l\ \forall l \in L$, with $L = \{pr, con, com, dr\}$. Therefore, an assignment is considered feasible if $I_l(e) \geq \tilde{I}_l\ \forall l \in L$. We associate a weight with every edge of the graph $e \in E$. The weight is computed using a convex combination of performance indicators, i.e., $w(e) = \sum_{l \in L} \alpha_l \cdot I_l(e)$, for any $\alpha_l\ l \in L| \sum_{l \in L} \alpha_l = 1$.

A matching is defined as a subset of edges $M \subseteq E$ such that $\forall x \in X$ at most one edge in M is incident to x. The size of a matching is $|M|$. The weight of the matching is computed as the sum of the weights of edges in the matching, i.e., $w(M) = \sum_{e \in M} w(e)$. A matching M is a maximum weight matching if there is no other matching M' such that $w(M') > w(M)$. We recall that a max-weight matching is perfect, i.e., a matching in which every vertex is adjacent to some edge in M. The maximum weight matching on bipartite graphs is a polynomially solvable problem. We implemented the algorithm by Kuhn-Munkres also known as the Hungarian algorithm which runs in $O(n^3)$.

We remark that if $|X| \neq |Y|$ we create dummy vertices x' or y' and dummy edges e' with $w(e') = 0$. When dummy vertices are selected for being part of the maximum weight matching, the associated non-dummy vertices are left unmatched in the solution proposed to the dispatcher.

In order to provide additional dispatching opportunities to the operator, we iteratively solve the maximum weight matching removing the set of optimal edges from the initial set, i.e., $E \Leftarrow E \setminus M$ for a fixed number of iterations J. Let M^j be the maximum weight matching obtained at iteration $j \in \{0..J\}$. Then, M^0 is the optimal matching while M^i with $i \in \{1..J\}$ represent alternative options.

Most of the times, train assignments are chosen following best practices which are not encoded in the input data. During execution, WHATIF collects the chosen disposition alternative among a set of feasible ones. Based on this data, we have embedded in WHATIF a statistical model, which considers the most frequent assignments matching the current network status. In the maximum weight matching we consider statistically relevant alternatives with a positive contribution to the edge weight.

3. EXAMPLES OF USE

Hupac Intermodal SA (www.hupac.ch) operates more than 100 trains a day at a continental scale (see Figure 2) with regular connections to the Far East and the Asian continent. Hupac served more than 700.000 road consignments in 2011.

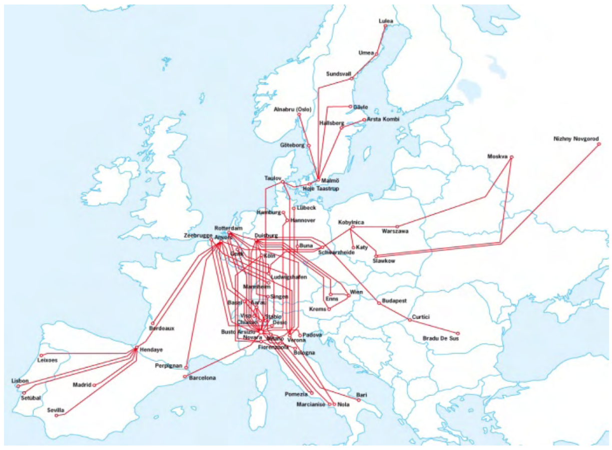

Figure 2: The continental network of Hupac Intermodal SA.

We illustrate the typical use of WHATIF Composer using a simple example. We consider the circulation plan between the terminals of Busto Arsizio (BUS2), Hamburg (HAMB) and Hannover (HANN) as shown in Figure 3. In a typical week, 6 trains circulate from north to south and vice versa. Given the timetable, WCO computes the needed rotations. (4 in the example). Using WCO, the network planner is able to evaluate schedule variations. For example, the service department detected a systematic lack of demand on Thursdays, the network planner can evaluate the effect of suppressing Thursday trains. For this network structure, suppressing the Thursday train does not produce any reduction in the number of requested compositions. Therefore, it is still convenient for Hupac to keep the Thursday train.

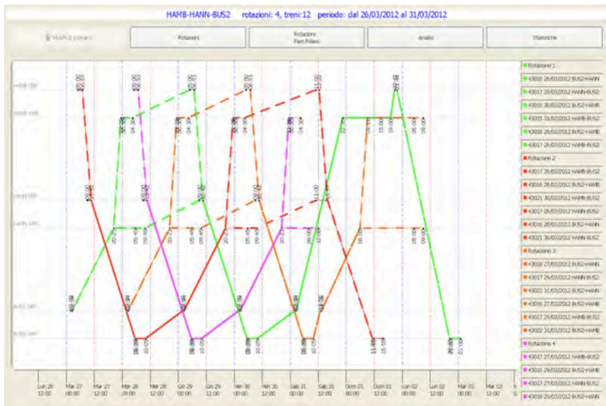

Figure 3: An illustration of the result of WHATIF Composer over a multi-terminal relationship (BUS2-HAMB/HANN) over a typical week: 12 trains are served with the rotation of 4 compositions.

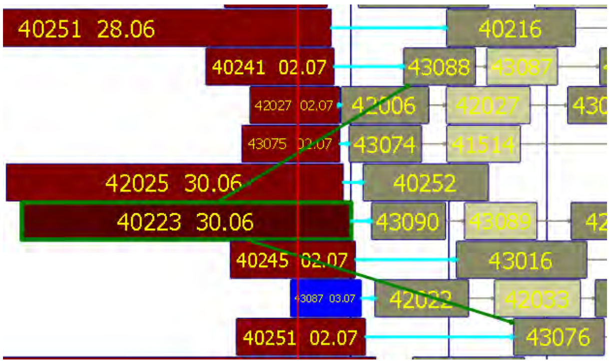

Figure 4: An illustration of the result of WHATIF Planner on a subset of trains heading to BUS2.

In Figure 4, we illustrate one of the uses of WPL. The illustration reports a subset of trains heading to Busto Arsizio terminal (BUS2, Hupac's main hub). We notice incoming trains on the left of the picture and train circulations originating in BUS2 on the right. Optimal assignments (M^0) computed by WPL are reported in light blue and are always visible. Alternative options are shown on demand to the dispatcher. In Figure 4, for example, alternative assignments for incoming train 40223 are reported in green. Alternative options can be shown for outgoing trains as well. Performance indicators for the optimal and alternative options are reported in a dedicated dialog (see Figure 5).

Dettagli associazione

LISTA ASSOCIAZIONI PER COMPOSIZIONE **910432**

Treno In-Out	Conf.	Comp.	Lung.	Rit.
40223 20120630 -> 43076 20120705	0.61	0.75	0.94	0.28
40223 20120630 -> 43088 20120704	0.61	0.75	0.94	0.27
40223 20120630 -> 43090 20120704	0.61	0.75	0.94	0.27

Figure 5. Report on performance indicators (in Italian). "Conf" is the conformity indicator, "Comp" is the compatibility indicator, "Lung" is the profitability indicator and finally "Rit" is the delay risk indicator, as described in Section 2.

4. CONCLUSIONS

The WHATIF decision support system is currently being deployed at Hupac Intermodal SA. The first feedback is encouraging, as it allows managers to solve their task in a more rapid and effective way. In particular, WHATIF is proving itself as a valuable tool for the strategic planning of commercial routes: testing whether a new connection is economically and technically viable can be done in a matter of minutes, against a previous effort of many hours. WHATIF is also demonstrating its validity as a support tool for dispatching trains from the wide network of terminals served by Hupac. The train dispatcher can easily consult the DSS to both easily access facts and figures on key train indicators, and also ask the DSS for support in making informed decisions.

ACKNOWLEDGMENTS

This research has been supported by the Commission for Technology and Innovation of the Federal Department of Economic Affairs of the Swiss Confederation, project number 10890.1 PFES-ES.

REFERENCES

European Commission, 2011. WHITE PAPER Roadmap to a Single European Transport Area – T owards a competitive and resource efficient transport system. Available from: http://ec.europa.eu/transport/strategies/2011_white_paper_en.htm [Accessed 2 July 2012].

Caprara, A., Fischetti, M., Toth, P. 2002. Modeling and solving the train timetabling problem. *Operations Research*, 50 (5), 851–861.

Peeters, L.W.P. 2003. Cyclic Railway Timetable Optimization. PhD Thesis. Erasmus Research Inst. of Management, Rotterdam, the Netherlands.

Cacchiani, V. 2007. Models and Algorithms for Combinatorial Optimization Problems arising in Railway Applications. PhD Thesis. University of Bologna, Bologna, Italy.

European Commission, 2012. Marco Polo. Available from: http://ec.europa.eu/transport/marcopolo/ [Accessed 6 May 2012].

Héritier, A., Kerwer, D., Knill, C., Lehmkuhl, D., Teutsch, M., Douillet, A-C. 2001. Differential Europe – The European Union Impact on Policymaking. Rowman & Littlefield. Oxford, UK.

Soncini-Sessa, R. Nardini, A., Gandolfi, C. and Kraszewski, A. 1990. Computer aided water reservoir management: a prototype two-level dss. In: Proceedings of the NATO ARW on Computer Aided Support Systems in Water Resources Research and Management.

D'Ariano, A., Pranzo, M. 2009. An Advanced Real-Time Train Dispatching System for Minimizing the Propagation of Delays in a Dispatching Area Under Severe Disturbances. *Networks and Spatial Economics* 9 (1), 63-84

Krasemann, J.T. 2012. Design of an effective algorithm for fast response to the re-scheduling of railway traffic during disturbances, *Transportation Research Part C: Emerging Technologies*, 20 (1), 62-78.

Corman, A., D'Ariano, A., Pranzo, M. 2011. Effectiveness of dynamic reordering and rerouting of trains in a complicated and densely occupied station area. *Transportation Planning and Technology*. 34 (4), 341-362.

Alfieri, A., Groot, R., Kroon, L., Schrijver, A. 2006. Efficient Circulation of Railway Rolling Stock. *Transportation Science*. 40 (3) 378-391.

Fioole, P.J., Kroon, L., Maróti, G., Schrijver, A. 2006. A rolling stock circulation model for combining and splitting of passenger trains. *European Journal of Operational Research*. 174 (2) 1281-1297