

AN LPV MODEL FOR A MARINE COOLING SYSTEM WITH TRANSPORT DELAYS

Michael Hansen^(a), Jakob Stoustrup^(b), Jan Dimon Bendtsen^(c)

^(a)A.P. Møller - Mærsk A/S, Esplanaden 50, DK-1098, Copenhagen, Denmark

^{(a)(b)(c)}Aalborg University, Fredrik Bajers Vej 7, DK-9320, Aalborg, Denmark

^(a)michael.hansen1@maersk.com, ^(b)jakob@es.aau.dk, ^(c)dimon@es.aau.dk

ABSTRACT

We address the problem of constructing a linear parameter varying (LPV) model for a nonlinear marine cooling system with flow dependent delays. We focus on the choice of scheduling variables for the LPV model to represent important nonlinear dynamics, and to preserve the flow dependency of the transport delays in the system. To this end, we redefine one of the system inputs to obtain a scheduling parameter that describes the time-varying operating point for this input, and also make some simple, but justifiable approximations in order to keep the number of scheduling variables low. A simulation example is provided to illustrate the performance of the LPV model compared to the original nonlinear model.

Keywords: Nonlinear systems, LPV modeling, Transport delay, Marine systems

1. INTRODUCTION

In this paper we consider the nonlinear marine cooling system with flow dependent delays that was first introduced in (Hansen, Stoustrup and Bendtsen 2011). The cooling system is used aboard container vessels for cooling the main engine and auxiliary components such as main engine scavenge air coolers, turbo chargers, diesel generators, etc. The motivation for considering this system is the potential energy savings that can be obtained by improving the currently implemented control, which is very energy inefficient due to an excessive use of the pumps in the cooling system. However, because of the structure of the system, the dynamic behavior includes transport delays and nonlinearities, which complicates the design of more advanced control laws. This entails that the models used for control design must describe the important dynamics of the system sufficiently accurate, but also has a form that fits the control design method.

One approach for dealing with the problem of control design for nonlinear systems is by use of linear parameter varying (LPV) control theory (Toth, 2010). LPV systems are characterized by being dependent on an unknown, but measurable time-varying parameter that describes the variations in the plant dynamics. When designing control for the LPV system, the time-varying parameter is used for scheduling the control

laws according to how the system dynamics changes. This makes LPV control applicable to a wide range of systems, including a large class of nonlinear systems that can be converted to an LPV form. With the combination of theory from optimal and robust control it is possible to guarantee stability, optimal performance and robustness of an LPV model in the entire field of operation. This is contrary to former gain scheduling approaches where a global nonlinear control design is obtained from interpolating local linear controllers, and where guarantees of performance and robustness cannot be made in general (Shamma and Athans, 1991), (Apkarian and Adams, 1998). Some results on the use of LPV control theory for systems with time-varying delays have been presented in (Zope, Mohammadpour, Grigoriadis, and Franchek, 2010), (Tan, Grigoriadis, and Wu, 2003), (Wu and Grigoriadis, 2001) and is part of the motivation for this work. However, the use of LPV control theory requires that the system model has an LPV representation which can be difficult to obtain (Jung and Glover, 2003).

The objective in this paper is to rewrite the nonlinear model from (Hansen, Stoustrup and Bendtsen, 2011) into the form of an LPV model that includes the flow dependent transport delays, and represents important dynamics sufficiently accurate. We only consider the thermodynamic part of the model, while appropriate control is assumed to be designed for the hydraulics such that the flows in the system can be considered as free input variables.

Related work is presented in (Jung and Glover, 2003) where a third order nonlinear model of the airpath of a turbocharged diesel engine is converted to an LPV model. However, delays are not a part of the nonlinear model considered (Jung and Glover, 2003), and the resulting LPV model is of the quasi-LPV type i.e. where scheduling variables depends on the system dynamics, which is somewhat different from what we seek here. The main contribution of this paper lies in the inclusion of transport delays when converting the nonlinear model to an LPV representation, and in the corresponding choice of scheduling parameters for adequately describing the transport delays as well as the nonlinear dynamics in the resulting LPV model.

The remaining paper is structured as follows: In Section 2 we make a brief presentation of the nonlinear

model considered in this paper. In Section 3 we bring the model into an LPV form and in Section 4 we compare the performance of the LPV model with the original nonlinear model. Finally, concluding remarks are presented in Section 5.

We make use of the following fairly standard notation: \mathbb{R} denotes the set of real numbers while \mathbb{R}_+ denotes the set of non-negative real numbers. $\mathbb{R}^{n \times m}$ is the set of real $n \times m$ matrices and $C^1(\mathcal{M}, \mathcal{N})$ is the set of continuous functions mapping from \mathcal{M} to \mathcal{N} with first order continuous derivatives.

2. NONLINEAR MODEL

The cooling system consists of three circuits; a sea water (SW) circuit, a low temperature (LT) circuit and a high temperature (HT) circuit. In this work the HT circuit is not of interest, and is therefore left out in the following. A simplified layout of the system considered in this work is illustrated in Figure 1.

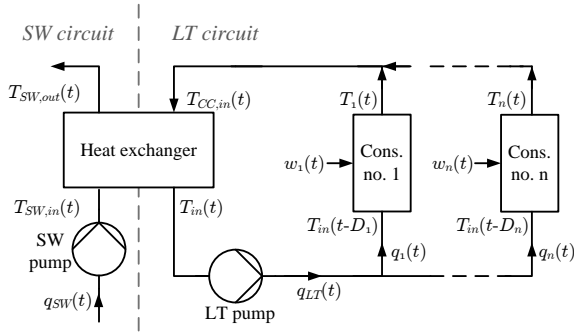


Figure 1: Simplified layout of the cooling system considered. The sea water (SW) circuit is to the left, while the low temperature (LT) circuit containing all the consumers is to the right.

The SW circuit pumps sea water through the cold side of the heat exchanger for lowering the temperature of the coolant in the LT circuit. The LT circuit contains all the main engine auxiliary components in a parallel configuration, and the supplied cooling is controlled through the flow rates in the system, $q_{SW}(t)$ and $q_{LT}(t)$.

The nonlinear thermodynamic model consists of two parts; one to describe the temperature change in the coolant out of each consumer in the LT circuit, and one to describe temperature change of the coolant out of the LT side of the heat exchanger.

The dynamics for the consumers $i = 1, \dots, n$, with $q = [q_1, q_2, \dots, q_n]^T$ is described by:

$$\dot{T}_i(t) = \frac{1}{V_i} \left[q_i(t)(T_{in}(t - D_i(q)) - T_i(t)) + \frac{w_i(t)}{\rho c_p} \right], \quad (1)$$

where $q_i(t)$ is the volumetric flow rate through the consumer, V_i is the internal volume of the consumer,

$T_i(t)$ is the outlet temperature of the consumer and $T_{in}(t)$ is the outlet temperature of the heat exchanger (into the LT circuit). Also, $w_i(t)$ is the heat transfer from the consumer to the coolant, ρ is the density of the coolant and c_p is the specific heat of the coolant.

We here consider the case where the flows $q_1(t), q_2(t), \dots, q_n(t)$ are not independents, but satisfy the relation:

$$\begin{aligned} q_1(t) &= c_1 q_{LT}(t) \\ q_2(t) &= c_2 q_{LT}(t) \\ &\vdots \\ q_n(t) &= c_n q_{LT}(t), \end{aligned}$$

where $\{c_i\}_{i=1}^n$ are positive constants, subject to:

$$\sum_{i=1}^n c_i = 1.$$

For the dynamics of $T_{in}(t)$ we have that:

$$\begin{aligned} \dot{T}_{in}(t) = \frac{1}{V_{CC}} \left[q_{LT}(t)(T_{CC,in}(t) - T_{in}(t)) \right. \\ \left. + q_{SW}(t) \frac{\rho_{sw} c_{p,sw}}{\rho c_p} T_{SW,in}(t) \right. \\ \left. - q_{SW}(t) \frac{\rho_{sw} c_{p,sw}}{\rho c_p} T_{SW,out}(t) \right], \quad (2) \end{aligned}$$

where $c_{p,sw}$ is the specific heat of sea water, ρ_{sw} is the density of sea water, and $T_{CC,in}(t)$ is the temperature of the coolant into the LT side of the heat exchanger. Also, $T_{SW,in}(t)$ and $T_{SW,out}(t)$ are the temperatures of the sea water in and out of the SW side of the heat exchanger. The transport delays are described by the relation:

$$D_i(q) = \sum_{j=1}^i \left(a_{m,j} \sum_{k=j}^n \frac{1}{q_k} \right) + \frac{a_{c,i}}{q_i}, \quad (3)$$

where $a_{m,i}$ and $a_{c,i}$ are system specific positive constants.

It is assumed that the temperatures $T_{CC,in}(t)$, $T_{SW,in}(t)$ and $T_{SW,out}(t)$ are measurable while the heat transfers $w_1(t), \dots, w_n(t)$ are unknown but belongs to the set:

$$\mathcal{W} := \{w \in C(\mathbb{R}, \mathbb{R}); 0 < \underline{W} \leq w(t) \leq \overline{W} < \infty\}. \quad (4)$$

i.e., the heat transfer from each consumer is continuous, positive and bounded from below by \underline{W} while bounded from above by \overline{W} . Also, we assume that delays D_1, \dots, D_n belongs to the set:

$$\begin{aligned} \mathcal{D} := \{D \in C(\mathbb{R}, \mathbb{R}); 0 \leq D(q) \leq \overline{D} < \infty, \\ \dot{D}(q) < 1 \forall t \in \mathbb{R}_+\}, \quad (5) \end{aligned}$$

which ensures that $t - D_i(q)$ is monotonically increasing for all D_i . The requirement that the first order derivative of the delays must be less than one is a necessary requirement, but obviously causes restrictions to how fast the input is allowed to change due to the relation between the inputs and the delays. In other words this means that the flow rates in the system cannot be allowed to decrease arbitrarily fast. Also, since it is required that the delays are positive and bounded from above by \bar{D} , the flow rates $q_1(t), \dots, q_n(t)$ must be non-zero and positive, which is considered to be the case for all relevant operating conditions.

3. CONSTRUCTION OF LPV MODEL

We seek a representation of the input-affine time delay system given by (1) and (2) on the LPV form of (Wu, 2001):

$$\begin{aligned} \dot{x}(t) = & A(p(t))x(t) + \sum_{i=1}^k A_{D_i}(p(t))x(t - D_i(p(t))) \\ & + B_1(p(t))w(t) + B_2(p(t))u(t) \end{aligned} \quad (6)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $w(t) \in \mathbb{R}^{n_w}$, is the disturbance vector and $u(t) \in \mathbb{R}^{n_u}$ is the input vector. The initial condition for the delay system in (6) is given by:

$$x(\theta) = \phi(\theta), \quad \theta \in [-\bar{D}, 0]. \quad (7)$$

The time-varying parameter, $p(t)$, belongs to the set of allowable parameter trajectories defined as:

$$\begin{aligned} \mathcal{P} := & \{p \in \mathcal{C}(\mathbb{R}, \mathbb{R}^m); p(t) \subset \mathbb{R}^m, |\dot{p}_j(t)| \leq v_j, \\ & j = 1, 2, \dots, m, \forall t \in \mathbb{R}_+\} \end{aligned} \quad (8)$$

where $\{v_j\}_{j=1}^m$ are positive constants, which means that the parameters have bounded trajectories, and bounded variation rate.

Choosing scheduling variables is not a trivial matter as there are several factors that come into play. It is obviously desired to describe the important dynamics of the system adequately by the choice of parameters. However, it is also essential to keep the number of parameters as low as possible, as a high number of parameters complicate the control design for the system (Jung and Glover, 2003).

In this particular case, a reasonable choice for a scheduling variable is the temperature difference $(T_{SW,in}(t) - T_{SW,out}(t))$ which is measurable and satisfies the requirements for bounded trajectories and bounded variation rates as given by (8). However, in order to describe the dynamics of $T_i(t)$ on the linear form (6), as well as to preserve the flow dependency of the delays, we need an additional parameter. We therefore write $q_{LT}(t)$ as:

$$q_{LT}(t) = \tilde{q}_{LT}(t) + \hat{q}_{LT}(t), \quad (9)$$

where $\tilde{q}_{LT}(t)$ represents the time varying operating point of the flow, while $\hat{q}_{LT}(t)$ is a small perturbation from this operating point.

Choosing $\tilde{q}_{LT}(t)$ as a scheduling variable along with $(T_{SW,in}(t) - T_{SW,out}(t))$, we get that:

$$\begin{aligned} p(t) = & \begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix} \\ = & \begin{pmatrix} \tilde{q}_{LT}(t) \\ (T_{SW,in}(t) - T_{SW,out}(t)) \end{pmatrix}. \end{aligned} \quad (10)$$

This brings (1) and (2) to the form:

$$\begin{aligned} \dot{T}_i(t) = & \frac{1}{V_i} \left[c_i p_1(t) (T_{in}(t - D_i(p_1(t))) - T_i(t)) \right. \\ & + c_i \tilde{q}_{LT}(t) (T_{in}(t - D_i(p_1(t))) - T_i(t)) \\ & \left. + \frac{w_i(t)}{\rho c_p} \right] \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{T}_{in}(t) = & \frac{1}{V_{CC}} \left[p_1(t) (T_{CC,in}(t) - T_{in}(t)) \right. \\ & + \hat{q}_{LT}(t) (T_{CC,in}(t) - T_{in}(t)) \\ & \left. + q_{SW}(t) \frac{\rho_{sw} c_{p,sw}}{\rho c_p} p_2(t) \right]. \end{aligned} \quad (12)$$

We define the state, disturbance and input vectors, $x(t), w(t), u(t)$ from (6) as:

$$\begin{aligned} x(t) = & \begin{pmatrix} T_1(t) \\ T_2(t) \\ \vdots \\ T_n(t) \\ T_{in}(t) \end{pmatrix} \quad w(t) = \begin{pmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_n(t) \\ T_{CC,in}(t) \end{pmatrix} \\ u(t) = & \begin{pmatrix} \hat{q}_{LT}(t) \\ q_{SW}(t) \end{pmatrix}. \end{aligned} \quad (13)$$

It is clear that (11) and (12) cannot be brought directly to the form of (6) without simplifications or introducing additional scheduling variables. Since it is desired to keep the number of scheduling variables low, we make the following approximation for (11):

$$\hat{q}_{LT}(t) (T_{in}(t - D_i(p_1(t))) - T_i(t)) \approx \hat{q}_{LT}(t) (\bar{T}_{in} - \bar{T}_i)$$

where are \bar{T}_{in} and \bar{T}_i are constant set point values for $T_{in}(t)$ and $T_i(t)$, respectively. This approximation can be justified by the fact that the purpose of designing control laws for the system is to keep the temperatures at or close to predefined set points. This means that with properly designed control laws, the temperatures $T_{in}(t)$ and $T_i(t)$ should be close to \bar{T}_{in} and \bar{T}_i at all times, making the approximation small.

For (12) we make the approximation:

$$(p_1(t) + \hat{q}_{LT}(t)) \approx p_1(t) .$$

The argument here is that it is not desired to use $\hat{q}_{LT}(t)$ as an control input for $T_{in}(t)$, and since it only constitutes a small perturbation from $p_1(t)$ it is reasonable to discard it in this context, as it otherwise appears multiplicative with the disturbance $T_{CC,in}(t)$.

This results in approximated models given by:

$$\begin{aligned} \dot{T}_i(t) \approx & \frac{1}{V_i} \left[c_i p_1(t) (T_{in}(t - D_i(p_1(t))) - T_i(t)) \right. \\ & \left. + c_i \hat{q}_{LT}(t) (\bar{T}_{in} - \bar{T}_i) + \frac{w_i(t)}{\rho c_p} \right] \end{aligned} \quad (14)$$

$$\begin{aligned} \dot{T}_{in}(t) \approx & \frac{1}{V_{CC}} \left[p_1(t) (T_{CC,in}(t) - T_{in}(t)) \right. \\ & \left. + q_{SW}(t) \frac{\rho_{sw} c_{p,sw}}{\rho c_p} p_2(t) \right] . \end{aligned} \quad (15)$$

The system given by (14) and (15) can now be written in the form of (6). With the choice of input, state and disturbance vectors as given by (13) we get that $A(p(t))$ can be written as:

$$A(p(t)) = \begin{bmatrix} \frac{-c_1}{V_1} p_1(t) & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \frac{-c_n}{V_n} p_1(t) & 0 \\ 0 & \cdots & 0 & \frac{-1}{V_{CC}} p_1(t) \end{bmatrix} \quad (16)$$

For matrices $A_{D_i}(p(t))$ we have that:

$$A_{D_i}(p(t)) = \begin{bmatrix} 0 & \cdots & 0 & \delta_{i,1} \frac{c_1}{V_1} p_1(t) \\ \vdots & \ddots & \vdots & \\ 0 & \cdots & 0 & \delta_{i,n} \frac{c_n}{V_n} p_1(t) \\ 0 & \cdots & 0 & 0 \end{bmatrix} , \quad (17)$$

for $i = 1, \dots, n$ and where δ is defined as:

$$\delta_{i,j} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

Furthermore, for $B_1(p(t))$ and $B_2(p(t))$ we have that:

$$B_1(p(t)) = \begin{bmatrix} \frac{1}{V_1 c_p \rho} & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \frac{1}{V_n c_p \rho} & 0 \\ 0 & \cdots & 0 & \frac{1}{V_{CC}} p_1(t) \end{bmatrix} \quad (18)$$

$$B_2(p(t)) = \begin{bmatrix} \frac{c_1(\bar{T}_{in} - \bar{T}_1)}{V_1} & 0 \\ \vdots & 0 \\ \frac{c_n(\bar{T}_{in} - \bar{T}_n)}{V_n} & 0 \\ 0 & \frac{\rho_{sw} c_{p,sw}}{\rho c_p} p_2(t) \end{bmatrix} . \quad (19)$$

Equations (16)-(19) constitutes the generic LPV model for the cooling system, and with the definitions of scheduling variables in (10) we have that delays are written as:

$$D_i(p(t)) = \sum_{j=1}^i \left(a_{m,j} \sum_{k=j}^n \frac{1}{c_k p_1(t)} \right) + \frac{a_{c,i}}{c_i p_1(t)} . \quad (20)$$

To make the structure of the LPV model clear, as well as to illustrate how the LPV model compares to the original nonlinear model, we construct a fictitious simulation example in the following section.

4. SIMULATION STUDIES

We consider a simulation example for a system with two consumers i.e., where $n = 2$. According to (16)-(19) we have that:

$$\begin{aligned} \dot{x}(t) = & \begin{bmatrix} \frac{-c_1}{V_1} p_1(t) & 0 & 0 \\ 0 & \frac{-c_2}{V_2} p_1(t) & 0 \\ 0 & 0 & \frac{-1}{V_{CC}} p_1(t) \end{bmatrix} x(t) \\ & + \begin{bmatrix} 0 & 0 & \frac{c_1}{V_1} p_1(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(t - D_1(p(t))) \\ & + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{c_2}{V_2} p_1(t) \\ 0 & 0 & 0 \end{bmatrix} x(t - D_2(p(t))) \\ & + \begin{bmatrix} \frac{1}{V_1 c_p \rho} & 0 & 0 \\ 0 & \frac{1}{V_2 c_p \rho} & 0 \\ 0 & 0 & \frac{1}{V_{CC}} p_1(t) \end{bmatrix} w(t) \\ & + \begin{bmatrix} \frac{c_1(\bar{T}_{in} - \bar{T}_1)}{V_1} & 0 \\ \frac{c_2(\bar{T}_{in} - \bar{T}_2)}{V_2} & 0 \\ 0 & \frac{\rho_{sw} c_{p,sw}}{\rho c_p} p_2(t) \end{bmatrix} u(t) . \end{aligned} \quad (21)$$

Accordingly, delays $D_1(p(t))$ and $D_2(p(t))$ are given by:

$$\begin{aligned} D_1(p(t)) &= \frac{a_{m,1}}{p_1(t)} + \frac{a_{c,1}}{c_1 p_1(t)} \\ D_2(p(t)) &= \frac{a_{m,1}}{p_1(t)} + \frac{a_{m,2}}{c_2 p_1(t)} + \frac{a_{c,2}}{c_2 p_1(t)} . \end{aligned} \quad (22)$$

Thermodynamic parameters are shown in Table 1 while system parameters are illustrated in Table 2. Be aware that only the V_1 and V_2 parameters have been estimated from an actual cooling system, while other

system specific parameters have been chosen for this example. The reason for not having more parameters for the actual system is simply the lack of available measurement data for parameter estimation.

Table 1: Thermodynamic parameters.

| c_p | $c_{p,sw}$ | ρ | ρ_{sw} |
|-------|------------|--------|-------------|
| 4181 | 3993 | 1000 | 1025 |

Table 2: System parameters for simulation examples.

| V_{cc} | V_1 | V_2 | $a_{m,1}$ | $a_{m,2}$ |
|-----------|-----------|-------|-----------|-----------|
| 20 | 13.5 | 13.5 | 30 | 10 |
| $a_{c,1}$ | $a_{m,2}$ | c_1 | c_2 | |
| 10 | 10 | 0.51 | 0.49 | |

The LPV model represented by (21) and the corresponding nonlinear model, which we will not state here, are subjected to the same input and disturbances as well as changes in scheduling variables. The model outputs are then compared to illustrate how well the LPV model approximates the nonlinear model. The simulation scenario is constructed such that the system is in steady state with the chosen initial conditions. The division of the input into a time varying set point and a perturbation from the set point as given by (9), is implemented using a simple first order discrete low pass filter with a cut off frequency of 0.002 rad/s. Initial conditions for the simulation are illustrated in Table 3 and the responses for both the LPV and nonlinear model are shown in Figure 2. Figure 3 shows the input signals, $\hat{q}_{LT}(t)$, $q_{LT}(t)$ and $q_{SW}(t)$, while Figure 4 illustrates the disturbances in terms of $w_1(t)$, $w_2(t)$ and $T_{CC,in}(t)$. Finally, Figure 5 shows the scheduling variables, $p_1(t)$ and $p_2(t)$, where $p_1(t)$ is the low pass filtered input, $q_{LT}(t)$.

Table 3: Initial conditions for the simulation example.

| $T_{sw,in}(0)$ | $T_1(0)$ | $q_{LT}(0)$ | $T_{in}(0)$ | $w_1(0)$ | $w_2(0)$ |
|-----------------|----------|-------------|--|-----------------|-----------------|
| 24 | 45 | 0.21 | 36 | 4×10^6 | 6×10^6 |
| $T_{sw,out}(0)$ | $T_2(0)$ | $q_{SW}(0)$ | $x(\theta)$ for $\theta \in [-\bar{D}, 0]$ | | |
| 40 | 50 | 0.59 | $[45 \ 50 \ 36]^T$ | | |

The purpose of the simulation example is not to illustrate a real world scenario, but rather to excite the models in a way that shows how well the LPV model approximates the original nonlinear model. It is expected that the model outputs will differ only when the approximated part of the LPV model is excited. As can be seen from Figure 2, deviations between the LPV model and the nonlinear model occurs in the transitions of the input $q_{LT}(t)$, which is expected since all approximation in the LPV model has to do with $q_{LT}(t)$. Despite the deviations, the LPV model captures all important dynamics and is considered to be sufficiently accurate for control design.

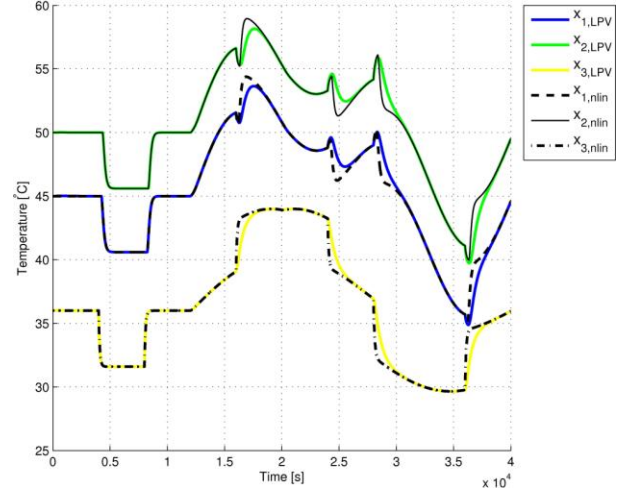


Figure 2: Comparison between LPV and nonlinear model outputs. Index 'LPV' denotes LPV model output, while 'nlin' denotes nonlinear model output.

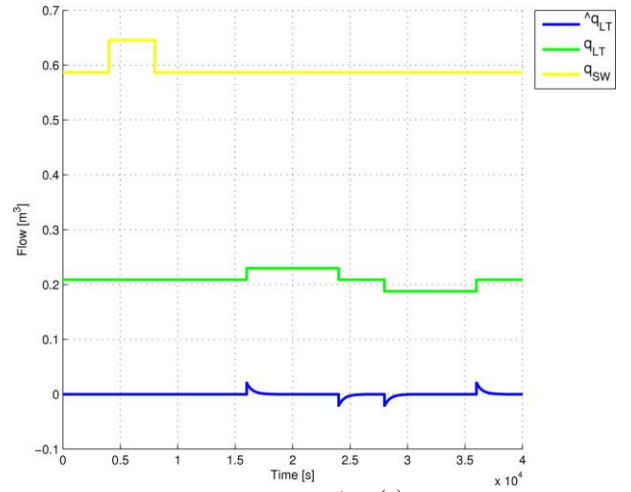


Figure 3: Plot of input signal $\hat{q}_{LT}(t)$ for the LPV model and $q_{LT}(t)$ for the nonlinear model. The input $q_{SW}(t)$ is the same for both models.

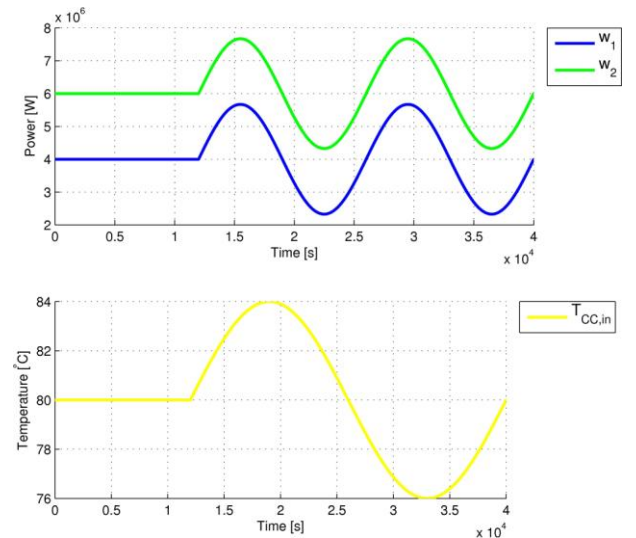


Figure 4: Top plot illustrates disturbances $w_1(t)$ and $w_2(t)$, while the bottom plot shows the disturbance $T_{CC,in}(t)$.

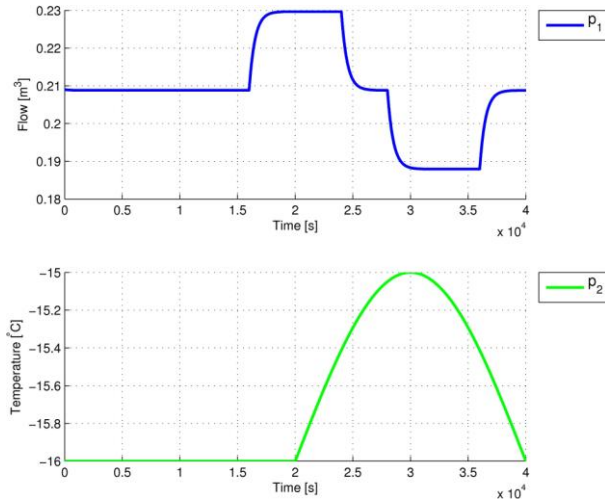


Figure 5: Top plot illustrates parameter $p_1(t)$ which is the low pass filtered input $q_{LT}(t)$, while the bottom plot shows the parameter $p_2(t)$ which is the temperature difference $(T_{SW,in}(t) - T_{SW,out}(t))$.

5. CONCLUDING REMARKS

We have presented the conversion of a nonlinear model to an LPV model for a marine cooling system with transport delays. The choice of scheduling variables for the LPV model was based on an attempt to keep the number of scheduling variables as low as possible, while still capturing the important nonlinear dynamics of the system and preserving the flow dependency of the delays. To illustrate the performance of the LPV model compared to the original nonlinear model, a simulation example was constructed. The simulation showed that the LPV model output only differed from the original nonlinear model when the input $q_{LT}(t)$ was excited, which was expected since all approximations in the LPV model was related to this input. The simulations indicate that the LPV model is sufficiently accurate for control design, and future work involves design of energy optimizing control laws that ensures robustness with the presence of disturbances and transport delays.

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