IMC CONTROL APPROACH FOR SUPPLY CHAIN MANAGEMENT

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ABSTRACT
In this work, a discrete time series model of a supply chain system is derived using material balances and information flow. Transfer functions for each unit in the supply chain are obtained by z-transform. The entire chain can be modelled by combining these transfer functions into a close loop transfer function for the network. Controllers are designed using frequency analysis. The bullwhip effect, magnification of amplitudes of demand perturbations from the tail to upstream levels of the supply chain, is a very important phenomenon for supply chain systems. Furthermore, we show that by implementing a proportional integral or an IMC inventory control and properly synthesizing the controller parameters, we can effectively suppress the bullwhip effect. Moreover, the IMC control structure is superior in meeting customer demand due to its better tracking of long term trends of customer demand.

Keywords: supply chain, beer game, z-transform bullwhip effect, PI control.

1. INTRODUCTION

A supply chain includes all the participants and processes involved in the satisfaction of customer demand: transportation, storages, wholesales, distributors and factories (Dejonckheere, Disney, Lambrecht and Towill 2002, Dejonckheere, Disney, Lambrecht and Towill 2004). A large number of participants, a variety of relations and processes, dynamics and the randomness in material and information flow prove that supply chains are complex systems in which coordination is one of the key elements of management.

A trouble and important phenomenon in supply chain management, known as the bullwhip effect, suggests that demand variability increases as one goes up in the supply chain (Hoberg, James, Bradley, Ulrich and Thonemann 2007, Marko, Rusjan 2008)

The causes of the bullwhip effect can be due to the forecasting demand, the lead times, order batching, supply shortages, or price fluctuations. We will mainly address the non-zero lead times and particularly the forecasting demand. The supply chain has attracted much attention among process system engineering researchers recently. There are many aspects in supply chain research.

One area is the analysis of the logistic problem of a supply chain using system control theory. In this paper the focus is on the analysis and control of the material balance and information flow among the system. In order to demonstrate the existence of bullwhip effect the beer game was created at the beginning of the sixties at the School of Management, Massachusetts Institute of Technology (MIT) (Ilhyung and Mark 2007). The game simulates a multi-echelon serial supply chain consisting of a Retailer (R), a Wholesaler (W), a Distributor (D) and a Factory/Manufacturer (M) We model the basic protocol of the ”beer distribution game”. The mathematical model used is the transfer function which represents the relation between the input and output of a linear time invariant system (LTI). In supply chain case we are dealing with discrete signals and system, therefore our theoretical analysis was performed by the z-transform. In control systems engineering, the transfer function of a system represents the relationship describing the dynamics of the system under consideration. It algebraically relates a systems output to its input and in this paper is defined as the ratio of the z-transform of the output variable to the z-transform of the input variable.

The Proportional-Integrative-Derivative (PID) controllers are without doubt the most extensive option that can be found on industrial control applications Ramon (2008). Their success is mainly due to their simple structure and meaning of the corresponding three parameters. This act makes the PID control easier to understand by the control engineer than most other advanced control techniques. An analysis of the effect of a P and PI controllers approach as an ordering strategy was made. Then, parameters values for stability and mitigate bullwhip was obtained. The derivate action is not analysed because this is not appropriate for noise systems. The internal model control structure has been introduced as an alternative to the feedback structure. Its main advantage is that closed-loop stability is assured simply by choosing a stable IMC controller. An analysis of the different controllers was made respect to its tuning simplicity, performance and mitigating of bullwhip effect.

2. BEER GAME MODEL

Let us consider a basic beer supply chain as shown in Figure (1). There are four logistic echelons: Retailer(R), wholesales (W), distributor (D), and Factory (F). We assume the following sequence of events: in each period t, the retailer first receives goods, then demand is observed and satisfied (if not backlogged), next, the retailer observes the new inventory level and finally places an order on the manufacturer. Let $I_i(k)$ denote the net stock inventory (the difference between on-hand

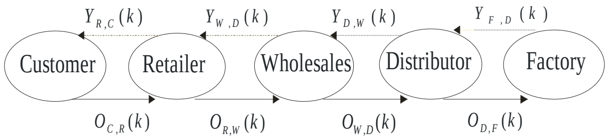

Figure 1. The Block Diagram Of Supply Chain

inventory and backorders) of a echelon of chain I(R,W,D, F) at any discrete time instant k. We also let $Y_{i,i-1}(k)$ indicate the amount of goods to be delivered to node $i-1$ by the upstream node i at the instant k, $O_{i-1,i}(k)$ indicates the demand received by node i from downstream $i-1$. A time delay of L due to transport is assumed for all delivery of goods so that goods dispatched for an upstream $i+1$ at time k will arrive at time $k+L$ at node i. However, due to the need for examination and administrative processing, this new delivery is only available to the node i at $k+L$. The orders placed in the upstream for the i node is denoted for $O_{i,i+1}(k)$

Coupling between two nodes through the ordering policy is eliminated when there is insufficient stock in any one node along the chain (Pin, David, Shan, Shi, Jang, Shyan and Zheng 2004). Therefore, propagation of demand fluctuations is only possible when every node has sufficient stock. Then in this work assume that the upstream $i+1$ supplier has sufficient inventory so that the orders of node i are always satisfied so that the amount of goods delivered by upstream $i+1$ in instant k. $Y_{i+1,i}(k)$ is equal to the order made for the downstream i in a previous time $O_{i,i+1}(k+L)$. It's also assumed the amount of goods delivered by upstream i at instant k. $Y_{i,i-1}(k)$ is equal to the order made for the downstream $i-1$ in a previous time $O_{i-1,i}(k)$.

The result of the integration of the difference between amount goods that entry from upstream node $i+1$ and amount goods that dispatched for downstream node $i-1$ is known as inventory balance, this has a role as a buffer to absorb the demand variability. In other words, the inventories should have stabilizing effect in material flow patterns. The equation for inventory balance at node i is given by:

$$I_i(k) = I_i(k-1) + O_{i,i+1}(k-L) - O_{i-1,i}(k) \qquad (1)$$

The manager aims at maintaining a certain inventory level without generating aggressive changes on orders, using the information signals. The control signal denoted by $O_{i,i+1}(k)$ can be the result of a feedback of any output and a proposed controller. For example, a inventory feedback and a simple P-control can be used. We assume that ordering information is communicated instantly. Hence, the order placed by the node i at the upstream $i+1$ is given by:

$$O_{i,i+1}(k) = C(k)(I_i^T(k) - I_i(k)) \qquad (2)$$

Where $C(k)$ denote the controller function and $I_I^T(k)$ is the inventory target at instant k.

The z-transform is a powerful operational method when one works with discrete control systems because the differential equation is converted to an algebraic problem. One important property of the Z-transform, which we use extensively, is the translation theorem Ogata (1996)

$$Z(x(k-n)) = z^{-n} X(z) \qquad (3)$$

Where z^{-n} is the operator of a time delay in space z and corresponds to a time delay of n time sampling periods T.

Using the translation theorem (3) , on equations (2) and(2), we obtained the z-transform of the above discrete time model, this is given by the equations (4) and (5).

$$I_i(z) - I_i(z)z^{-1} = O_{i,i+1}(z)z^{-L} - O_{i-1,i}(z) \qquad (4)$$

$$O_{i,i+1}(z) = C(z)(I_i^T(z) - I_i(z)) \qquad (5)$$

The resulting model in (4) is thus amenable to implement some controllers that exist in literature.

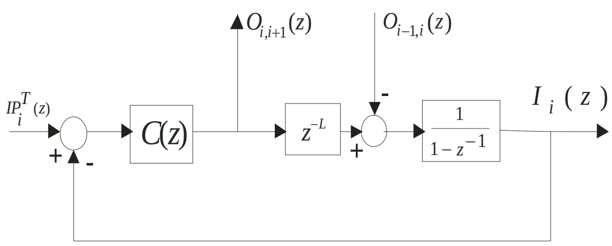

Figure 2. The Block Diagram Of Node i Of A Supply Chain.

The simplified block diagram is given in Figure (2). $C(z)$ represent the transfer function of anyone controller as replenishment policies. The above model seems simple. Nevertheless, it captures the basic dynamic feature of a supply chain system. A real supply chain usually has many customers, suppliers, and products. In a decentralized system, the inventory dynamics does not really depend on how many customers the node has, since all customer demands can be lumped into an aggregate demand. Obviously, if every node has sufficient inventory and has the same transportation delay, the distribution of order would not affect the dynamic behaviour of the system. We can assume that different suppliers have different inventory levels and different transportation delays.

2.1 Uncertainty description

The last model to describe an approximation of the dynamic of a supply chain, the real procces is nonlinearity and has uncertainty on the parameters.

To account for model uncertainty we will assume that the dynamic behaviour of a plant is described not by a single linear time invariant model but by a family $F(z)$ of LTI models. Algebraically, the family of plants is defined by:

$$F(z) = \tilde{P}(z)(1 + l_m(z)) : |l_m(z)| < |\bar{l}_m(z) \tag{6}$$

Where $\tilde{P}(z)$ denotes the nominal model, $l_m(z)$ is the relative (multiplicative) model error

$$\bar{l}_m(z) = \frac{P(z) - \tilde{P}(z)}{\tilde{P}(z)} \tag{7}$$

And $\bar{l}_m(z)$ is a frequency dependent weight that defines the family of plants by bounding the modelling error.

3. TRENSFER FUNCTIONS FOR A SUPPLY CHAIN MANAGEMENT

In order to derive the transfer function for a particular controller as replenishment inventory policy, let's refer to Figure (2). Here the control strategy $C(z)$ must be tuned in order to the system will be stable, keep the inventory $I_i(z)$ close to inventory target $I_i^T(z)$ and avoiding the bullwhip effect.

The transfers function that relates the physical inventory with the desired inventory were obtained

$$\frac{I_i(z)}{I_i^T(z)} = \frac{C(z)z^{-L}}{1 - z^{-1} + C(z)z^{-L}} \tag{8}$$

$$\frac{O_{i,i+1}(z)}{I_I^T(z)} = \frac{C(z)(z-1)}{1 - z^{-1} + C(z)z^{-L_i}} \tag{9}$$

Assuming that there is no change in the set point, the ratio of orders to successive nodes can be expressed as

$$\frac{O_{i,i+1}(z)}{O_{i-1,i}(z)} = \frac{C(z)}{1 - z^{-1} + C(z)z^{-L_i}} \tag{10}$$

The bullwhip effect can be represented as an amplification of demand fluctuations from downstream to upstream, it is translate to de amplification of amplitude of the relation (10) will be greater than 1.

All relations contain the same characteristic equation; hence it is possible to do stability analysis for both transfer functions using this equation. Then the characteristic equation is:

$$1 - z^{-1} + C(z)z^{-L_i} = 0 \tag{11}$$

With (8), (9), (10) and (11) the stability analysis and tuning for bullwhip mitigation can be realised for any controller.

3.1 Controller tuning criterion for bullwhip mitigation

Because of the widespread use of PID controllers, it is interesting to have simple but efficient methods for tuning the controller. In fact, since Ziegler Nichols proposed their first tuning rules Ramon (2008), an intensive research has been done. In this case an analytical tuning technique is used in order to mitigate the bullwhip effect. We assume that the customer demand is stochastic. However, the average of demand may be subjected to a low frequency disturbance such as a step change or seasonal cyclic changes. The objective of a simple inventory level controller is to maintain a given inventory position in the presence of such a low frequency disturbance. However, in addition to achieving the inventory position target, the objectives of a supply chain manager also include setting an inventory position target that is not too high (resulting inexcess inventory costs) or too low (resulting in customer dissatisfaction due to backorder) compared to the current average demand. Therefore, a manager should aim to create a fast response of the order to low frequency demand changes so that the inventory level can be maintained, but limit the ratio of order to demand to less than 1 at high frequency. The frequency response of $|O_{i,i+1}(z)| / |O_{i-1,i}(z)|$ of a closed loop supply chain node should be used for controller design. Standard textbooks (Pin, David, Shan, Shi, Jang, Shyan and Zheng 2004). suggest the following two factors to be considered:

- A wide bandwidth (The frequency at which the magnitude ratio is reduced to below 0.7.) indicates a faster response but poorer noise rejection capabilities. As we only discuss a discrete system; therefore, the highest frequency is at $\omega = \pi$. Therefore, a design ruler is choosing a controller than setting magnitude ratio less than 0.7 for this frequency.
- A higher resonance peak (RP) indicates a faster response but may be more oscillatory. Then the second rule is choosing suitability a controller which setting RP less than 1.

4 ANALISIS OF THE EFFECT OF PICONTROLLER AS REPLENISMENT INVENTORY

A control system is a combination of elements (components of the system) which enable us to control the dynamics of the selected process in a certain way. The PID (Proportional, Integral and Derivative) is the controller most commonly used in control engineering because of its flexibility and simplicity. Therefore there will by introductory analysis of this controller as inventory replenishment policy. In this section we show that by implementing a proportional integral inventory control and properly synthesizing the controller parameters, we can effectively suppress the bullwhip effect. We perform simulations with different values of parameters (k_p, k_i) and finally we compare the behaviour of the inventory and orders signals using PI actions. The derivate action is discarded because of its high noise sensitivity.

$$C(z) = k_p + \frac{k_i}{z-1} \tag{12}$$

4.1 Proportional Control

The main interest is management the of inventory level. Therefore we use a feedback-level inventory and a proportional controller. When there is sufficient supply and high stock, substituting the P-control into the transfer functions (8),(9) and (10) it is obtain the transfer functions of the system in close loop with the proportional action, then the stability analysis, the controller performance and the bullwhip effect can be analysed. Stability criterions and simulations have been used in previous works (Carlos , Pedro and Ramón 2008) by evaluate the stability and the behaviour of the orders $O_{i,i+1}(z)$ and the inventory level $I_i(z)$. With the proportional controller is possible to stabilize the system whit certain values of k_p but a phenomenon (offset) that is a mistake of steady state is inevitable because if the error is constant the control action is constant too.

One factor that bullwhip is usually attributed to aggressive ordering. We have demonstrated in last one work (Carlos, Pedro and Ramón 2008) that the system would become unstable when $k_p < 1$. When there is sufficient supply and high stock, substituting the P-control into the transfer Function (10) we get

$$\frac{O_{i,i+1}(z)}{O_{i-1,i}(z)} = \frac{k_p}{1 - z^{-1} + k_p z^{-L_i}} \tag{13}$$

Figure 3. gives the frequency response of an echelon with a proportional only controller to its inventory position. with several different controller gains. It can be shown that with a controller gain lower than one, the bullwhip effect of the echelon is suppressed. Thus $k_p = 0.2$ should be appropriate according to controller tuning criterion.

There is a large offset between the inventory position and set point (Carlos , Pedro and Ramón 2008). This offset will lead to accumulation of a large amount of backorder and low customer satisfaction. Since an offset cannot be avoided, customer dissatisfaction is inevitable for a P-only controller. To avoid this offset, a PI controller can be used.

4.2 Proportional Integral (PI) control

The integral action has some characteristics that mprove the response of the system in close loop.

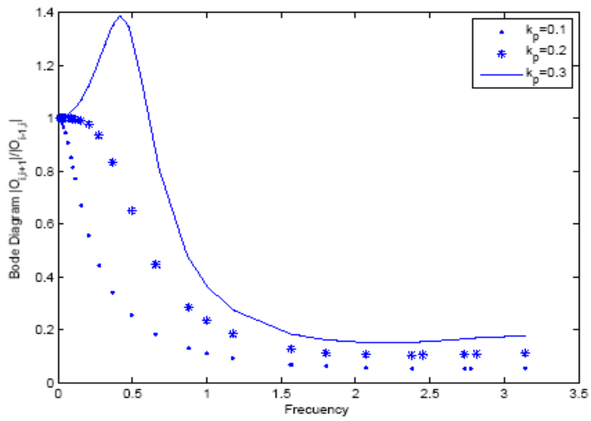

Figure 3. Frequency Responses Of $|O_{i,i+1}(z)|/|O_{i-1,i}(z)|$ with Different k_p Values Of The P Controller.

4.2 Proportional Integral (PI) control

The integral action has some characteristics that improve the response of the system in close loop. One of these properties is that it removes the offset because the control increases although the error remains constant (integrates the error), hence we analyze its behaviour in a supply chain. In this section we analyze the frequency response of the expression that related the orders an demand introducing this controller as replenishment inventory in order to tuning the PI parameters so that exist a trade off between inventory tracking and the mitigation of bullwhip effect. As well as we analyze the inventory and orders response. Substituting the PI-control give by (12) into the transfer function (10) we get the relation between the orders and the demand

The closed loop Bode plot of one supply Chain echelon with a PI controller is shown in Figure (4). It can see that for a lead time $L = 3$ and a given $k_p = 0.2$, the bullwhip effect still appears even the controller gain, k_i is close to zero. Hence there is a trade-off to be made between being responsive and following the demand changes very closely (large k_i-values) on the one hand and avoiding bullwhip (small k_i -values) on the other hand.Therefore we approach an alternative IMC scheme in order to improve the tracking and robust stability without generating bullwhip and following the demand changes.

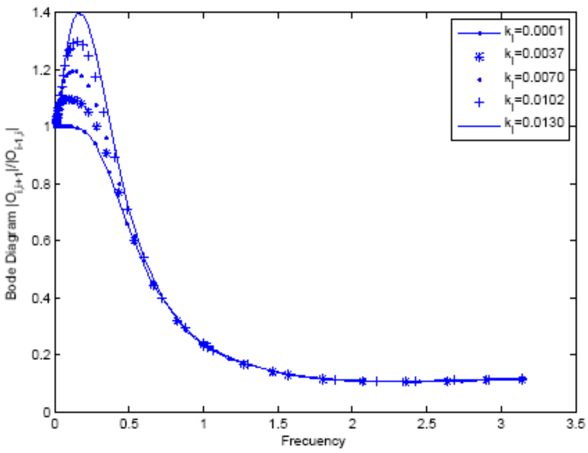

Figure 4. Frequency Responses Of $|O_{i,i+1}(z)|/|O_{i-1,i}(z)|$ with Different k_i Values Of The P Controller.

5 INTERNAL MODEL CONTROL BASED APPROACH

The internal model control structure has been introduced as an alternative to the feedback structure. Its main advantage is that closed-loop stability is assured simply by choosing a stable IMC controller (Manfred and Zafiriou 1989). Closed-loop performance characteristics (like settling time) are also, related directly to controller parameters which make on-line tuning of the IMC controller very convenient. A two-step design procedure was proposed. In the first step the controller is designed for optimal setpoint tracking (disturbance rejection) without regards for input saturation or model uncertainty. In the second step the controller is detuned for robust performance.

5.1 IMC structure for stable plants

The simplified block diagram of the IMC loop is shown in Figure (5). Where $P(z)$ denotes the plant and $\tilde{P}(z)$ is the nominal model. The controller $Q(z)$ determines the value of the input (manipulated variable) $u(z)$. The control objective is to keep $y(z)$ close to the reference (setpoint) $r(z)$.

If the model is exact ($\tilde{P}(z) = P(z)$) and there are no disturbances, then the output $\tilde{y}(z)$ and $y(z)$ are the same and the feedback signal $\tilde{d}(z)$ is zero. Thus, the control system is open-loop when there is no disturbance and no plant/model mismatch. The feedback signal $\tilde{d}(z)$ expresses the uncertainty about the process.

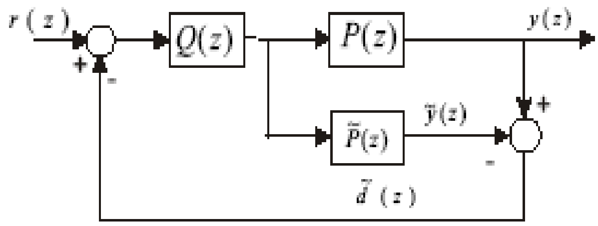

Figure 5. Internal Model Control Structure.

5.1.1 An overview of IMC design

The ultimate objective of control system design is clearly that the controller works well when implemented on the real plant. This goal can be best understood if decompose it into a series of subobjectives (nominal stability, Nominal performance, Robust Stability and Robust Performance) (Manfred and Zafiriou 1989) for this case, avoiding of bullwhip effect is introduced as the design criteria. The IMC has the property that allows us to discompose the design in steps in order to solve the objectives separately.

5.1.2 Stability conditions for IMC

Because it is assumed that the model is an approximate description of the true plant it is reasonable to require stability when the controller is applied to the plant model. Thus, the minimal requirement on the closed-loop system is nominal stability. Then, in order to test for internal stability the transfer functions between all possible system inputs and outputs are examined. From the block diagram in Figure (5). If there is no model error $\tilde{P}(z) = P(z)$, the inputs and outputs are related through the transfer matrix (15). Then the IMC system is internally stable if and only if both the plant $P(z)$ and the controller $Q(z)$ are stable

$$\begin{pmatrix} y(z) \\ u(z) \end{pmatrix} = (H) \begin{pmatrix} r(z) \\ u'(z) \end{pmatrix} \qquad (14)$$

Where

$$H = \begin{pmatrix} \tilde{P}(z)Q(z) & P(z)(1-\tilde{P}(z)Q(z)) \\ Q(z) & 1-\tilde{P}(z)Q(z) \end{pmatrix} \qquad (15)$$

On the other hand for good nominal performance and robust stability and performance the IMC uses a two-step approach which has no inherent optimality characteristic but should provide a good approximation to the optimal solution. It guarantees robustness but the performance is generally not optimal.

STEP1: For the Nominal performance the controller $\tilde{Q}(z)$ is selected to yield a good system response for the input(s) of interest, disregarding constrains and model uncertainty, in other words a weighted norm of the sensitivity function ε should be made small. Generally $\tilde{Q}(z)$ is chosen such that it is Integral Square Error or H_2-optimal.

$$\min \| 1-\tilde{P}(z)\tilde{Q}(z) \|_2 \tag{16}$$

The optimal sensitivity function becomes

$$\varepsilon \doteq 1-\tilde{P}(z)\tilde{Q}(z) \tag{17}$$

And the optimal complementary sensitivity function

$$\eta \doteq \tilde{P}(z)\tilde{Q}(z) \tag{18}$$

The model inverse is an acceptable solution only for Minimum-Phase (MP) systems. For Nonminimum-Phase (NMP) systems the exact inverse model is unstable or noncausal then an approximate inverse of $\tilde{P}(z)$, must be found such that the weighted 2-norm of the sensitivity function is minimized. According to this, let's assume that $\tilde{P}(z)$ is stable.

Factor $\tilde{P}(z)$ into an allpass portion $\tilde{P}_A(z)$ and a MP portion $\tilde{P}_M(z)$

$$\tilde{P}(z)=\tilde{P}_M(z)\tilde{P}_A(z) \tag{19}$$

So that $\tilde{P}_A(z)$ includes all zeros outside the unite circle and delays and has the form

$$\tilde{P}_A(z)=z^N \prod_{j=1}^{h} \frac{(1-(\xi_j^H)^{-1})(z-\xi_j)}{(z-\xi_j)(1-(\xi_j^H)^{-1})} \tag{20}$$

Where the integer N is chosen such that $\tilde{P}_M(z)$ is semiproper and $\xi_j,\ j=1,....h$ are the unstable zeros. Therefore H_2-optimal optimal controller $\tilde{Q}(z)$ is given by

$$\tilde{Q}(z)=\frac{1}{\tilde{P}_M(z)} \tag{21}$$

STEP2: For robust stability and performance at high frequencies when $l_m(z)$ exceeds unity $\eta(z)$ has to be rolled off. Therefore $\tilde{Q}(z)$ is augmented with low-pass filter $f(z)$.

$$Q(z) \doteq \tilde{Q}(z)f(z) \tag{22}$$

The order of $f(z)$ is such that $Q(z)$ is proper and its roll-off frequency low enough to satisfy the robust stability constrains. With the filter, $\varepsilon(z)$ and $\eta(z)$ become

$$\varepsilon \doteq 1-\tilde{P}(z)\tilde{Q}(z)f(z) \tag{23}$$

And the resulting complementary sensitivity function

$$\eta \doteq \tilde{P}(z)\tilde{Q}(z)f(z) \tag{24}$$

The function of f(z) is to detune the controller, to sacrifice performance (increase$|\varepsilon|$) for robustness (decrease $|\eta|$). In principle both the structure and parameters should be determined such that an optimal compromise between performance and robustness is reached. To simplify the design task the filter structure is fixed and search over a small number of filter parameters (just one) to obtain desired robustness characteristics. Typically the filter is chosen of the form given by

$$f(z)=\frac{1-\lambda}{1-\lambda z^{-1}} \tag{25}$$

Here the λ is an adjustable parameter.

5.1.3 Relationship with Classic Feedback

If we combine the two blocks $\tilde{Q}(z)$ and $\tilde{P}(z)$ in Figure (5), which are both part of the control system, into the one block $C(z)$ we obtain a single controller block.

$$C(z)=\frac{\tilde{Q}(z)}{1-\tilde{P}(z)\tilde{Q}(z)} \tag{26}$$

On the other hand its possible obtain $\tilde{Q}(z)$ from $C(z)$ used the following equation

$$\tilde{Q}(z)=\frac{C(z)}{1-\tilde{P}(z)C(z)} \tag{27}$$

5.1.4 Numerical Example

Consider the system given by

$$\tilde{P}(z)=0.483\frac{z^2+1.01+0.0597}{z^3-0.116z^2+0.118z-0.00315} \tag{28}$$

It has two zeros, both inside to unit circle, at $z=-0.95$ and $z=-0.95$. Therefore, the H_2-optimal controller $\tilde{Q}(z)$ is determined by (21) as follow

$$\tilde{P}(z)=1.001\frac{z^3-0.116z^2+0.118z-0.00315}{z^3} \tag{29}$$

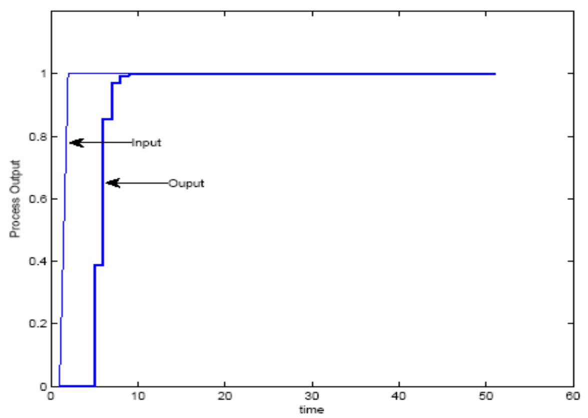

Figure 6. Output Response Of Close Loop System With A Control Based On IMC Design.

5.2 Modified IMC structure for unstable systems

We can see that the supply chain is an unstable processes with time delays, therefore the design of a conventional IMC is difficult, then a IMC modified structure (Wen, Horacio, Marquez and Tongwen 2003) can be an easy form to overcome the implementation problem as shown in Figure (7).

The procedure consists of first designing a compensator to stabilize the nominal plant, and then designing an IMC controller for the stabilized model (Wen, Horacio, Marquez and Tongwen 2003) with uncertainty. The block diagram of the modified IMC loop is shown in Figure (7). Here k is used to stabilize $\tilde{P}(z)$, the original (unstable) plant, ignoring the uncertainty in the time-delay and $Q(z)$ is designed to mitigate the bullwhip effect and robustly stabilize of new process.

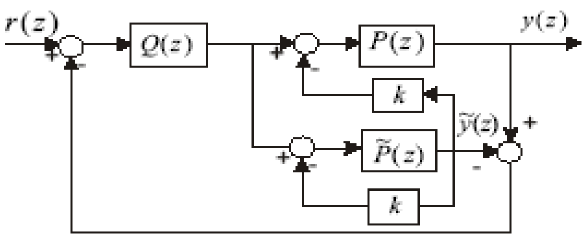

Figure 7. Modified Internal Model Control For Unstable Process.

5.2.1 Modified IMC design for an echelon of a Supply Chain

Let us consider the nominal model of the classical feedback ignoring uncertainty of an echelon as shown in Figure (8). It has an internal close-loop which is stabilized with a k controller. The transfer function is given by (30). Due to Lead time and the controller parameters the system poles can change, then is important to find the mathematical relation between the k controller and the lead time so that the system will be stable.

$$\tilde{P}(z) = \frac{I_i(z)}{I_i^T * (z)} = \frac{1}{1 - z^{-1} + kz^{-L_i}} z^{-L_i} \quad (30)$$

Any root or pole is a complex number then each of these can be represented as

$$z = re^{j\Omega} \quad (31)$$

Our interest is to find a value for the controller so that the roots are into the unit circle $|r| < 1$, then if we replace (32) on the characteristic equation we have

$$|k| = -r^{L_i} e^{jL_i\Omega} + r^{(L_i-1)} e^{j(L_i-1)\Omega} \quad (32)$$

So the relationship between the lead time and the controller that guarantees the system is stable is given by

$$|k| < 2\cos\left(\frac{(L_i-1)\pi}{2L_i-1}\right) \quad (33)$$

Using (33) we can chose a k value in order to stabilise the nominal plant. With the stabilized plant we can therefore apply the IMC design for stable plants.

STEP1: For the nominal performance the model inverse is an acceptable solution only for MP systems. Otherwise the exact inverse is unstable and/or no causal. Thus an approximate inverse of $\tilde{P}(z)$ must be found. Then, factor the nominal plant according to (19),(20) we have

$$\tilde{P}_A(z) = z^{-L_i} \quad (34)$$

And

$$\tilde{P}_M(z) = \frac{1}{1 - z^{-1} + kz^{-L_i}} \quad (35)$$

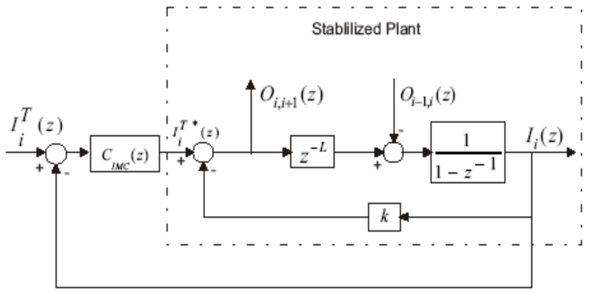

Figure 8. The Block Diagram Of Node i Stabilized With k Controller.

Thus we chose $\tilde{Q}(z)$ as shown bellow.

$$\tilde{Q}(z) = \frac{1}{\tilde{P}_M(z)} = 1 - z^{-1} + kz^{-L_i} \tag{36}$$

STEP2: For robust stability and performance, $\tilde{Q}(z)$ is augmented by a low-pass filter $f(z)$ in this case we used the parameter for tuned so that the bullwhip is mitigate. Finally the structure of controller IMC would be

$$Q(z) = \frac{(1-\lambda)}{1-\lambda z^{-1}}(1 - z^{-1} + kz^{-L_i}) \tag{37}$$

5.2.2 Tuning of for mitigate the bullwhip effect

$Q(z)$ is intended to be an IMC controller that mitigates the bullwhip effect, therefore it is necessary to obtain the expression that determines the behaviour of the orders vs demand with the IMC controller. The relationship between $|O_{i,i+1}(z)| / |O_{i-1,i}(z)|$ in close loop including the k controller and classical feedback controller $C(z)$ has been

$$\left|\frac{O_{i,i+1}(z)}{O_{i-1,i}(z)}\right| = \left|\frac{C(z)+k}{1 - z^{-1} + (C(z)k\ )z^{-L_i}}\right| \le 1 \tag{38}$$

Using the relation (26) we obtain the expression for the bullwhip effect with an IMC controller as

$$\left|\frac{Q(z) + k(1 - Q(z)\tilde{P}(z))}{1 - z^{-1} + k\ z^{-L_i}}\right| \le 1 \tag{39}$$

This expression is so complicated to analyze, so we used the triangle inequality to separate it in two functions T_1 and T_2 as is shown bellow.

$$\overbrace{\left|Q(z)\frac{1 - k\tilde{P}(z)}{1 - z^{-1} + k\ z^{-L_i}}\right|}^{T_2} \le 1 - \overbrace{\left|\frac{k}{1 - z^{-1} + k\ z^{-L_i}}\right|}^{T_1} \tag{40}$$

In the light of the above equation we can see that once k is designed to stabilise the plant at hand using (33) T_1 gets fixed. Then, the λ parameter in $Q(z)$ can be used to satisfy (40) through T_2. Let's assume a nominal lead time $L = 3$, It has tuned the k controller in order to stabilize the plant and mitigate the bullwhip effect using the $T_1 < 1$ condition, then we tune the λ parameter of $Q(z)$ controller for improve the relation between tracking of inventory position and robust stability, the bullwhip effect is avoided if the relationship (40) is fulfilled, As shown on Figure (9), setting $\lambda = 0.212$ we have a wide bandwidth that indicates a faster response.

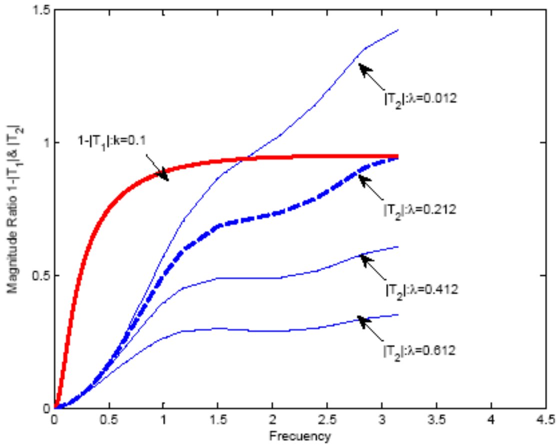

Figure 9. Magnitude Of $1-|T_1|$ and $|T_2|$ With Different Values Of λ.

Figure (10) shows that the IMC controller approach with the analytical design, improve the response with respect to PI controller. This implies that there is a faster replenishment inventory as well as Figure (11) shows that the bullwhip effect is avoided.

5.2.3 Demand forecasting

The forecasting demand will lead to bullwhip effect Pin, David, Shan, Shi, Jang, Shyan and Zheng 2004). We assume that the retailer is using a common method of simple exponential smoothing to estimate a demand forecast for the next period, that is

$$F(z) = \frac{\alpha}{1 - \alpha z^{-1}} \tag{41}$$

Then if we attempt to forecast the customer demand $F(z)$ with $\alpha = 1$ and set the inventory position target according to $L+2$, as shown in Figure (12), the closed loop responses of order to supplier $|O_{i,i+1}(z)| / |O_{i-1,i}(z)|$ become

$$\overbrace{\left|Q(z)\frac{F(z)(1-z^{-1}) + 1 - k\tilde{P}(z)}{1 - z^{-1} + k\ z^{-L_i}}\right|}^{T_2} \le 1 - \overbrace{\left|\frac{k}{1 - z^{-1} + k\ z^{-L_i}}\right|}^{T_1} \tag{42}$$

The forecasting demand function $F(z)$ does not affect the stability so the tuning criterion is the same. The Figure (13) shows the tuning for λ in order to improve the performance avoiding the bullwhip effect.

We simulated the orders for an echelon with the IMC scheme and forecasting demand as shown in Figure (14). IMC control scheme provides efficient control of the inventory position of a supply chain echelon avoiding the bullwhip effect.

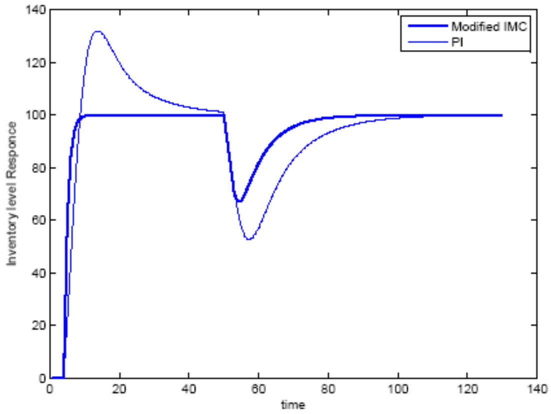

Figure 10. Inventory Level Response

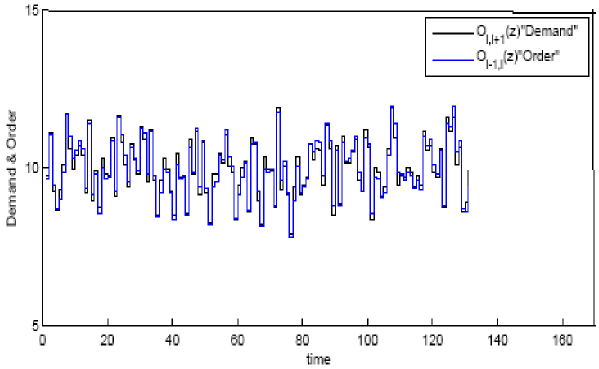

Figure 11. Simulation Results Of a Supply Chain Unit With A IMC ($\lambda = 0.212$) Controller.

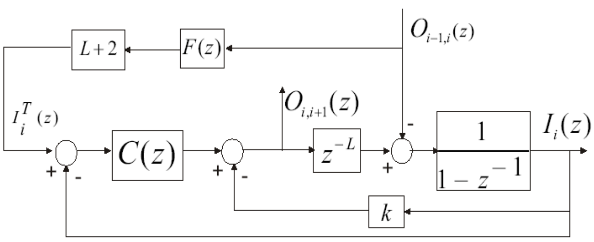

Figure 12. The Block Diagram Of Node i Of A Supply Chain With Forecasting Demand.

6 CONCLUSIONS

The discrete model for an echelon of the beer game has been derived using the z-transform. We obtain the characteristic equations of the closed loop. The bullwhip effect is also analyzed through frequency response. Some alternative ordering policies were formulated as P, PI and IMC control schemes and we can conclude that using P and PI controllers the bullwhip effect of a supply chain unit can be suppressed.

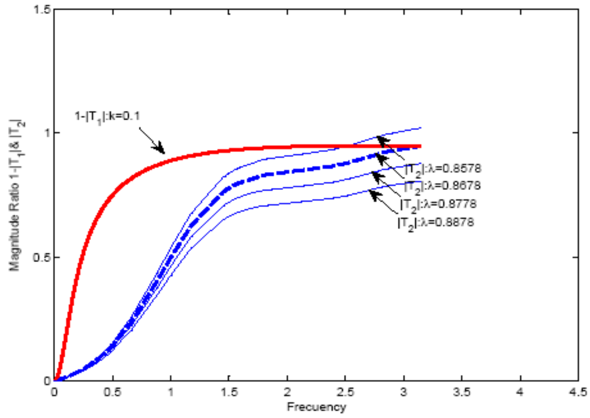

Figure 13. Magnitude Of $1-|T_1|$ and $|T_2|$ With Different Values Of λ

Moreover, the IMC control structure is superior in meeting customer demand due to its better tracking of long term trends of customer demand. We can therefore conclude that control theory is applicable to analysis of supply chains and that is would be possible to improve results using more efficient controllers as PI and IMC.

7 ACKNOWLEDGMENTS

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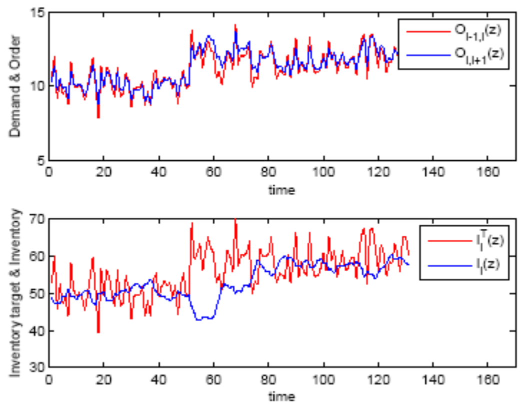

Figure 14. Simulation Results Of A Supply Chain Unit With A IMC Controller ($\lambda = 0.88$) And Stochastic Demand From Downstream.

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