ABSTRACT
Transportation management is a crucial issue in today’s business environment. Firms pay attention to their core business and focus their competences and resources to improve and strengthen their competitive advantages. Consequently, firms find themselves integrated in a network of agents, since they purchase products and services to complete their processes. Logistics activities and services are committed to specialized companies. This changes the standard formulations of delivery and routing problems. This paper focuses the attention in a new formulation of the transportation problem accounting the supplier relationships of freight and groupage and the transportation problem. Transportation Management includes load planning and delivery route planning, respectively referred as Container Loading Problem and Vehicle Routing Problem. The two problems are very investigated but deal with optimization separately. This paper focuses the attention on an integrated approach considering also the outsourcing of logistic.

Keywords: container loading, vehicle routing, outsourcing, decision support system

1. INTRODUCTION
Outsourcing is a strategic and tactical tool of management. In the last decade of the last century, managers have increased its use in the business. The trend on the future is similar, and it is not related to a particular kind of firm, size or business.

Logistics activities and services are committed to specialized companies. In the literature, these actors of the supply chain are known as “third-party logistics (3PL) providers” or “logistics service providers (LSPs)” (Hertz and Alfredsson 2003; Rabinovich and Knemeyer 2006). They provide different types of logistics services (Larson and Gammelgaard 2001; Hong, Chin, and Liu 2004; Lai and Cheng 2004), and the carriage of goods seems to be the crucial service for the supply chain efficiency and effectiveness. More than 50% of the firms use logistics providers for transportation goods (Dapiran, Lieb, Millen, and Sohal 1996; Lieb and Bentz 2004).

The main aim of logistics outsourcing is the minimization of the transportation costs, without forcing the final client to receive a lower level of service. On the contrary, managers have to find ways to increase the customer service level. Generally, two types of managerial problems are related to logistics decisions. The strategic decisions deal with location, production, transportation and inventory problems, according to the specific configuration of the logistics networks. The operational decisions deal with scheduling, lead time, routing, truck loading problems, according to the coordination in the logistic networks.

Different costs could affect the optimization of the shipments. Inventory costs affect the frequency of the shipments, and they are also related to the production process. Transportation and delivery costs affect the design of the flow of goods and the number of travels. The importance of transportation is underlined by the effort made on the networks design and management - point to point, corridor, hub and spoke systems (Tavasszy, Ruijgrok, and Thissen 2003; Lapiere, Ruiz, and Soriano 2004; Hesse and Rodrigue 2004).

The use of logistics outsourcing is sensitive to the mechanisms of coordination among the actors in the supply chain. Logistics is a typical activity where coordination is very important, and the consolidation of orders (Galbraith 1977) is a suitable technique to perform it. Consolidation is the process of combining different items, produced and used at different locations (spatial consolidation) and/or at different times (temporal consolidation) into single vehicle load (Mintzberg 1979).

2. LITERATURE REVIEW

2.1. Three-dimensional Container Loading
The container loading optimization problem is a central problem in the industry. Common problem formulations are Bin-Packing, Knapsack Packing, Container Loading and Multi-Container Loading (Pisinger 2002). Many variants of the Container Loading Problem have been studied.

We consider the three-dimensional knapsack problem with irregular shapes. While two-dimensional packing of irregular shapes is a topic which has been
thoroughly investigated, three-dimensional packing has received far less attention. Generally, polygons are often used to represent the contour of shapes in the two-dimensional problems, whereas triangles mesh structures are often used in three-dimensional problems.

Moreover there are two overall placement strategies: relaxed placement and legal placement. In legal placement no overlapping is allowed.

Ikonen et al. were among the first to consider optimization problems with irregular three-dimensional shapes. They used genetic algorithms with a relaxed placement method based on triangle intersection (Ikonen, Biles, Kumar, Wissel, and Ragade 1997; Ikonen, Biles, Lewis, Kumar, and Ragade 1998).

Dickinson and Knopf used a legal placement method where every item is sequentially placed through an individual optimization heuristics (Dickinson and Knopf 1998; Dickinson and Knopf 2002).

Cagan et al. used a relaxed placement method (Cagan, Degentesh, and Yin 1998). Simulated annealing and spatial octrees (de Berg, Van Kreveld, Overmarks, and Schvarzkopf 2000) were used to quickly determine overlap.

Egeblad et al. generalize their 2D relaxed placement method to three-dimensional problems, improving the results of Ikonen et al. and Dickinson and Knopf both in speed and quality (Egeblad, Nielsen, and Odgaard 2006).

Egeblad et al. use a several consecutively applied heuristics to optimize container loading of furniture (Egeblad, Garavelli, Lisi, and Pisinger 2009).

2.2. Integration of Routing and Loading

Managing the distribution of goods is an important issue in Transportation Management. The vehicle routing problem is a topic which has been thoroughly investigated, but only in the last years integrated approaches have been presented to jointly solve the two problems.

Iori et al. used an exact approach to solve a two-dimensional capacitated vehicle routing problem (2L-CVRP) based on a branch and cut algorithm, for the minimization of the routing cost and on a branch and bound algorithm for checking the feasibility of the loadings. Actually, it was a three-dimensional problem, but the items were often transported on top of rectangular bases (e.g., large pallets of suitable size), and, due to their fragility or shape, they could not be stacked one over the other. In this case, the general three-dimensional loading problem reduces to a suitably defined two-dimensional loading problem of the rectangular items on the floor of the vehicle (Iori, Gonzalez, and Vigo 2006).

Gendreau et al. developed, implemented, and tested a Tabu Search algorithm to solve a 3L-CVRP (Gendreau, Iori, Laporte, and Martello 2006). The authors considered a combination of capacitated vehicle routing and three-dimensional loading, with additional constraints frequently encountered in freight transportation. They proposed a Tabu search algorithm that iteratively invokes an inner Tabu search procedure for the solution of the loading sub-problems. The vehicle had a rectangular loading space. The items were parallelepipeds and had a fixed orientation with respect to the height. Moreover, there were fragility constraints. A 2L-CVRP is solved in (Gendreau, Iori, Laporte, and Martello 2008).

Moura and Oliveira considered an integrated approach to solve a three-dimensional loading and a vehicle routing problem with time windows (Moura and Oliveira 2004). They proposed two different approaches. The first one was a sequential approach; the second used a hierarchical approach to integrate the two problems.

3. PRELIMINARY CONSIDERATIONS

Consolidation requires that the placement of the items in a vehicle is verified before the real loading. Otherwise, some items could be left out with a negative impact on the backorder and service level for the final client. Consequently, a three dimensional container loading problem needs to be solved.

To carry out an internal consolidation in the shipment, two ways can be followed. First, goods of different orders and clients could be loaded in the same vehicle to carry out a route with many destinations. Second, goods of different orders and clients could be loaded in the same vehicle to carry out a route to a hub point where groupage services are asked to 3PL. In both cases a three dimensional container loading problem must be solved to verify that the selected route can be conducted.

Transportation and groupage services are given in outsourcing, and an optimization of the network utilization needs to be carried out.

In this paper, we present a model of distribution where there are three actors: a manufactory company that produce items for the clients, a provider of freight, and a provider of groupage. In the model, the company is both shipper and consolidator, while the providers are the carriers.

In the model we solve a vehicle routing problem integrated with a three-dimensional container loading of items with irregular shapes. The cost formulation of the Vehicle routing problem takes into account an innovative objective function, consistent with the specific outsourcing of the logistic services. An Adaptive Large Neighbourhood Search (Pisinger and Ropke 2007) is then used to solve the distribution model, and some real tests are carried out with the distribution logistics of an Italian furniture manufacturer.

3.1. Furnishing Company Loading

The container loading of furniture problem has special features that were considered in Egeblad et al. 2009. Conventional three-dimensional container loading problems consider placement of boxes with fixed dimensions inside a box-shaped container.
Since sofas and armchairs have irregular shapes, they are usually coupled in order to create cuboids to mimic rectangular placement. Items can be squeezed slightly during transport, which makes a prediction of the final cuboid dimensions difficult. Moreover the shapes can be very different and the items are not rigid. These conditions affect the strategy of integration of routing and loading, because the evaluation of volume of set of items is affected by combination of the products in the loading configurations. Let’s say $v_a$ the volume of item $a$ and $v_b$ the volume of item $b$, $v_{(a,b)}$ the volume of a configuration with item $a$ and $b$. Then,

$$v_{a,b} \leq v_a + v_b.$$  \hfill (1)

Generally the volume of configuration is lower than sum of volumes of single items.

Let’s say $v_{c1}$ the volume of bounding box of configuration $c_1$ and $v_{c2}$ the volume of bounding box of configuration $c_2$. $v_{(c_1,c_2)}$ the volume of the two configuration placed one nearest the other.

$$v_{\text{overlap}(c_1,c_2)} = v_{c_1} + v_{c_2} - v_{c_1,c_2} \geq 0.$$  \hfill (2)

Generally, sofas and armchairs are loaded with their arms resting on the container floor. To solve the container loading problem we use the heuristics presented in (Egeblad, Garavelli, Lisi, and Pisinger 2009) in which more details of the difficulties involved with container loading of furniture are also presented.

### 3.2. Delivery Features

In the model of shipment that we have analyzed there are four kinds of actors:

- Shopkeeper (consignee)
- Manufacturer (shipper and internal consolidator)
- Transportation suppliers (carrier)
- Groupage suppliers (carrier and external consolidator)

Shopkeeper specifies the orders of products to manufacturer. After the production manufacturer plan the shipments, define the service mode of shipments and buy the services they need from the logistic providers. The freight are transported by road. Transportation suppliers execute the routes organized by the manufacturer. They need to supply the trucks and all of the resources required to execute the routes, and to plan appointments with shopkeepers or groupage suppliers for delivery. Groupage suppliers organize the delivery of the products not sent by the manufacturer directly to the shopkeepers via the transportation suppliers.

Manufacturers can choose different service modes to deliver the products to the shopkeepers:

- Direct
- Multi Drop
- Groupage
- Mix

Direct shipment consists in a route supplied by carrier. It takes the products from the warehouse of manufacturer and delivers them to warehouse of the shopkeeper. The cost of service is fixed and it is related to a subarea of the shopkeeper’s country. It’s not related to effective distance and time: so if there are two warehouses in the same subarea and one is more distant than the other, the cost of service is the same for both warehouses. However there is a variability in the profits of carrier. This transport is used in Full Truck Load and Full Container Load.

Multi Drop shipment consists in a route supplied by carrier. It takes the product from the warehouse of manufactures and delivers them to warehouse of shopkeepers. The shopkeepers can be localized in the same subarea or in a different subarea. So the cost of service is related to the cost of single direct shipments for every shopkeeper. In fact, it is the biggest of all. Fixed costs are provided for each un-load location. Additional costs are related to specific travel and are related to distance and time of travel.

Groupage shipment consists of two steps. In the first step a carrier supplies a route from warehouse of manufacturer to warehouse of a groupage provider. The cost of service is fixed and it is related to a subarea of the groupage provider’s country. This transport is used in Full Truck Load and Full Container Load. After the groupage supplier provides delivery of the products to shopkeepers, in according to Less than Truck Load deliveries. The cost of service is related to volume and weight of items and on locations of destinations.

Mix shipment is a combination of groupage shipment and multi drop; exactly one unload point is a groupage point.

Often, the routing optimization is “static”: for each customer is a-priori defined the service mode (point-to-point, groupage, direct-to-unload, etc…). Therefore, each customer is considering lying on a pre-defined path or, at least, each one belongs to a specific area and no optimization of the route is made during the
optimization referring to the loading. Likewise no optimization of loading is made during the optimization of routing. Generally the problem is solved in more steps: first of all the service mode is defined. After the route is organized and finally a container loading is performed. This could lead to an inefficient optimization because:

- it is easy to define the optimal service mode for a single travel but it’s not easy to choose the optimal service mode for all orders and customers;
- not considering the loading during optimization of routing can lead to infeasible truck loadings or truck loadings with not good level of filling;
- if the loading is organized by a LIFO rule, not considering the routing during the optimization of loading can lead to not feasible truck loading.

4. MODEL OF ANALYSIS

4.1. Working hypothesis

Consolidation can be performed in internal way or in external way. Internal consolidation is performed with products of same company shifted in the time or in the space. External consolidation is performed with products of different firms. In the model we describe how the consolidation is performed by a shipper and by a groupage provider. The shipper performs an internal consolidation group in same trucks products of different shopkeepers and delivers them buying freight services. An external consolidation when shipper acquires groupage service from logistic provider.

More logistic providers can be selected by the shipper. This means that we can consider that there are enough resources in the markets to supply service to shippers. In the analysis of this paper, no attention is involved in vendor rating and supplier selection. A hypothesis is that all suppliers are equal; this means that we can consider just one carrier and one provider of groupage service.

The previous hypothesis permits to classify the routing problem as a routing problem with an unlimited fleet of vehicles. One kind of vehicles at the time is considered in the problem.

The planning of delivery is built by shippers, only execution is performed by carrier. From this point of view, the vehicle routing problem can also be classified as an open vehicle routing problem.

Only capacity of vehicles is take in analysis, so the problem is a capacitated routing problem. There are no explicit time windows constraints.

To verify that vehicles are not over-filled a three dimensional loading problem is solved. Loading constraints are considered in the model and products with irregular shapes are taken in account. The routing is a three dimensional loading capacitated vehicle routing problem.

To choose the products to deliver, some rules and constraints are considered. There isn’t the capacity to ship all products of warehouse, because the loading operations are manual. Oldest produced orders have a higher priority in the delivery as orders to seasonal sales. In the model, only two kinds of priority are considered.

Generally the warehouse costs affect the priority of shipments. Since the delivery is a daily activity, the warehouse costs are not taken in account in the objective function. Besides an unfilled vehicle can be stopped in the warehouse for few days to allow filling with work in progress products of same shopkeepers. The problem is an open problem and to manage it we assume that two average filled trucks are worse than one filled truck and one no filled truck. In this way an objective will be to find solutions with more filled trucks.

4.2. Objective function and constraints

Since the transportation is in outsourcing the routing problem has to reflect a different formulation of the objective function. There is a transaction between carrier and shipper based on master contracts to delivery items to shopkeepers. The planning of deliveries performed by shippers has to contain more objectives.

The principle objective of the manufacturer is to minimize the cost of the delivery plan. The delivery plan is defined by the all routes need to perform the shipments. \( R \) is the whole of route and \( r_i \) is the \( i \)-th route.

\[
C(R) = \sum_{i=1}^{n} c(r_i) \tag{3}
\]

The cost of single route is made up of three parts: link cost, unload cost and groupage cost. Link costs and unload costs are defined in the contracts between manufacturer and freight providers, groupage costs are defined in the contract between manufacturer and groupage providers.

A route \( r_i \) is a sequence of points (source, \( d_{i1}, d_{i2}, \ldots, d_{ir}, \ldots,d_{in} \)), \( d_j \) is the \( j \)-th point of delivery in the route \( r \). \( d_l \) is a location and for each location a subarea \( I_l \) is defined.

\[
d_j \in I_{l_j} \subset \{l_1,l_2,\ldots,l_i,\ldots,l_n\} = L \tag{4}
\]

\[
d_{j,r} \in I_{l_r} \subset \{l_{1r},l_{2r},\ldots,l_{ir},\ldots,l_{nr}\} = L(r) \tag{5}
\]

\[
m \leq n \tag{6}
\]
Figure 2 – Supply Chain in the outbound of logistic of the model: shopkeeper orders product to manufacturer. After the production, manufacturer delivery the products buying transport and groupage services from logistic providers.

\[ C(l_i) = f(source, l_i) \]  

(7)

In real contracts the cost of link is also due to specific carrier and kind of vehicle. Since we consider just one kind of vehicle and no selection of carrier, we can assume valid the expression (7).

\[ c(l_i) = f(source, l_i) \]

(7)

The second part of route’s cost is related to number of delivery points in the route. Since that the distance and the time to perform a route with more delivery point are bigger than in a route with one delivery, appropriately costs are imputed. The cost is made up of two parts. One is related to number of stops: for each stop except one a fixed cost \( c_{stop} \) is imputed. This cost is imputed because the carrier has to plan more appointments and queues and has to cover more distance. The second one is a variable cost and is related to the extra distance involved in a multi point travel \( r_i \) compared to a single point travel \( r_i \). Since that some kilometres are covered in fixed part, a threshold \( k \) is defined.

\[ C_u(r_i) = C_{u1}(r_i) + C_{u2}(r_i) \]

(8)

\[ C_{u1}(r_i) = (n_{stop}(r_i) - 1) \times c_{stop} \]

(9)

\[ C_{u2}(r_i) = (d(r_i) - d(r_i^*) + \left[ (n_{stop}(r_i) - 1) \times k \right] \times c_{km} \]

(10)

\[ C_{u1}(r_i) = \max_{i \in L(r_i)} c(l_i) \]

(11)

\[ \text{route } r_i \] is the travel between source and the last delivery point in the route.

\[ L3 \]

\[ L2 \]

\[ L1 \]

\[ L4 \]

\[ L5 \]

\[ L6 \]

\[ r1 \]

\[ r2 \]

\[ r3 \]

\[ r*(r_i) \] is the travel between source and the last delivery point in the route.

The last part of route’s cost is the groupage cost. It is applied to all products if the service mode is groupage or to products in the groupage point of mix. Exactly this part of cost is related to second phase of groupage service. The groupage provider organizes the shipment to shopkeepers. It benefits from filling a vehicle with products of a specific couple shopkeeper- manufacturer with products of other couples supplier-customer, generally in the same subarea of shopkeeper or nearest. In this sense the manufacturer benefits from external consolidation. The cost of service is related to subarea of the shopkeeper’s location, volume and weight of products. Here the structure of costs is related only to
volume of products and a not standardized unit of measure is used; it is typical in Italian furniture manufacturer and it measures the number of seats in the products. Let’s say s to be the number of seats in the route to ship at groupage for location li. The cost per seats is:

\[ c^s_i(l_i) \text{ for } 0 \leq s < d_1 \]
\[ c^s_i(l_i) \text{ for } d_1 \leq s < d_2 \]
\[ \ldots \]
\[ c^s_i(l_i) \text{ for } d_n < s \] \hspace{1cm} (12)

Then the total cost of groupage in the route is:

\[ C_g(r_i) = \sum_{l_i} s_i(r_i) \times c^g_i(l_i, s_i) \] \hspace{1cm} (13)

Since that the groupage provider and the transport provider can be different, not necessarily the location defined in the groupage formulation are equal in the link cost formulation.

The total cost of route is:

\[ C_t(r_i) = C_t(r_i) + C_u(r_i) + C_g(r_i) \] \hspace{1cm} (14)

Minimizing the total cost of route is one of objectives of manufacturer. The shipment is travelled on long distance, so only truck with high utilization are used to delivery the products. High levels of consolidation are required. Often high level of consolidation means more use of groupage service. Since the delivery lead time using groupage service is larger, a trade-off between utilization of volumes and delivery lead time occurred.

The model has not a time windows formulation, but the orders are split in two groups:

- High priority of delivery
- Normal priority of delivery

Orders with high priority have to be placed in full vehicles and no groupage service has to be used. The first condition is modelled as a soft constraint: if the products of a high priority order are placed in an unfilled vehicle, a penalty cost is added to objective function. If the volume of truck is \( v_t \), the available volume \( v_f \) is:

\[ v_f = v_t \times f_r \] \hspace{1cm} (15)

Since that not all space can be used to fill the truck, \( f_r \) is estimation on real useful space. It is related to variety of product’s dimensions. The utilization of space is:

\[ u(r_i) = \frac{v(r_i)}{v_f} \] \hspace{1cm} (16)

\( t_t \) is a threshold value to identify a full vehicle. If the \( u(r_i) \) is bigger than \( t_t \), the route is a full truck. Then priority cost is:

\[ pc(o, r_i) = \begin{cases} 0 & t_f < u(r_i) \\ c_p & u(r_i) \leq t_f \end{cases} \] \hspace{1cm} (17)

If order o is a priority order \( y_o \) is 0, otherwise is 1. If \( O(r_i) \) is the set of orders o in the route \( r_i \), the penalty cost of priority in the route \( r_i \) is:

\[ C_p(r_i) = \sum_{o \in O(r_i)} y_o \times pc(o, r_i) \] \hspace{1cm} (18)

Since the delivery lead time using groupage is bigger than direct or multi drop service mode, the second condition is a hard constraint. Planning products of high priority orders with groupage service makes the solution not feasible.

As explained in Section 4, a sub-objective is to planning more filled trucks. If two solutions are equal as regard the total cost of route (expression 14th), the solution with more filled trucks is preferred. Besides, unit cost transport is generally larger in an unfilled vehicle than in filled vehicle. There is a soft constraint in the objective function, adding a penalty for whenever an unfilled truck is used.

Let’ say \( t'^{\text{inf}}_n \) and \( t'^{\text{inf}}_n \) to be two threshold values to identify an average filled truck. Then if \( u(r_i) \) is between the two threshold values, the cost of not full vehicle is:

\[ C_{nf}(r_i) = \text{Min}[\left( t'^{\text{inf}}_n - u(r_i) \right)^2 \times c_{nf} \times \left( u(r_i) - t'^{\text{inf}}_n \right)^2 \times c_{nf} ] \] \hspace{1cm} (19)

Otherwise it is zero. The penalty cost has a quadratic function and it penalize the solutions with many average full trucks in comparison to solution with filled and empty trucks.

The described objective function takes in account the aims of manufacturer. Since the formulation is different in comparison to standard vehicle routing problem formulation, the planning of the route could be far from objective of transport provider and not feasible.

In the contract there are some constraints. They refer to:

- max number of stops in a route;
- max distance between first and last stop in the route;
- max distance in the route;
- max distance between stops.

The constraints allow at transport provider to plan a feasible execution of travels performing the
appointments with shopkeepers or groupage providers to delivery the products. Since that the cost are related to locations and not on distances, nothing ensures that the route is cheapest for the carrier. He would like to change the order of visits to shopkeeper but he has to unload and load all products at every stop. Since a LIFO rules is used to load and unload the products, the manufacturer have to take in account it in the routes’ planning. A standard formulation of vehicle routing problem needs to meet provider’s objectives.

The provider’s cost to perform a route is made up of three parts. One is related to fixed costs as assurances, hires, amortizations, etc. The second one is related to distance and the other one is related to time, as expressed in the formula below.

\[
C_p(r) = C_{fp} + \sum_{i=0}^{n-1} c_{im} \times d(i, i+1) + c_h \times f(i, i+1) + e(i, i+1)
\]  

In the previous formula the return travel is not considered because the provider generally performs a not empty return travel, supplying services to other companies.

4.3. Heuristics description

To solve the model some heuristics are used to solve:

- the vehicle routing problem with outsourcing of logistic formulation (we call it Capacitated Vehicle Routing Problem with Contract Optimization, CCVRPCO);
- the Three Dimensional Container Loading of furniture with Irregular Shapes;
- the integration between Container Loading and Vehicle Routing.

Due to the intrinsic hardness of three-dimensional (3D) packing, it is intractable to verify feasibility of a packing in every step of the algorithm in reasonable time. Besides, it’s not possible a pre-computation of all combinations of packages of products. Then a sequential integration approach can not be used, a hierarchical method is used. In the hierarchical approach the CLP is considered as a sub-problem of CVRPCO. The three dimensional CLP is solved with heuristics showed in (Egeblad et al. 2009).

To solve the CVRPCO an Adaptive Large Neighbourhood Search was implemented.

A heuristic was implemented to solve the integration problem. The principle problem on integration is that the volumes of packages of products is related to real placement as explained in §3. In the figure 5 the interaction of the three heuristics is showed.

ALNS combines various functions of removal and insertion of package of orders allowing an exploration of large neighborhoods of solution. After a definition of starting solution, the insertion heuristics and the removal heuristics are selected using a roulette mechanism of selection.

Randomly the size of action is defined and the removal and insertion heuristics are applied to define a new feasible solution of 3DL-CVRPCO. The new solution is accepted with a Simulating Annealing Criteria. Exactly the probability is:

\[
\begin{align*}
& \begin{cases}
1 & f(s') \leq f(s) \\
\exp\left(\frac{f(s) - f(s')}{T}\right) & f(s') > f(s)
\end{cases}
\end{align*}
\]  

To manage the removal of orders three kinds of heuristics are used: random removal, shaw removal, worst removal.

In random removal q orders are randomly selected and removed from routes, allowing an escape from a local optimum and introducing a diversifying effect.

In shaw removal q orders are selected if they are closely related. Different criteria define the closeness. The idea is that no closely related orders could generate
worse solutions. The q orders that give the minimum sum of relatedness value are selected:

\[
\min \sum_{m \in O} \sum_{n \in O} r_{mn} x_m x_n \\
\sum_{o \in O} x_o = q \\
x_o \in \{0, 1\} \quad \forall o \in O
\]  

(22)

\(r_{mn}\) is the measure of relatedness. In the model there are four different measurements:

- **Distance**: it measures the distance between orders. The idea is that the management of nearest orders could generate better solutions;
- **Route**: it measures if the orders are placed in the same truck. The idea is to move more orders from the same route to generate new good solutions.
- **Volume**: it measures the capability of orders to fill a truck. The idea is to use the orders from the least filled trucks to fill the most filled trucks.
- **Groupage**: it measures the service mode of orders. Orders that are placed in same truck and are delivered to groupage will be selected.

\[\Delta(o) = f(s'(o)) - f(s)\]  

(23)

The saving is negative if removal of order decreases the objective function. So the order with lowest saving is selected. After removal, the procedure is iterated starting from solution s'.

Similarly some insertion heuristics are defined: random, best first insert, regret insert groupage and regret insert.

Random insert is a random insertion of the removed q orders and the advantages are the same of random removal.

Best first insert is an insertion heuristics that works as worst removal. The difference is that also the position in route has to be defined. Solution \(s'(o,r,i)\) has a new solution in which order o is inserted in the route r at position i. The saving is:

\[\Delta(o, r, i) = f(s'(o,r,i)) - f(s)\]  

(24)

If an insertion of an order decreases the objective value of solution s', the saving is negative. So the order o in the route r at position i is selected. After insertion the procedure is iterated until all orders q are re-inserted.

The regret insert is similar to best first insert, but it is based on regret value. The regret value is the difference between the saving as defined in (24). Exactly the difference is between the saving of 2nd best position of order o and his best position:

\[\delta^0_o = \Delta^{0(r)}_o - \Delta^{0(r)}_{i}\]  

(25)

High value of regret value means to insert order in his best position is priority. It’s possible to extend the regret insert, considering more insertions. The aim is to preview the difficulties.

\[\delta^0_o = \sum_{j=1}^k \Delta^{0(r)}_{j} - \Delta^{0(r)}_{i}\]  

(26)

\(s'_{t+1}\) is the solution without the order o. The saving is:

\[\Delta(o) = f(s'(o)) - f(s)\]  

(23)

The saving is negative if removal of order decreases the objective function. So the order with lowest saving is selected. After removal, the procedure is iterated starting from solution s'.

Similarly some insertion heuristics are defined: random, best first insert, regret insert groupage and regret insert.

Random insert is a random insertion of the removed q orders and the advantages are the same of random removal.

Best first insert is an insertion heuristics that works as worst removal. The difference is that also the position in route has to be defined. Solution \(s'(o,r,i)\) has a new solution in which order o is inserted in the route r at position i. The saving is:

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If an insertion of an order decreases the objective value of solution s’, the saving is negative. So the order o in the route r at position i is selected. After insertion the procedure is iterated until all orders q are re-inserted.

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High value of regret value means to insert order in his best position is priority. It’s possible to extend the regret insert, considering more insertions. The aim is to preview the difficulties.

\[\delta^0_o = \sum_{j=1}^k \Delta^{0(r)}_{j} - \Delta^{0(r)}_{i}\]  

(26)

In all iterations of ALNS a removal heuristics and an insertion heuristics are selected to modify the solutions. A roulette mechanism is designed to select the heuristics. Each heuristics has a score. The scores of couple of heuristics used in iteration are updated with:

- \(\sigma_1\): a new best solution is found;
- \(\sigma_2\): a new solution better than current solution is found;
- \(\sigma_3\): a worse solution is found and it is accepted due to simulated annealing criteria.

After x iterations the scores are updated with the formula below:
\[ w_i(1-r) + \frac{r \varphi_i}{\phi} \]  

(27)

The probability to choose heuristics is related to the scores since that \( p_i \) is:

\[ p_i = \frac{\varphi_i}{\sum_{j=1}^{h} w_j} \]  

(28)

Every time a new solution of 1LCVRPCO has been generated, the loading heuristics is used to verify the feasibility of loading of trucks. If all trucks are feasible the procedure stops and the loadings define the 3LCVRPCO. Otherwise a new solution 1LCVRPCO will have to be generated after a penalty volume has been added to product volumes. As explained in §3.1, there are two reasons on difference between sum of volume of single products and volume of bounding box of placed products: product in cuboids arrangements and cuboids arrangements in the package of products.

A utility function estimates the volume used by a product in cuboids arrangements. Since that the loading heuristics uses different cuboids arrangements, each time a three-dimensional problem is solved, statistical data are collected. Let \( C(i) \) be the set of encountered configurations with product \( i \) from all previously solved three-dimensional problems. Then the utility of product \( i \) is:

\[ \frac{1}{|C(i)|} \sum_{c \in C(i)} \sum_{c \subseteq j} w(j) \times h(j) \times d(j) \]  

(29)

In the next stage of 1LCVRPCO the estimate of the amount of volume that will be occupied by \( i \) will be:

\[ v_e(i) = u(i) \times w(i) \times h(i) \times d(i) \]  

(30)

Waste space and occupied space in the truck are affected by lost space above and between products due to the related positions between cuboids arrangements. To consider it, a penalty factor is applied to volume of package of products. The penalty factor is a measurement of how hard it is to place an individual product in the truck.

At end of every iteration and after the loading checking, if the truck is overfilled the package’s penalty factor is increased.

If the truck is not overfilled a ratio \( r \) is calculated:

\[ r = \frac{\text{OccupiedVolume} + \text{WasteVolume}}{\text{ItemVolume} + \text{PenalVolume}} \]  

(31)

\[ 0.9 \times \text{IterationCount} \leq r \leq 1.02 \]  

(32)

If expression (32) is verified the penalty factor is not updated, otherwise it will increase or decrease due to value of \( r \).

5. EXPERIMENTS

To test the model some instances were randomly built. Precisely, a random instance generator was used to select a subset of clients from a large set of clients with same probability of extraction. In the same way a group of orders is selected until to the specified total number of seats of products. Randomly the orders are assigned to clients. The only condition is that each client has at least one order.

<table>
<thead>
<tr>
<th>Table 1: Experiment’s Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>service mode - prov. constraints</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Total # route</td>
</tr>
<tr>
<td>average filling index %</td>
</tr>
<tr>
<td># unloads</td>
</tr>
<tr>
<td>Total %group.</td>
</tr>
<tr>
<td>Manufacturer’s cost in O.F. [E]</td>
</tr>
<tr>
<td>Total # route</td>
</tr>
<tr>
<td>average filling index %</td>
</tr>
<tr>
<td># unloads</td>
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<tr>
<td># unloads</td>
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<tr>
<td>average filling index %</td>
</tr>
<tr>
<td># unloads</td>
</tr>
<tr>
<td>Total %group.</td>
</tr>
<tr>
<td>Manufacturer’s cost in O.F. [E]</td>
</tr>
</tbody>
</table>

The parameters of the random instance generator are the number of clients and the number of seats. The number of clients is 25 or 50 or 75 or 100, the number of seats is close to 500 or 1000 or 1500 or 2000. Sixteen instances are so defined.

The instances are solved considering three different capacity criteria:
• three dimensional loading, using the integrated model;
• one dimensional loading considering the volume of single products;
• one dimensional loading considering the seats of products.

The aim of the tests is to analyze the performance of models changing the service mode and the consolidation opportunity:

• no consolidation with only direct service mode;
• maximum level of consolidation with only groupage service mode;
• internal consolidation with more destinations in the routes and without the provider’s constraints and objectives (multi drop service mode);
• internal consolidation with more destinations in the routes and managing the provider’s constraints and objectives (multi drop service mode);
• internal and external consolidation giving freedom to model to select the product to delivery using groupage and accounting the provider’s constraints and objectives.

In total 240 instances are solved using two Quad Core Intel Xeon E5430 2.66 Ghz processors. All routes defined by 1L-CCVRPCO, with both seats and volume capacity criteria and with all allowed service modes are been separately solved by Loading Heuristics (see table 2). Some tests are conducted in an Italian Furniture Company just using the seats capacity. Nine randomly generated instances are been solved by both experts and 1L-VRPCO using seats capacity criteria (see table 3).

Table 2: Loading Checks on the 16 instances with all allowed service modes and provider’s constraints and objectives solved with 1L-CVRPCO

<table>
<thead>
<tr>
<th>Capacity criteria</th>
<th># cases</th>
<th>3D loading filling index %</th>
<th># cases loading check: ok [%]</th>
<th>3D loading filling index %</th>
<th># cases loading check: not ok [%]</th>
<th>3D loading filling index %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol</td>
<td>110</td>
<td>84</td>
<td>49</td>
<td>89</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>Seats</td>
<td>145</td>
<td>79</td>
<td>74</td>
<td>77</td>
<td>26</td>
<td>84</td>
</tr>
</tbody>
</table>

Table 3: a comparison between heuristics and expertise solutions for 9 randomly generated instances

<table>
<thead>
<tr>
<th>Instances</th>
<th># Clients</th>
<th>Seats</th>
<th>Improvements</th>
</tr>
</thead>
<tbody>
<tr>
<td>t01</td>
<td>67</td>
<td>2761.4</td>
<td>7%</td>
</tr>
<tr>
<td>t02</td>
<td>63</td>
<td>2211.1</td>
<td>2%</td>
</tr>
<tr>
<td>t03</td>
<td>14</td>
<td>554.3</td>
<td>7%</td>
</tr>
<tr>
<td>t04</td>
<td>36</td>
<td>986.1</td>
<td>4%</td>
</tr>
<tr>
<td>t05</td>
<td>99</td>
<td>1141.6</td>
<td>6%</td>
</tr>
<tr>
<td>t06</td>
<td>35</td>
<td>274.7</td>
<td>4%</td>
</tr>
<tr>
<td>t07</td>
<td>242</td>
<td>2965.6</td>
<td>5%</td>
</tr>
<tr>
<td>t08</td>
<td>48</td>
<td>1754.0</td>
<td>7%</td>
</tr>
<tr>
<td>t09</td>
<td>82</td>
<td>3970.0</td>
<td>3%</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

The experiments show how the developed meta-heuristics correctly solves the model.

Allowing only direct service mode in the model, it performs a consolidation between orders of same clients. An upper-bound of the number of routes has been defined but the filling indexes are lower. The filling indexes are calculated considering the relative applied capacity criteria. If a capacity criterion is Full3D, a bounding box volume of placed products is defined and it is divided per available volume of truck. If capacity criterion is seats, the total number of seats of products is divided by the seats capacity of truck. If the capacity criterion is Vol, the sum of volume of each product is divided per volume of truck. In all cases the average filling indexes are lower, a different use of shipment network need to be lower the cost of shipment. Since that the route has only one delivery, the objective of transport provider is achieved. Allowing only groupage service mode, the max level of consolidation could be achieved. In fact a lower number of routes and high level of utilization are achieved. Since that the route has only one delivery, the objective of transport provider is achieved. The cost of shipment is very high and also the delivery lead time is high considering other service modes as direct and multi drop service modes. Allowing direct and multi drop service modes, without to accounts the aims of transport providers, a reasonable delivery cost and high filling indexes are achieved in 1L-CVRPCO model. In 3L-CVRPCO the filling index is lower but this is reasonable considering the effective loading of products. However the index increase seeing only at route with more seats (>60 or >100 in table1). Anyway the routes are not feasible because the aims of transport providers are not respected. This is confirmed by applying the aims of providers to the model: the number of routes increases as the costs of the routes, the filling indexes decreases. Finally good and reasonable solutions are achieved allowing internal and external consolidation using all service modes. The heuristics finds the optimal assignment of orders, routes and service mode.

It’s interesting to notice the impact of loading in the solutions. In the tests with all allowed service modes the cost of 3L-CVRPCO solutions increases about 20% compared to only using volume as a criterion and 3% compared to using seats as a criterion. The seat is a no standard measurements unit that experience has suggested in the furniture companies. It looks to be a better measurement than volume of single items: the difference between the 3L-CVRPCO and 1L-CVRPCO(seat) is lower. Besides, seeing at number of routes in the solutions with high consolidation (only groupage service mode) it is equal in both Full3D and seats capacity criteria. This suggests using 1L-CVRPCO to solve the model, without the integration of loading. However, this is not true, since some routes could be infeasible. In the table 2 we report data from
the verification process of the routes. Using seats about 26% of routes don’t pass the loading verification. Clearly using vol, the percentage is bigger.

Finally the solutions of model are been compared with solutions of companies’ experts. Since that the expert cannot check the loading, the comparison is between 1L-CVRPCO(seat) and experts. In all instances there are significant improvements and the previous tests suggest that 3L-CVRPCO warrant good improvements with certainty in loading.

Further developments of this study seem to be promising. An improvement could be made on the working hypothesis about warehouses’ costs and delivery times required by customers. Other improvements could be made considering splitting of orders in overfilled routes.

REFERENCES


