CONTROLLED TRIANGULAR BATCHES PETRI NETS: A HIGHWAY CASE STUDY

Leonardo Brenner[a], **Isabel Demongodin**[b]

[a],[b]Aix Marseille Université, CNRS, ENSAM, Université de Toulon
LSIS UMR 7296, 13397, Marseille, France

[a]Leonardo.Brenner@lsis.org, [b]Isabel.Demongodin@lsis.org

ABSTRACT

In the discrete event and hybrid systems theory, Triangular Batches Petri Nets (TBPN) have been defined as an extension of Batches Petri Nets (BPN) to represent conges- tion/decongestion phenomena. Applied to mesoscopic modeling of traffic road networks, TBPN define a triangular flow-density relation, that allow to modeling these phenomena. An other extension, Controlled Triangular Batches Petri Nets (CTBPN) define controlled events in the formalism that allow to represent variation speed limit and flow when a control is applied on the traffic road networks. In this paper, we present the hybrid formalism defined in CTBPN, i.e., the hybrid behavior of a batch in free, congestion or decongestion behavior. A simulation method is introduced and a case study of a real highway is modeled. A comparison of the collected data and simulation results shows the accuracy of the model.

keywords: Petri Nets, Discrete Event Systems, Hybrid systems, Flow, Traffic road

1. INTRODUCTION

Traffic congestion is a growing problem in many metropolitan areas . Congestion increases travel time, air pollution, carbon dioxide (CO2) emissions and fuel use because cars cannot run efficiently. In fact, when the traffic demand is greater than the traffic capacity of the road, a phenomenon of traffic congestion appears. Mathematical models can simulate the real world scenario to predict the behavior of traffic in proper planning and design of the road network. These same models can be used to test different control strategies to reduce the congestion. In these contexts, mathematical description of traffic flow has been and is always a lively subject of research (Lighthill and Whitham 1955).

In this paper, we model and simulate a real highway road section using the Controlled Triangular Batches Petri Nets (CTBPN) formalism (Gaddouri, Brenner, and Demongodin 2014; Gaddouri, Brenner, and Demongodin 2016). This extension of the Generalized Batches Petri Nets (BTN) formalism (Demongodin 2001, Demongodin 2009) integrates the flow-density relation observed on the traffic flow.

The section 2. introduces the concepts of the CTBPN formalism. The section 3. presents the simulation method that compute the dynamic evolution of the model. A real highway section is simulated in the section 4. and the results are compared with collected data.

2. CONTROLLED TRIANGULAR BATCHES PETRI NETS

We present in this section the main concepts of the Controlled Triangular Batches Petri Nets (CTBPN) formalism. This formalism extends the flow-density relation in a batch place to represent the triangular fundamental diagram used in traffic road models (Daganzo 1994) to describe the flow-density relation.

2.1. Triangular Batches Petri Nets

A Triangular Batches Petri Net (TBPN) is an extension of a Generalized Batches Petri Net (see (Demongodin 2001, Demongodin 2009, Demongodin and Giua 2014) for more details on this formalism), where some new characteristics related to the batch place have been added.

TBPN is an hybrid Petri Nets formalism defined by six kind of nodes (places and transitions) (Figure 1).

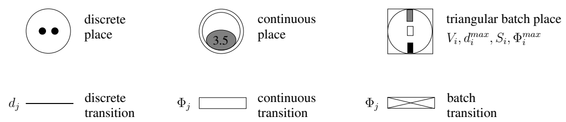

Figure 1: Nodes of Triangular Batches Petri Nets

Definition 1 *A* Triangular Batches Petri Net *(TBPN) is a 6-tuple* $N = (P, T, Pre, Post, \gamma, Time)$ *where:*

- $P = P^D \cup P^C \cup P^{TB}$ *is a finite set of places partitioned into the three classes of discrete, continuous and triangular batch places.*
- $T = T^D \cup T^C \cup T^B$ *is a finite set of transitions partitioned into the three classes of discrete, continuous and batch transitions.*
- $Pre, Post : (P^D \times T \to \mathbb{N}) \cup ((P^C \cup P^B) \times T \to \mathbb{R}_{\geq 0})$ *are, respectively, the pre-incidence and post-incidence matrices, denoting the weight of the arcs from places to transitions and from transitions to places.*
- $\gamma : P^{TB} \to \mathbb{R}^4_{\geq 0}$ *is the* triangular batch place function. *It associates with each triangular batch place* $p_i \in P^{TB}$ *the quadruple* $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$ *that represents, respectively, the maximum speed, the maximum density, the length and the maximum flow.*
- $Time : T \to \mathbb{R}_{\geq 0}$ *associates a non negative number with every transition:*
  - *if* $t_j \in T^D$, *then* $Time(t_j) = d_j$ *denotes the* firing delay *associated with the discrete transition;*
  - *if* $t_j \in T^C \cup T^{TB}$, *then* $Time(t_j) = \Phi_j$ *denotes the* maximal firing flow *associated with the continuous or batch transition.* ■

2.2. Triangular Batch Place

A triangular batch place (TB-place) $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$ is a transfer zone and is defined by four characteristics: *maximum speed*, *maximum density*, *length* and *maximum flow*.

Definition 2 *Let a triangular batch place* p_i, *with* $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$. *A* propagation speed of congestion, *denoted* W_i, *and a* critical density d_i^{cri}, *are associated with* p_i, *defined respectively by:*

$$W_i = \frac{\Phi_i^{max} \cdot V_i}{d_i^{max} \cdot V_i - \Phi_i^{max}} \tag{1}$$

$$d_i^{cri} = \frac{W_i . d_i^{max}}{V_i + W_i} \tag{2}$$

The flow-density relation *that governs the dynamics of TB-place* p_i *is defined as follows:*

$$\phi = \begin{cases} d.V_i & \text{if} \quad 0 \leq d \leq d_i^{cri} \\ W_i.(d_i^{max} - d) & \text{if} \quad d_i^{cri} < d \leq d_i^{max} \end{cases} \tag{3}$$

where d *denotes density and* ϕ *denotes flow.* ■

Figure 2 represents these definitions.

Let us now introduced some definitions needed for the rest of this paper. Let a Triangular Batch place p_i, with $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$. An *input flow* $\phi_i^{in}(\tau)$ and an *output flow* $\phi_i^{out}(\tau)$ of place p_i are respectively: $\phi_i^{\text{in}}(\tau) = Post(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$ and $\phi_i^{\text{out}}(\tau) = Pre(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$ where $\varphi(\tau)$ is the instantaneous firing vector of continuous and batch transitions (see (Demongodin and Giua 2010) for more details).

2.3. Controlled Triangular Batches Petri Nets

A Controlled Triangular Batches Petri Net (CTBPN) has the same syntax than TBPN. However we associate with CTBPN a different semantics, assuming that the maximal firing flow of continuous and batch transitions and, the maximal transfer speed of triangular batch places are control inputs.

Definition 3 *A Controlled Triangular Batches Petri Net (CTBPN) is a TBPN where the maximal transfer speed of TB-place* $p_i \in P^{TB}$ *and, the maximal firing flow associated with a continuous or batch transition* $t_j \in T^C \cup T^B$, *can varied. We denote respectively these variables:* $v_i(\tau)$, *with* $0 \leq v_i(\tau) \leq V_i$, *and* $\phi_j(\tau)$, *with* $0 \leq \phi_j(\tau) \leq \Phi_j$. ■

It should be noted that the variation of the speed of TB-places imposes a variation of the critical density and of the maximum flow of TB-place while the propagation speed of congestion, W_i stays constant as shown in Figure 2.

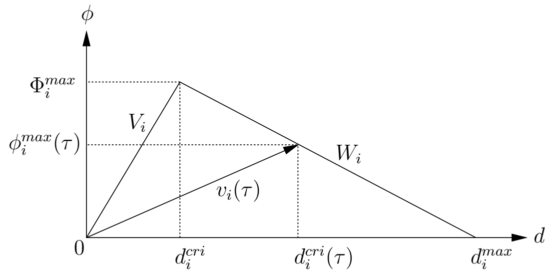

Figure 2: Flow-density relation of a TB-place

Definition 4 *Let TB-place* p_i *with* $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$ *with a maximal transfer speed* $v_i(\tau)$ *such that* $0 \leq v_i(\tau) \leq V_i$. *At time* τ, *the* controlled critical density $d_i^{cri}(\tau)$ *and the* controlled maximum flow $\phi_i^{max}(\tau)$ *are respectively defined by:*

$$d_i^{cri}(\tau) = \frac{W_i . d_i^{max}}{v_i(\tau) + W_i}, \tag{4}$$

$$\phi_i^{max}(\tau) = v_i(\tau) . d_i^{cri}(\tau) \tag{5}$$

ISBN 978-88-97999-85-0; Affenzeller, Bruzzone, Jiménez, Longo and Piera Eds.

with $0 \leq \phi_i^{max}(\tau) \leq \Phi_i^{max}$ *and* $\frac{\Phi_i^{max}}{V_i} \leq d_i^{cri}(\tau) \leq d_i^{max}$.

2.3.1. Controllable Batches

Each TB-place contains a series of controllable batches ordered by their head positions.

A controllable batch, i.e., a group of discrete entities characterized by continuous variables, is a four continuous variables defined as follows (Demongodin 2009).

Definition 5 *A* controllable batch $C\beta_r(\tau)$ *at time* τ*, is defined by a quadruple,* $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ *where* $l_r(\tau) \in \mathbb{R}_{\geq 0}$ *is the* length*,* $d_r(\tau) \in \mathbb{R}_{\geq 0}$ *is the* density*,* $x_r(\tau) \in \mathbb{R}_{\geq 0}$ *is the* head position *and* $v_r(\tau) \in \mathbb{R}_{\geq 0}$ *is the* speed*. An* instantaneous batch flow *of* $C\beta_r(\tau)$ *is defined by:* $\varphi_r(\tau) = v_r(\tau).d_r(\tau)$. ■

Definition 6 *The* marking of a TB-place *at time* τ *is a series of controllable batches. If* $p_i \in P^{TB}$ *then* $m_i = \{C\beta_h, \cdots, C\beta_r\}$.

Definition 7 *A controllable batch* $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ *of TB-place* p_i *with* $\gamma(p_i) = (V_i, d_i^{max}, S_i, \Phi_i^{max})$*, where its head position equals to the length of* p_i*, i.e.,* $x_r(\tau) = S_i$*, is called an* output controllable batch*, denoted* $OC\beta_r(\tau)$*. The* output density*,* $d_i^{out}(\tau)$*, of a TB-place is defined as follows. If at time* τ*, TB-place* p_i *has an output controllable batch* $OC\beta_r(\tau)$*, then* $d_i^{out}(\tau) = d_r(\tau)$*, else* $d_i^{out}(\tau) = 0$. ■

All controllable batches composing the marking of a TB-place must respect the triangular flow-density relation (see eq.3). This condition allows us to define states of controllable batches.

Definition 8 *(States of batches) Let* $C\beta_r(\tau) = (l_r(\tau), d_r(\tau), x_r(\tau), v_r(\tau))$ *be a controllable batch of TB-place* p_i*, with* $v_i(\tau)$ *variable speed and* V_i *maximum speed of* p_i *(*$v_i(\tau) \leq V_i$*).*

- $C\beta_r$ *is in a* free state *if its density is lower or equal to the critical density of* p_i*:* $d_r(\tau) \leq d_i^{cri}(\tau)$*;*
- $C\beta_r$ *is in a* congested state *if its density is greater to the critical density of* p_i*:* $d_r(\tau) > d_i^{cri}(\tau)$. ■

More details concerning the continuous-time dynamics of controllable batches inside a TB-place are explained in (Gaddouri, Brenner, and Demongodin 2016).

3. SIMULATION

The simulation of a CTBPN model is based on a discrete event approach with linear or constant continuous evolutions between timed events. Between two timed events, the state of the model has an invariant behavior state (*IB-state*). The state of the system is calculated only when it undergoes discontinuity. Three kind of events change the *IB-state*.

- Internal events (timed events inside a TB-place)
  - i.1 a batch becomes an output batch $C\beta_r = OC\beta_r$;
  - i.2 two batches meet;
  - i.3 an output batch is destroyed $OC\beta_r = \mathbf{0}$.
- External events
  - e.1 a discrete transition is fired: t_j;
  - e.2 a continuous place becomes empty: $m_i^n = 0$;
  - e.3 a discrete transition becomes enabled $m_i^n = a$;
  - e.4 a batch becomes an output batch (i.e. event i.1 above);
  - e.5 an output batch is destroyed (i.e. event i.3 above);
- Controlled events
  - c.1 the flow of a batch transitions is modified: $\phi_j(\tau) = \phi'_j(\tau)$;
  - c.2 the speed of a BB-place is modified: $v_i(\tau) = v'_i(\tau)$.

The dynamic evolution of the model is computed by the algorithm in Figure 3.

4. CASE STUDY: HIGHWAY

We consider in our case study the highway section between Marseille and Aix-en-Provence of the A51 highway in France.

4.1. Physical Model

The highway section studied has $1.207km$ long. The section start at the kilometer 2.3 and end at 3.507. The section has no input or output ramps, maximum speed is limited to $90km/h$ and only one direction traffic from Aix-en-Provence to Marseille will be modeled here.

Figure 4 represent this road section.

The data are collected by inductive loops at three points: at the begin of the section (B1), $0.9km$ away (B2) and at the end of the section (B3) for $1:36$ hours starting October, 15, 2015 at $15:58$. The inductive loops in each points send the flow (vehicles/hours), density, average speed and the state (congested or free) each six minutes.

The initial state of the section is a congestion between the point B1 and B2 and free traffic between B2 and B3.

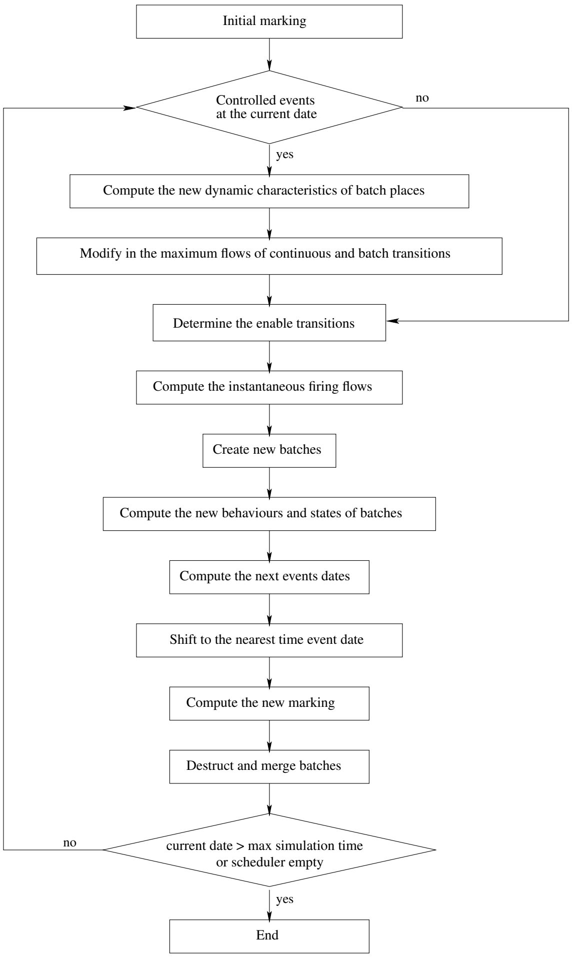

Figure 3: Dynamics of a triangular Batches Petri Nets

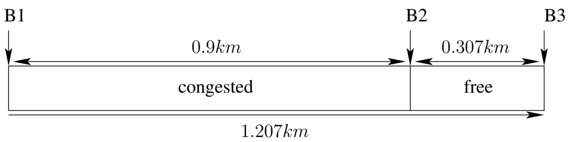

Figure 4: Road section physical model

4.2. CTBPN Model

The CTBPN model (Figure 5f this highway section is composed of two places : continuous place p_1 used to model the section capacity and triangular batch place p_2 to model the section. The characteristics of the place p_2 are computed based on observed flow, speed and number of ways of the road section.

The critical density of this road section is $\frac{4450}{90} = 49.44$ and we can define the initial marking of the place p_2 with two batches. The first batch $OC\beta_1 = (0.307, 34.33, 1.207, 90)$ is in free state. The second batch $C\beta_2 = (0.9, 122.58, 0.9, 31)$ is in congested state.

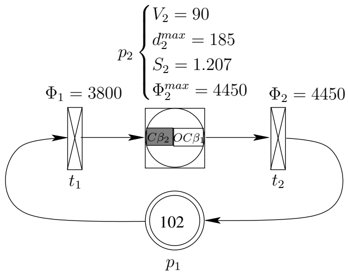

Figure 5: CTBPN model

4.3. Simulation Results

Ours interest here is to compare the output flow measured at the point B3 and the simulation results (flow at the transition t_2). All long the simulation, the variations of the input flow (measured by B1) is applied also in our model. We use controlled events to increase and decrease the maximum flow of the transition t_1 .

The simulation results are presented in Figure 6.

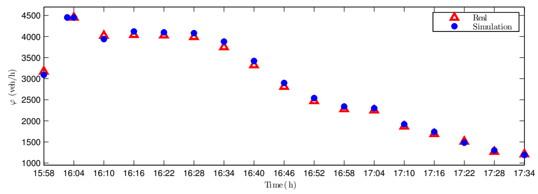

Figure 6: Output flow in decongestion behaviour

We can see at the beginning of the simulation an important increase of the output flow. This flow increasing (from 3090 to 4450 veh/h) is due the decongestion of the section between the points B1 and B2. We can also determine the exact instant when the congestion is finished. This instant is represented by the second round point (simulation) at 16 : 03 hours in the graph.

The maximum error measured between the simulation results and the collected data are 3.20% and can be justified by the no respect of the maximum speed that was observed at the collected data.

5. CONCLUSION

Firstly, we presented the CTBPN formalism that was used to model the highway section. A schematic algorithm based on discrete events approach was shown in section 3.

The main contribution of the paper is the case study of a highway section. We proposed a simple CTBPN model that describe a physical highway section. The parameters of the model used in the simulation were extracted from a set of collected data.

The comparison between the simulation results and collected data show a good accuracy between our model and behaviour observed in the traffic road.

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