# OPTIMAL STOCHASTIC CONTROL OF AN ALUMINIUM RECYCLING UNIT IN REVERSE LOGISTICS

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**ABSTRACT**
This paper deals with the problem of control of an aluminum recycling unit in reverse logistics. The unit studied is in open loop. It's assimilated to a machine producing a single type of products and is subject to random failures and repairs, fueled by a random return rate of aluminum and supplying a constant customer demand rate. Due to the presence of random breakdowns of the machine and the constraints related to customer satisfaction, it is imperative to determine an optimal policy of production in order to insure the customer demand satisfaction. The objective of this study is to determine the production and disposal policies in order to minimize the overall production cost. A corresponding optimal stochastic model has been developed and leads to Hamilton-Jacobi-Bellman equations describing the optimality conditions. A numerical solving method has been used and led to an optimal policy which is of the Hedging Point Policy type (HPP).

**Keywords**: Reverse logistics; Stochastic processes; Dynamic programming; Supply chain management: Numerical methods.

## 1. INTRODUCTION

This paper addresses the problem of optimal control in reverse logistics system in open loop with random return of end of life aluminum products. Previously, consumers were concerned only with quantity, quality and price of products consumed. The conventional product cycle ranged from production sites to landfills (disposal). However, mentalities have changed today, and they are increasingly concerned about the preservation of the environment and the possibility of recycling products made available to them, as mentioned by Thomas et Wirtz (1994). The authors showed the advantages and the importance of the recycling of aluminum against their complete destruction because it allows recovering their potential wealth. Many authors have shown that it's better to recapture the value of end of life products than dispose them (Rogers and Tibben-Lembke, 1998). The need for recycling of aluminum doesn't only increase for economic, environmental and legislative reasons, but also for social reasons. The optimal management of the logistics network and their activities is very complex in reverse logistics due to the wide variety of decisions of different scopes, disturbance factors and attention to the desired time horizon.

One of the challenges facing manufacturing companies is achieving production targets that meet customer demand. Although it is often possible to predict the state of machines (operational or failed) over a short time horizon with a degree of confidence, it is difficult to predict their behavior over a long time horizon given that they are generally prone to breakdowns and repairs. The need for optimal production planning tools to deal with these hazards has prompted and several authors addressed the corresponding issues.

Stochastic dynamic programming method has been applied for many systems under different initial considerations. Indeed, the joint optimization of production and preventive maintenance for a non-homogeneous Markov process was treated by Gharbi and Kenne (2000). The extension of production policy optimization to larger systems subjected to random phenomena such as machine failures and repairs have also been studied by Gharbi and Kenne (2003). Out of the control of machine failure and repair, Kenne (2004) integrated tool wear and rejection rates in order to determine the optimal production policies and preventive replacement of tools to minimize the incurred costs (storage, repairs and replacements). Nodem et al. (2011) developed a policy of production, repair and preventive maintenance of a manufacturing system subject to random failures and repairs; They also obtained a suboptimal policy of critical threshold type.

In order to optimize production for a closed loop hybrid system, Kenné et al. (2012) studied a production unit consisting of two machines in parallel producing a single type of products. They determined the optimal production policies of each machine in order to supplement customer demand while minimizing the incurred costs (inventory and shortage).

The rest of this paper is organized as follows: in section 2, the problem statement is presented. In section 3, the formulation of the control problem and the numerical approach used to solve it are presented. In section 4, the results of a numerical case are presented. In section 5, the conclusion of the paper is presented.

## 2. PROBLEM STATEMENT

This section presents the problem statement of the stochastic optimal control problem under study. The recycling unit under our study consists of a single machine producing a single product so as to supply a constant demand rate of aluminum ingot customer.

End of life aluminum products in the form of bales arrive at the recycling unit at variable rates $r_1$ and $r_2$ as shown in figure 1 and then stored in the warehouse so as to constitute stock $x_1$ of material necessary for the production. The return rate is described by the stochastic process $\alpha_1(t)$. Sometimes, the storage costs of these collected products become very high or the amount of material collected reaches the maximum storage capacity of the recycling unit denoted by x1max. In these cases, returned products collected are disposed from the warehouse at a variable and unknown rate denoted $u_{d_\alpha}$.

The manufacturing machine is supplied with raw material of the stock $x_1$ so as to build stock $x_2$ of finished products (aluminum ingots). It produces with an unknown variable production rate $u_\alpha$ in order to meet the constant demand d of the customers. However, the machine (M) is subject to random breakdowns and repairs. The availability of the machine is described by the stochastic process $\alpha_2$ (t). The decision variables are respectively the production rate uα and also the elimination rate $u_{d_\alpha}$ while state variables are respectively the inventory level $x_1$ of returned products and the inventory level $x_2$ of the finished products.

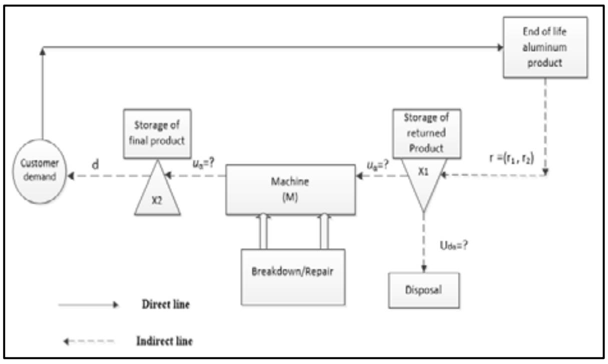

Figure 1: Structure of the recycling unit fueled by a variable return rate

The main assumptions that support our model are: (1). The process is a homogeneous Markov chain process; (2). The machine is prone to random breakdowns and repairs described by known constant rates; (3). The customer demand rate is constant and known; (4). The maximum production rate of the system is known and constant; (5). Different return rates are known and also transition rate between them; (6). When the machine breaks down, a corrective maintenance activity is immediately implemented.

## 3. PROBLEM FORMULATIONS

Considering a discrete-state stochastic process $\{\alpha_1(t), t \geq 0\}$ which describes the return rate of end of life products at each time t taking values $\{1, 2\}$ such that: $\alpha_1(t) = 1$ when $r = r_1$ and $\alpha_1(t) = 2$ when $r = r_2$.

Considering a discrete-state stochastic process $\{\alpha_2(t), t \geq 0\}$ which describes the availability of the machine at each time t taking values $\{0, 1\}$ such that: $\alpha_2(t) = 1$ when the machine is available and $\alpha_2(t) = 0$ when the machine is broken down.

Finally, $\{\alpha(t) = \alpha_1(t) \times \alpha_2(t),\ t \geq 0\}$ is the discrete-stochastic state process that describes the whole state of the system at each time with value in B = {1, 2, 3, 4} depending on whether the machine is available or not and supplied at a certain return rate as resume in the table 1 below.

Table 1 Dynamic of the system

| | | | | |
|---|---|---|---|---|
| $\alpha_1(t)$ | 1 | 1 | 0 | 0 |
| $\alpha_2(t)$ | 1 | 2 | 1 | 2 |
| $\alpha(t)$ | 1 | 2 | 3 | 4 |

The state of the reverse logistic unit is modeled by a discrete-state continuous-time Markov chain with an ergotic $4 \times 4$ matrix of transition rates $Q = [\lambda_{ij}]$. The relationship between the transition rate $\lambda_{ij}$ and transition probability from mode i to mode j is given by equation (1).

$$\Pr[\alpha(t+\delta t) = j / \alpha(t) = i] = \begin{cases} \lambda_{ij}\delta t + o(\delta t) & \text{if } i \neq j \\ 1 + \lambda_{ij}\delta t + o(\delta t) & \text{if } i = j \end{cases} \quad i, j \in B = \{1, 2, 3, 4\} \tag{1}$$

With $\lambda_{ij} \geq 0$ for all $i \neq j$, $\lambda_{ii} = -\sum_{i \neq j} \lambda_{ij}$ and $\lim_{t \to 0} \frac{o(\delta t)}{t} = 0$

Different possible transitions among the state of the system are given by fig. 2.

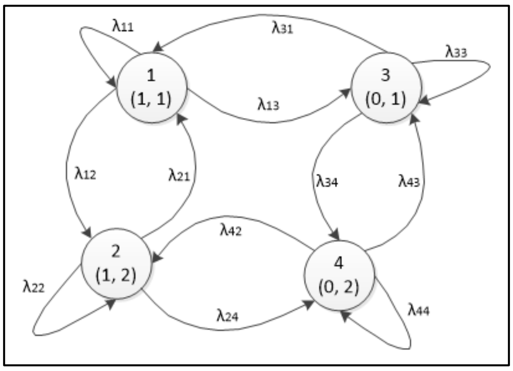

Figure 2: State transition diagram

Let $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ be the vector of limiting probabilities at each mode α (α =1,..,4) of the system. Those limiting probabilities $\pi_i$ (i=1, 2, 3, 4) are the solution of the equation (2).

$$\begin{cases} \sum_{i=1}^{4} \pi_i \times \lambda_{ij} = 0 \quad \textit{for all } j = 1,..,4 \\ \sum_{i=1}^{4} \pi_i = 1 \end{cases} \tag{2}$$

The feasible control policies are given by the set $\Gamma(\alpha)$ define as follow:

$$\Gamma(\alpha) = \begin{Bmatrix} (u_\alpha,\ u_{d_\alpha}) \in R^2 / \\ 0 \leq u_\alpha \leq u_{\max},\ 0 \leq u_{d_\alpha} \leq r_\alpha,\ \alpha = 1,..,4 \end{Bmatrix}$$

For example, $\pi_2$ represent the probability that the machine is available and fueled at the returned rate $r_2$. Considering the fact that in our study, returned rate $r_2$ is lower than the maximal production rate $u_{max}$, the system is feasible only if the equation (3) is verified so as:

$$(\pi_1 \times u_{\max} + \pi_2 \times r_2) \geq d \quad (3)$$

The variations of different stocks are described by equations (4). These equations take in account the fact that, at each time t, in the given state of the system, the inventory level of the storage x1 is increased by the return rate $r_\alpha$ (α = 1,..,4) while the same stock is decreased by the rate of elimination $u_{d_\alpha}$ and also by the production rate of the machine $u_\alpha$. Similarly, the stock level of finished products is increased by the production rate $u_\alpha$ and decreased by the demand rate d.

$$\begin{cases} \dfrac{dx_1(t)}{dt} = r_\alpha - u_{d_\alpha} - u_\alpha, & x_1(0) = x_{10} \\ \dfrac{dx_2(t)}{dt} = u_\alpha - d, & x_2(0) = x_{20} \end{cases} \quad (4)$$

where $x_{10}$ and $x_{20}$ are respectively the initial stock of returned and final products.

The running cost of our model depends on the storage costs of the returned products c1, the storage cost of finished products c2p, the shortage cost of finished products c2m, the disposal cost of excess returned product cd, the manufacturing cost cm, the corrective maintenance cost cα, the environmental cost $c_{env}$ and the cost penalizing the lack of raw materials during shortages of finished products. Its expression is given by equation (5).

$$\begin{aligned} g(x_1,x_2,\alpha) &= c_1 x_1 + (c_{2p} x_2^+ + c_{2m} x_2^-) \\ &+ c_d u_{d_\alpha} + c_m u_\alpha + c^\alpha (ind\{\alpha(t)=3\} + ind\{\alpha(t)=4\}) \\ &+ c_{env}(x_{1\max} - x_1) + c_{ecart} \left|(|x_2| - x_1)\right| ind\{x_2<0\} \end{aligned} \quad (5)$$

where $x_2^+ = \max(0, x_2)$ ; $x_2^- = \max(-x_2, 0)$ and

$$ind\{\Theta(.)\} = \begin{cases} 1, & if \;\; \Theta(.) \;\; is\ true \\ 0, & otherwise \end{cases}$$

By taking in account the discount rate ρ (Garceau, 1996), the discounted total cost is given by the relation (6):

$$J(x_1,x_2,\alpha) = \mathrm{E}\left\{ \int_0^\infty e^{-\rho t} g(x_1,x_2,\alpha)\mathrm{d}t \,\middle|\, x(0)=x, \alpha(0)=\alpha \right\} \quad (6)$$

In this equation, E(A/B) symbolizes the conditional expectation operator and $x=(x_1, x_2)$ is the vector of stock levels.

Our objective is to obtain the optimal control policies that will minimize the discounted cost (6). In another word, the objective is to find the production rate $u_\alpha$ and the disposal rate $u_{d_\alpha}$ so as to minimize the expected discounted cost given by (6). The value function of such a problem is defined by:

$$v(x_1,x_2,\alpha) = \inf_{(u_\alpha,\ u_{d\alpha}) \in \Gamma(\alpha)} J(x_1,x_2,\alpha), \quad \forall \alpha \in \mathrm{B} \quad (7)$$

The value function (7) satisfies the Hamilton–Jacobi–Bellman (HJB) equations (8) which describes the optimality conditions and can be found in chapter 8 of Gershwin (1994). In the work of Rivera-Gómez et al. (2016), they showed that the value function is continuously differentiable and viscosity solution to the Hamilton-Jacobi Bellman (HJB) equations. Such HJB equations, which integrate the dynamics of the stock as well as the machine modes, are given by:

$$\rho v(x_1,x_2,\alpha) = \min_{(u_\alpha, u_{d_\alpha}) \in \Gamma(\alpha)} \begin{bmatrix} g(x_1,x_2,\alpha) + \\ \sum_{j\in B} \lambda_{\alpha j} v(x_1,x_2,j) + (r_\alpha - u_{d_\alpha} - u_\alpha) \dfrac{\partial v(x_1,x_2,\alpha)}{\partial x_1} \\ +(u_\alpha - d) \dfrac{\partial v(x_1,x_2,\alpha)}{\partial x_2} \end{bmatrix} \quad (8)$$

The solution of equation (8) has been obtained by Akella et Kumar (1986) but, for the simplest single machine under certain assumptions which are not valid for our model. Since an analytical solving method does not exist for the HJB equations, we adopted a numerical methods methodology to obtain the structure of the optimal control policies.

Numerical approach based on Kushner et Dupuis (1992) method is used to obtain a sub-optimal solution which is a good approximation of value function at a predetermined precision degree δ (in our study, we fixed δ=0.01).

That numerical method consists in simplifying the HJB equations by approximating the value function $v(x_1,x_2,\alpha)$ by the function $v^h(x_1,x_2,\alpha)$ and the approximation of the gradient of the value function $\partial v(x_1,x_2,\alpha)/\partial x_i$ (i = 1,2) respectively by the following expressions.

$$\frac{\partial v(x_1,x_2,\alpha)}{\partial x_1} = \begin{cases} \dfrac{v^{h_1}(x_1+h_1,x_2,\alpha) - v^{h_1}(x_1,x_2,\alpha)}{h_1}, & if \;\; r_\alpha - u_{d_\alpha} - u_\alpha \geq 0 \\ \dfrac{v^{h_1}(x_1,x_2,\alpha) - v^{h_1}(x_1-h_1,x_2,\alpha)}{h_1}, & otherwise \end{cases}$$

$$\frac{\partial v(x_1,x_2,\alpha)}{\partial x_2} = \begin{cases} \dfrac{v^{h_2}(x_1,x_2+h_2,\alpha) - v^{h_2}(x_1,x_2,\alpha)}{h_2}, & if \; u_\alpha - d \geq 0 \\ \dfrac{v^{h2}(x_1,x_2,\alpha) - v^{h_2}(x_1,x_2-h_2,\alpha)}{h_2}, & otherwise \end{cases}$$

where hi (i = 1, 2) is the discretization step of the state variable. Considering such approximations, equations (8) becomes equations (9):

$$\rho v^h(x_1,x_2,\alpha) = \min_{(u_\alpha,u_{d_\alpha})\in\Gamma(\alpha)} \begin{bmatrix} g(x_1,x_2,\alpha) + \lambda_{\alpha\alpha} v^h(x_1,x_2,\alpha) + \sum_{j\neq\alpha} \lambda_{\alpha j} v^h(x_1,x_2,j) \\ + \dfrac{|r_\alpha - u_{d_\alpha} - u_\alpha|}{h_1} v^{h_1}(x_1+h_1,x_2,\alpha) ind\{r_\alpha - u_{d_\alpha} - u_\alpha \geq 0\} \\ + \dfrac{|r_\alpha - u_{d_\alpha} - u_\alpha|}{h_1} v^{h_1}(x_1-h_1,x_2,\alpha) ind\{r_\alpha - u_{d_\alpha} - u_\alpha < 0\} \\ + \dfrac{|u_\alpha - d|}{h_2} v^{h_2}(x_1,x_2+h_2,\alpha) ind\{u_\alpha - d \geq 0\} \\ + \dfrac{|u_\alpha - d|}{h_2} v^{h_2}(x_1,x_2-h_2,\alpha) ind\{u_\alpha - d < 0\} \\ - \dfrac{|r_\alpha - u_{d_\alpha} - u_\alpha|}{h_1} v^{h_1}(x_1,x_2,\alpha) - \dfrac{|u_\alpha - d|}{h_2} v^{h_2}(x_1,x_2,\alpha) \end{bmatrix} \quad (9)$$

After several manipulations as appeared in the works of Kenne et al. (2003), equation (9) becomes:

$$v^h(x_1,x_2,\alpha)=\min_{(u_\alpha,u_{d_\alpha})\in\Gamma(\alpha)}\left[\begin{array}{l}(\rho+|\lambda_{\alpha\alpha}|+\frac{|r_\alpha-u_{d_\alpha}-u_\alpha|}{h_1}+\frac{|u_\alpha-d|}{h_2})^{-1}\times\\ \left\{\begin{array}{l}c_1x_1+c_{2p}x_2^{+}+c_{2m}x_2^{-}+c_du_d+c_mu_\alpha+c^\alpha(ind\{\xi(t)=3\}+ind\{\xi(t)=4\})\\ +c_{env}(x_{1max}-x_1)+c_{ecart}|(|x_2|-x_1)|ind\{x_2<0\}+\sum_{j\neq\alpha}\lambda_{\alpha j}v^h(x_1,x_2,j)\\ +\frac{|r_\alpha-u_{d_\alpha}-u_\alpha|}{h_1}v^{h_1}(x_1+h_1,x_2,\alpha)ind\{r_\alpha-u_{d_\alpha}-u_\alpha\geq 0\}\\ +\frac{|r_\alpha-u_{d_\alpha}-u_\alpha|}{h_1}v^{h_1}(x_1-h_1,x_2,\alpha)ind\{r_\alpha-u_{d_\alpha}-u_\alpha<0\}\\ +\frac{|u_\alpha-d|}{h_2}v^{h_2}(x_1,x_2+h_2,\alpha)ind\{u_\alpha-d\geq 0\}\\ +\frac{|u_\alpha-d|}{h_2}v^{h_2}(x_1,x_2-h_2,\alpha)ind\{u_\alpha-d<0\}\end{array}\right\}\end{array}\right] \quad (10)$$

## 4. NUMERICAL ANALYSIS AND CONTROL POLICIES

In this section, to solve numerically the HJB equations in order to determine the optimal control policy of the recycling unit, we are going to use algorithm of Yan et Zhang (1997) and also matlab software. Furthermore, discretization step used is $h_{x1} = h_{x2} = 0.5$. A finite grid D defined below is necessary to circumscribe the domain for the state variables.

$$D=\{0\leq x_1\leq 20;\ -10\leq x_2\leq 20\} \qquad (11)$$

The set of parameters needed during the simulation are presented in the table 2 below.

Table 2: Parameters of the numerical example

| Variables | $c_m$ | $c_d$ | $c_1$ | $c_{2p}$ | $c_{2m}$ | $c^\alpha$ | $c_{ecart}$ |
|---|---|---|---|---|---|---|---|
| Value | 14 | 1.5 | 1.8 | 3.5 | 55 | 13 | 55 |
| Units | $/product/UT | $/product/UT | $/product/UT | $/product/UT | $/product/UT | $/break/UT | $/product/UT |

| $C_{env}$ | ρ | $r_1$ | $r_2$ | d | $u_{max}$ |
|---|---|---|---|---|---|
| 1 | 0.1 | 0.55 | 0.45 | 0.4 | 0.5 |
| $/product/UT | /UT | produit/UT | produit/UT | produit/UT | produit/UT |

After running the simulation program, we have obtained the structure of optimal production and disposal policies, which are both, hedging point policy type. In all the following figures, $x_1$ represents the stock of returned product while $x_2$ represents the stock of finished products.

During mode 1 (machine available, return rate of end of life products is equal to $r_1$), the optimal production policy shown in fig.4 stipulates to produce at the maximum rate $u_{max}$ when the inventory level of final products is strictly below the critical threshold $\psi(x_1)$ which is not constant, but depends also on the inventory level of returned product.

It stipulates to set the production rate at demand rate d when the inventory level of final products is equal to the critical threshold $\psi(x_1)$

Finally, it is needed to set the production rate at zero rate (no production) if the inventory level of final products is bigger than the threshold $\psi(x_1)$. That policy is summarized by equation (12).

$$u_1(x_1,x_2,1)=\begin{cases}u_{max} & if\ \ x_2<\psi(x_1)\\ d & if\ \ x_2=\psi(x_1)\\ 0 & if\ \ x_2>\psi(x_1)\end{cases} \qquad (12)$$

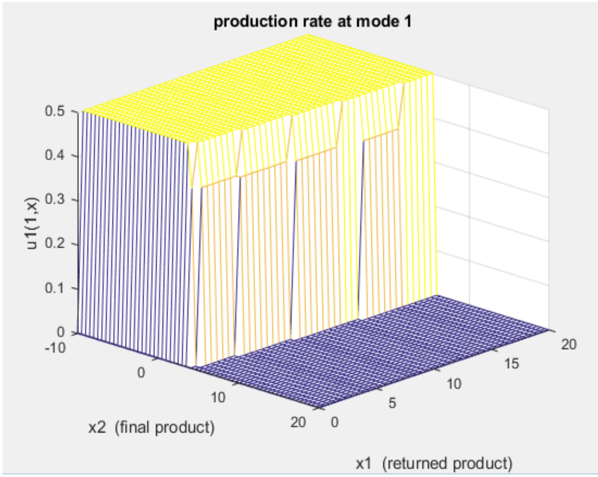


Figure 3 Production rate at mode 1

At the same mode 1, the optimal disposal policy $u_{d_1}$ shown in fig.5 stipulates to dispose the returned products at the maximum rate r1 when the returned products level is strictly higher than the critical threshold $z_{12}=\sigma(x_2)$, which is a function that gives the threshold depending on inventory level of final products $x_2$. It also stipulates to set the disposal rate at the rate r1-umax when the inventory level of returned products is equal to the critical threshold $z_{12}=\sigma(x_2)$.

Finally, it is needed to set the disposal rate at zero rates if the inventory level of returned product is lower than the threshold $z_{12}=\sigma(x_2)$; in other words to store the entire returned products that conveyed to the recycling unit. This disposal policy is summarized by equation (13).

$$u_{d_1}(x_1,x_2,1)=\begin{cases}r_1 & if\ \ x_1>\sigma(x_2)\\ r_1-u_{max} & if\ \ x_1=\sigma(x_2)\\ 0 & if\ \ x_1<\sigma(x_2)\end{cases} \qquad (13)$$


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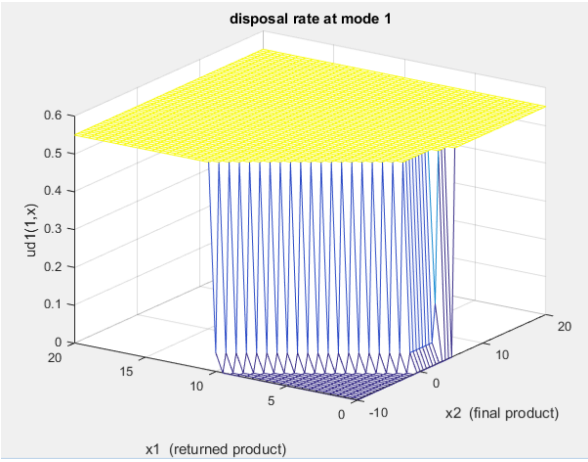

Figure 4 Disposal rate at mode 1

At the mode 2 of the unit (machine failed, return rate equal to $r_2$), the optimal production policy shows in fig.6 stipulate the following rule for the machine:

- Set the machine rate to $r_2$ when the stock level of final products x2 is under the threshold $z_{21} = \varpi(x_1)$, where function $\varpi(x_1)$ is a function depending on level of returned products $x_1$.
- Set the machine rate to the demand rate d when the level stock $x_2$ is equal to the threshold $z_{21} = \varpi(x_1)$.
- Stop the production of the machine when the stock level $x_2$ is greater than the threshold $z_{21} = \varpi(x_1)$. The production policy at state 2 is summarized by equation (14).

$$u(x_1, x_2, 2) = \begin{cases} r_2 & if \quad x_2 < \varpi(x_1) \\ d & if \quad x_2 = \varpi(x_1) \\ 0 & if \quad x_2 > \varpi(x_1) \end{cases} \qquad (14)$$

In the same way, disposal policy at mode 2 is shown in fig.7 and is summarized by equation (15). In this equation, $z_{22} = \delta(x_2)$ is the threshold for the disposal rate depending on the inventory of final products $x_2$.

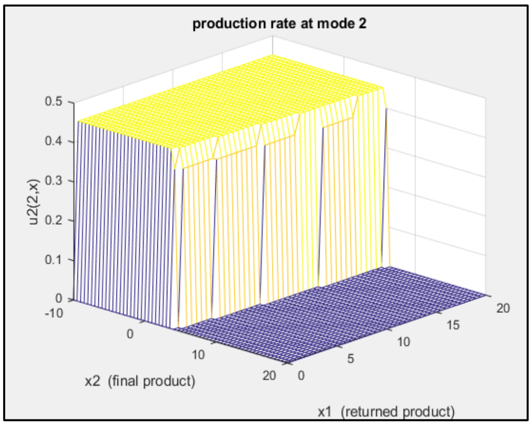

Figure 5 Production rate at mode 2

$$u_{d_2}(x_1, x_2, 2) = \begin{cases} r_2 & if \quad x_1 \geq \delta(x_2) \\ 0 & if \quad x_1 < \delta(x_2) \end{cases} \qquad (15)$$

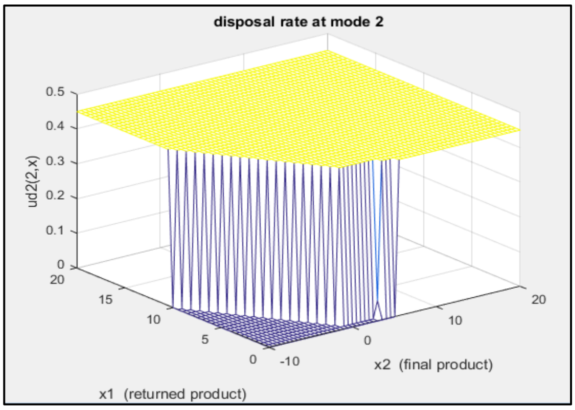

Figure 6 Disposal rate at mode 2

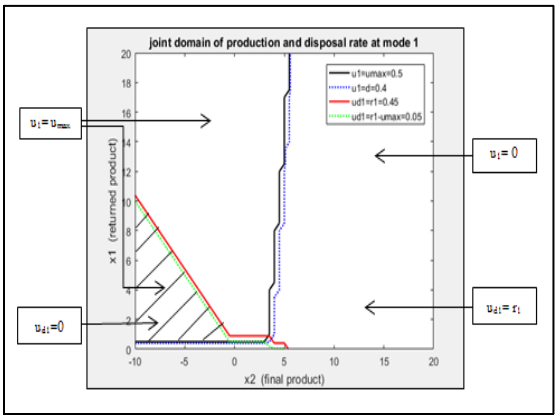

Figure 7: Feasible domain at mode 1

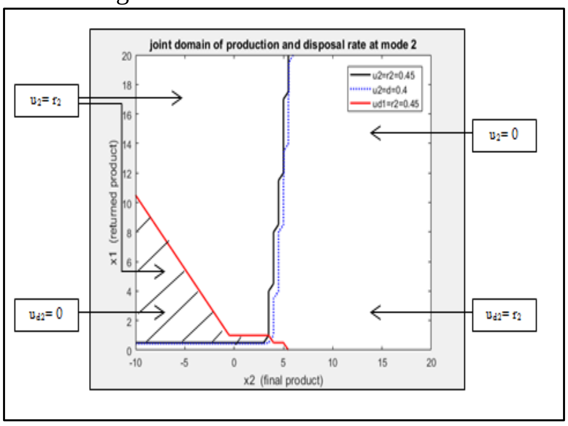

Figure 8: Feasible domain at mode 2

During the modes 3 and 4, the machine is broken, there is no possible production, and hence disposal policies are summarized respectively by equation 16 and 17 and can be interpreted as previously. In those equations, $z_3 = \eta(x_2)$ and $z_4 = \iota(x_2)$ represent respectively the function that give the threshold of disposal policy at mode 3 and 4 depending on the level of the final products storage.

$$u_{d_3}(x_1, x_2, 3) = \begin{cases} r_1 & if \quad x_1 \geq \eta(x_2) \\ 0 & if \quad x_1 < \eta(x_2) \end{cases} \qquad (16)$$

$$u_{d_4}(x_1, x_2, 4) = \begin{cases} r_2 & if \quad x_1 \geq \iota(x_2) \\ 0 & if \quad x_1 < \iota(x_2) \end{cases} \qquad (17)$$


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The hedging point policy structure obtained in this study is an extension of woks of Akella et Kumar (1986). Although they have obtained a fixed threshold, we obtained variable thresholds for the control policies obtained in this paper.

Having obtaining a disposal and a production policies simultaneously at the operational modes 1 and 2, we legitimately want to know where the raw material required for the production at those modes would come from since at the same modes, returned product is disposed ? To answer this question, we have studied the joint domain of the two policies (disposal and production rates) illustrated by fig. 8 and 9 (hatched area), which clearly shows the existence of a joint domain for production at a non-zero rate ($u_1 = u_{max}$, $u_2 = r_2$) and the elimination of returns at the zero rate ( $u_{d_1} = 0,\ u_{d_2} = 0$ ). This intersection corresponds to the feasibility domain, meaning the field where the machine will operate while being sufficiently fueled with material.

## 5. CONCLUSION

Attending to the end of our study, we can conclude that, the objectives have been achieved. We have considered the problem of control of production and disposal policies for an aluminum recycling unit in open loop. The unit considered was considered as a single machine producing a single type of product and subject to stochastic random breakdowns and repairs. After modeling the system using a homogeneous Markov chain, we have determined the optimality conditions through the HJB equations. We were able to solve these equations numerically by using the Kushner and Dupuis (1992) method and determined the optimal controls. We have obtained the optimal policies which are of critical type (Hedging Point Policy) at each mode of the machine (running or broken). In our study, we showed that, the optimal critical threshold of finished products depends on the inventory level of returned products which is quite realistic.

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