SIMULATION OF AN EVOLUTIONARY GAME FOR A WEALTH DISTRIBUTION MODEL STRUCTURED IN A SMALL WORLD NETWORK

Javier Lara de Paz[(a)], Gabriel Policroniades[(b)], Idalia Flores[(c)]
[(a)]EngineeringSchool
Universidad Nacional Autónoma de México
[(a)]javier_lara_depaz@comunidad.unam.mx
[(b)] Engineering School
Universidad Nacional Autónoma de México
gpolicroniadesch@gmail.com
[(c)] Engineering School
Universidad Nacional Autónoma de México
idalia@unam.mx

ABSTRACT
Wealth distribution studies have been reported for almost 200 years using different models to explain the dynamics involved. Also, many kinds of approaches have arisen to fit the registered data. Pareto´s distribution emerged as one of the best empirical model showing a good fitting with real data sets of wealth distribution all over the world and over different time ages. Theoretical models validate their assessments through this distribution. Souma asserted that wealth distribution interaction between agents could be modeled in a small-world network with different rules of wealth exchange. Garlaschelli, found that long-term shape of the empirical distribution strongly depends on the topology of the transaction networks among economic units. In the present work, an evolutionary game theory method was used to establish wealth exchange between economical agents structured in a small world network. The present project constructs a model based on México´s population income data sets dividing it in ten equal sized groups with different range of income stated as agents in the model. Agent based model simulations were performed using NetlLogo software, through different time intervals. Goodness of fit techniques was used to verify simulation results approaches to Pareto and log normal distribution.

Keywords: Complex Networks, Wealth Distribution,Econophysics, Evolutionary Game Theory,Small-World Networks, Pareto´s Law.

1. INTRODUCTION

1.1. Pareto´Law

Wealth distribution has been a controversial affair, that is of general interest. And it is about how the reality affects to everyone who can distinguish emerging economical inequalities. Is a fact which not only concerns to politicians or social science researchers but to many other different knowledge areas and deserves a certain systematic and methodical research. For many years, social sciences have shown more interest on this topic, but the methods employed tried to describe observed inequalities due to wealth distribution, showed inaccurate results unable to estimate different scenarios.

Wealth distribution formal studies (Piketty, 2014) date from about 18th century endings with the famous economist and philosopher Adam Smith, who in 1776 published his outstanding work *"An inquiry into the nature and causes of Wealth of Nations"* in which Smith established the first relations between wealth and means of production. There were also other researchers, who were interested in the behave of wealth among nations and societies. as for example Thomas Malthus, who in 1798 published his essay "Principle of Population" using the little information available, in which he postulated his famous exponential population grow law. Malthus research was based on Arthur's Young research published in 1792 about observations documented of France country life between 1787 -1788, before France Revolution. Another remarkable researcher of the XIX century was David Ricardo (1772-1823). In 1817 published his "Principles of Political Economy and Taxation", which replaced the famous Adam Smith´s "The Wealth of Nations" as a mandatory textbook in economic issues. The study of wealth distribution and economic disparities, were also of interest of Karl Marx and Friedrich Engels, but they focused mainly on how to structure the mentioned means of production in order to close wealth differences gap more than in a wealth distribution data study. At the end of the 19th century the Italian economist Vilfredo Pareto first pointed out a common feature among different wealth and income distributions studies, that emerge even if quantitative difference exists across the data obtained from various economies. Pareto´s investigation was based among household wealth and personal incomes modeling with statistical methods using a power-law distribution and even for the last hundred years data observations, this law seems to be a natural law, irrespective of many differences in culture, history, language, regime and the economic policies followed in different countries, the income distribution is found to follow a universal pattern in the upper tail a power law for the richest 1 - 5% of the population0 (Gaffeo et al., 2008). For the rest of the 95% of the population, wealth distribution fits to a conspicuous log normal decreasing distribution or an exponential fitting is used as well. The mentioned Pareto distribution describes by 'Pareto-tails', which decay as a power law for large wealth

$$\mathscr{P}_{>}(W) \sim (\frac{W_0}{W})^{\mu} \quad \ldots (1)$$

Where $\mathscr{P}_{>}(W)$ is the probability to find an agent with wealth greater than W, and μ is a certain exponent, of order 1 both for individual wealth or company sizes. Pareto estimated this parameter $\mu \approx 1.5$.

Today the Pareto law is usually quoted in terms of the probability density function, $\mathscr{P}(W)$,

$$\mathscr{P}(W) \sim W^{-(1+\mu)} \quad \ldots (2)$$

;for large W

Furthermore, Pareto observed that this distribution is universal, i.e., it "varies very little in space and time; different peoples and different eras yield very similar curves; there is a remarkable stability of form in this curve". This main achievement by Pareto could be seen from the perspective of today's economists and econophysists. (Chatterjee, 2009, 2008, 2005). Both approaches distant themselves from the most traditional one of mainstream economics, which does not consider statistical and heterogeneous aspects of the economy. Nowadays with the fast advances in data mining techniques these fields can analyze economic data with more accuracy, such as personal income, wage income, household wealth and so on, which have become increasingly available with the aids of fast computing and the Internet. More recently, statistical approaches have been used to provide insight on the origin of this law, even more, new methods from both economics and econophysics as well as sociology have emerged. By a much common ground with agent-based modeling and simulation, more recently, econophysists have tried to view this universal, two-class structure of income or wealth distribution as a nature law for a statistical many body-dynamical market system, by analogy with gases, liquids or solids in classical or quantum physical system. Both approximations have tried of all explore the exact shape of the real distribution from economic data, and secondly, to design theoretical models capable to reproduce such distributions.

As mentioned, works done in the field of statistical mechanics have bridged both fields, economic with physics, through the applications of Boltzmann-Gibbs function and the latest k-generalized statistical mechanic (Gaffeo, 2009).

Other paradigm applied is the Agent-Based Modeling (ABM) that uses computational modeling of systems as collections of autonomous interacting entities. This approach, differs from the classic mechanic reductionism considerations presented in economical mainstream models established in the latest 18th and 19th centuries, since ABM allows to represent the systems as a complex adaptive system (Tesfatsion, 2003) an idea introduced first by John von Neumann in the late 1940's. Remarkable ABM advantages are that one doesn´t need to restrict arbitrary assumptions, it is possible to ground the rules chosen for agent´s behaviors against reality, it is able to introduce additional complexity, simulations tools are simplifying any further analysis of the system as well. Using an ABM approach, besides the complex system consideration complex networks topologies are applied to structure agent's interactions. The present project focuses on the above-mentioned ABM approach, modeling the system as a set of economic agents exchanging wealth under an exchange algorithm, towards establishing an evolutionary game rule.

Many different works have been developed including the one´s done by Chakrabarti's group (Chatterjee et al., 2009). They consider in their model an element of proportionality|-saving propensity, thus allowing agents save a fraction of their money ¸m_i, whereas the rest of their money balance (1- λ) m_i is available for random exchanges. It was found the Gamma-like distribution for the case of a fixed saving propensity and a power-law tail in the case of a distributed saving propensity. Drăgulescu and Yakovenko (Drâgulescu, 2003) argued that wealth is equal to money m plus the other property that an agent i has. In the simplest model, it considers only one type of property, denoted as stocks s_i with a price p, and the wealth of an agent i is given by

$$w_i = m_i + p * s_i \quad \ldots (3)$$

It is assumed that the total amount of money and exchanged stocks remains constant. One of the most referenced wealth distribution theoretical modeling works corresponds to Bouchaud and Mézard0. They postulated a time evolution of the wealth w_i of an agent I described by a stochastic differential equation, capturing both exchange between individuals and random speculative trading.

$$\frac{dw_i}{dt} = \eta_i(t) w_i + \sum_{j(\neq i)} J_{ij} w_j - \sum_{j(\neq i)} J_{ji} w_i \quad \ldots (4)$$

The component $\eta_i(t) w_i$ is a Gaussian multiplicative process that simulates the investment dynamics, and the last two terms describe the trade interaction network between the agent i and all other agents in the society. J_{ij} is the exchange rate between agent i and agent j.

Besides the above two types of modeling wealth distribution, there were other wealth exchange models for wealth distribution based on complex networks (Iglesias et al., 2003).

The present work focuses mainly in wealth distribution simulation under an ABM approach and the following papers have been analyzed to establish a background based on economic agents interacting on a small world complex network, under an exchange algorithm as part of a development of an evolutionary game decision rules. Previously among others, Iglesias and his team (Iglesias et al., 2003) presented a simplified model for the exploitation of resources by interacting agents, in an economy with small-world properties having a Gaussian distribution of wealth. However they didn´t show any Pareto´s law distribution for the GNP, they found through network´s mean degree and exchange probability variation that the number of rich agents increases, but poor agents wealth decreases. Garlaschelli and Loffredo (Garlaschelli, 2009) studied Bouchard-Mézard wealth distribution model performing numerical simulations to test the model on more complex network topologies showing that a log-normal or a power-law distribution emerges depending on link's density variation, generally present in real systems. Souma, Fujiwara and Aoyama (Souma et al., 2004) 0 constructed a model of wealth distribution, based on an interactive multiplicative stochastic process on static complex networks. Through numerical simulations they showed that a decrease in the number of links discourages equality in wealth distribution, while the rewiring of links in small-world networks encourages it. Based on the previous results we considered that a small-world network topology can show real patterns on economic agent's interaction. Mao Bin Hu and colleagues (Hua et al., 2008) applied the prisoner's dilemma and snowdrift game cooperation model in a multi-agents system to establish wealth distribution in a population, obtaining good fitting in the simulation results with a log-normal adjustment.

1.2. Evolutionary Game Theory
Adami (Adami et al., 2016) mentions that in game theory, the objective is to find an appropriate strategy to resolve arising conflicts, or alternatively to find the optimal sequence of decisions that leads to the highest payoff. This optimal decision technique takes part through a set of individuals who develop a action, which is called "Movement"; this action, is codified within the genes of each individual granting the possibility of inheriting this characteristic. These actions allow the individual to make decisions in the face of certain events; which are stored in a set of equations; and where each of them preserves those genes that presented a better response to those events. In other words, only those genes most capable of solving a problem are preserved. As well, Nishino (Nishino et al., 2017) considered that one of the great advantages of mixing the agent-based simulation with the theory of evolutionary games is that the different agents obtain the ability to realize cooperation between them if they increase the capacity to obtain a higher performance; a question that otherwise would have been eliminated.

1.3. Data Sources
Present work is based in Mexico´s economic wealth distribution data, obtained from the Statistics and Information Bureau, which systematically present data gathered out from populations profile. Information used is about population average income organized in the so-called *deciles*, that means that population was divided into ten equal population sized groups, each one having the same number of people and ranging the corresponding average income, see table 1.1 below (del Castillo Negrete, 2016).

Table1.1 Mexico´s Population monthly average income distribution in ten equal sized groups (deciles). Simulation color codes for each group are represented in the extreme right column.

Population decile group from a total amount of 127 million	Monthly Average Income (in USD)	Percentage of the total national income	Agents color code
I	147.7	0,3	
II	391.9	0,9	
III	634.4	1,5	
IV	850.3	2,1	
V	1089.4	2,7	
VI	1353.6	3,5	
VII	1678.0	4,7	
VIII	2140.7	6,5	
IX	2956.2	10,8	
X	6930.3	67,0	
Total	18172.5	100	

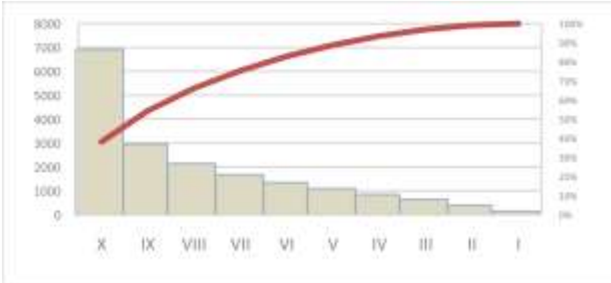

Figure 1.1. Distribution plot for table 1.1. with cumulative tendency line.

2. METHODOLOGY

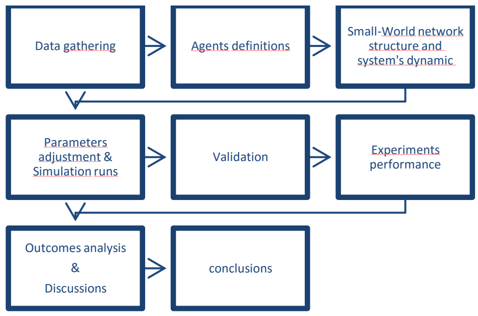

Figure 2.1. Methodology diagram used in the present project. Corresponding sections are described below.

The model is developed using multi-agents based on the income distribution deciles from table 1.1. Agents are set in a small-world network structured in a ring as Strogratz proposed (Watts & Strogratz, 1998). The random network was built upon a topological ring with N vertices and coordination number 2K. Each link connecting a vertex to a neighbor, considering clockwise sense, is then rewired at random, with probability $\boldsymbol{p}$, to any vertex of the system. It is assumed any self-connection and multiple connection as prohibited, obtaining a regular lattice at $\boldsymbol{p} = 0$ and progressively random graphs for $\boldsymbol{p}>0$. The system model was programmed in NetLogo6.0.1. a "rewiring link" button was setup for rewiring any link in every step, as (Watss and Strogratz, 1998). Rewiring a link corresponds to long social contacts made shorter by this rewired link, representing what Milgram established (Milgram, 1967), a Small-World. The complex network represents a connected economic system were nodes are agents exchanging an amount of money in every step (tick). Ten different agent sets have been constructed, corresponding to a monthly wealth income *"decil"*, and were spread out randomly over the network as nodes. For an easier visualization, every *decil* agent set was setup as circles with different colors and radius size proportional to its monthly income (figure 2.2). Nodes radius will increase or decrease whether the node gains or loses money due to exchange process. At start up certain amount of wealth based on official data (see table 1.1) is set up for each agent group, and an interaction rule is established following an exchange algorithm (see equation 6).

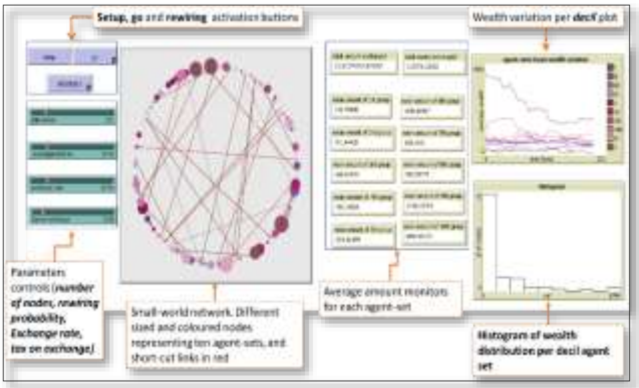

Figure 2.2. Model interface programed in NetLogo 6.1.0. Figure describes elements included.

Once system's topology was structured in a Small-World complex network the go process is constructed as follows, as a Watts & Strogratz Small-World network, system's network has a rewiring probability changing a random number of links wiring them with other different pair of nodes. There are two types of links, active and inactive links, through active links flows exchanged amount of money from a node to recipient, depending on interacting nodes wealth. Number of active or inactive links is randomly defined by "exchange-rate", which corresponds to the opportunity each node has for exchanging money with its corresponding neighbors. This parameter defines the dynamism as in a real economic system. Through every time step, set default as ticks in NetLogo software, a new connection between a pair of nodes is set, new economic interaction starts, based on algorithm equation 6, a node will gain and the other will lose money. Winning node will increase its size proportionally to money gained, as looser decreases its radius. During process running average amount of money each *decil* has, is monitored to determine systems behavior. The process continues with every tick until it is arbitrarily stopped. Data obtained from simulations are collected and analyzed. System's dynamic chart is show below (figure 2.3).

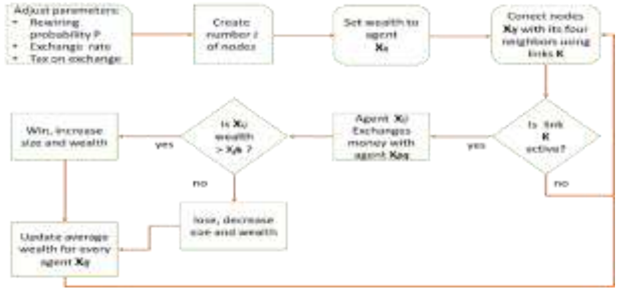

Figure 2.3. System's dynamic process

2.3. Parameters setting

Parameters included represent real economic and social systems behaviors. Rewiring probability, shows heterogeneous agents dynamic in social contact system. In real situation, a contact network is not a full random network nor a regular graph, but intermediate to both. In this system rewiring probability represents long path interactions between two agents theoretically far separated. As Milgram demonstrated, these long-distance relations are possible as we live in a Small-World. For simulation performance, this parameter is set to 30% due to social contacts models perform more realistic around this value (Lara & Flores de la Mota, 2014). "Exchange rate" parameter creates a random number of directed-active links less than or equal to adjusted value connecting agents through directed flow edges enabling an agent financial interaction. It is possible that an agent is connected to its four neighbors, this node will exchange with its connected neighbors winning or losing money with any of them therefore increasing or decreasing its wealth, changing visual agent size as well.

This parameter represents opportunities a person has to exchange money for commodities. If there is a bigger exchange rate, the economic system has a better dynamism. By this project an insight in rate exchange variation scope is being look for and its affectation to wealth distribution. "Tax-on-exchange" parameter establishes tax rate paid in every carried-out transaction. System is not adjusted to reload the initial quantity so that as time steps keep on running agent's money tends to cero owing to tax amount paid. The other parameter present in model is the "number of nodes". By this control, nodes are created considering ten different equal sized *decil* groups.

Figure 2.4. Parameters controls panel

2.4. Money exchange process

If an agent is linked with another by a directed edge exchange action is executed following a trade rule established by an exchange algorithm 0

$$m_i' = m_i - \Delta m$$
$$m_j' = m_j + \Delta m \quad \dots (5)$$

where m_i is starting money each ***agent**$_i$* has at the beginning of period (see table 1.1). Being Δm the transaction amount of money or wealth.

$$\Delta m = profits - (profits * tax) \dots (6)$$

Where profits correspond to exchanged money gained in a transaction between agents *i* and *j* .

2.5. Simulation

To perform simulations, once the network was set up with a certain number of agents and wealth exchange interaction algorithm, NetLogo software is used to program the model, with agents representing nodes and the small-world network topology, wealth exchange between agents flows through diffusion links. In each time step, tick, money or wealth is permitted to flow across specified links between agents, having more preferential flow agents with bigger wealth concentration due to connectivity, every link has a preferential attachment with the richer link. Network has a rewiring probability as well, and every step a rewiring is performed. After simulation took part each node could earn more money, or lose it, and the new outcoming wealth distribution scenario is then analyzed. First system is setup with fixed rate-exchange and tax-rates values, as well as rewiring probability, to fit with Pareto´s distribution (equation 2) for validation. Five hundred agents were created, with a rewiring-probability P = 0.30, and an exchange - rate = 0.4, tax-on-exchanged value is set to 0.16 value (Mexico's corresponding VAT). A sample of 150 runs were performed, and each simulation had about 30 ticks of duration, the wage earned and described in table 1.1 is the amount earned monthly and the system doesn´t consider a reload of money. Results and validation analysis are shown in next section.

Figure 2.5 A simulation framework

3. VALIDATION

Since the system is based on Mexico´s income distribution the starting money set in the model based on Table 1.1. outcomes are adjusted to this distribution parameters. For validation, a goodness of fit was performed using Chi-square distribution, using simulation outcomes of final mean value of remaining money each *decil* agent-set has at the end of 30 days (ticks), also maximum and minimum values are included (see table 3.1). Average is obtained to construct *decil* income distribution obtained from simulations. As main objective of present project is to represent an agent based model income distribution with an exchange rule, adjustment to theoretical Pareto Function is also performed with μ = 1.5. Results are shown in the analysis below.

Table 3.1. Data outcomes from 150 simulation runs with 500 nodes. Columns show mean, maximal and minimal value of money after 30 ticks (a month).Extreme right column represent average of the other columns.

Decil group	**mean value**	**max value**	**min value**	**average value (Max-Mean-min)**
I	**418.13**	**518.82**	**0.00402**	**312.32**
II	**502.36**	**757.35**	**0.00447**	**419.91**
III	**654.51**	**859.04**	**0.00361**	**504.52**
IV	**833.22**	**1099.61**	**0.00299**	**644.28**
V	**804.75**	**1025.13**	**0.00389**	**609.96**
VI	**963.83**	**1278.29**	**0.00323**	**747.37**
VII	**826.87**	**1015.21**	**0.00450**	**614.03**
VIII	**972.98**	**1622.86**	**0.00320**	**865.28**
IX	**1332.51**	**1685.83**	**0.00196**	**1006.11**
X	**4158.16**	**4789.32**	**0.00410**	**2982.49**

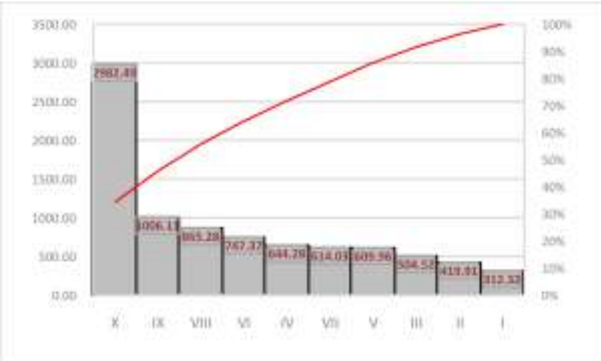

Figure 3.1. Decil-sets distribution plot with cumulative tendency line from table 3.1.

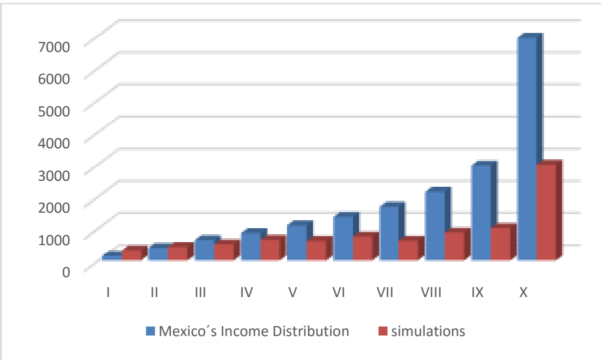

Figure 3.2. Bar-Plot for both data-set. It is shown a decreasing in simulation data outcomes. Decil-sets Distribution remains in simulation data's as shown in previous graphs.

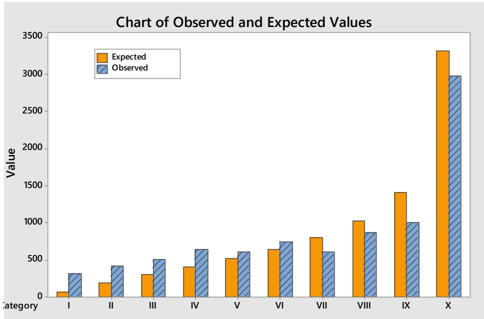

Figure 3.3. Plot for contrasting both distributions, historical Mexico's income distribution and observed simulation results, for a Chi-Squared Goodness of fit validation Analysis.

Table 3.2. Chi-Squared Goodness of fit Analysis, for 150 Simulation Outcomes vs Table 1.1. information. At the bottom of the table adjustments parameters are shown (Chi-Squared Value aand P-Value)

Decil set	Simulation data's	Table 1.1 information	Proportion	Expected	Contribution to Chi-Sq
I	312.32	147.7	0.00813	70.76	824.585
II	419.91	391.9	0.02157	187.76	287.045
III	504.52	634.4	0.03491	303.93	132.374
IV	644.28	850.3	0.04679	407.37	137.776
V	609.96	1089.4	0.05995	521.92	14.852
VI	747.37	1353.6	0.07449	648.5	15.076
VII	614.03	1678	0.09234	803.91	44.852
VIII	865.28	2140.7	0.1178	1025.59	25.058
IX	1006.11	2956.2	0.16267	1416.29	118.791
X	2982.49	6930.3	0.38136	3320.24	34.357

Degrees of Freedom	Chi-Sq	P-Value
9	*1634.77*	*0*

As seen on Analysis shown above, it exists a good fit between Mexicós Income Distribution information used to perform the model and our simulation results. Next a Pareto´s Distribution Adjustment is carried out to verify if our model corresponds to a Pareto Income Distribution. Analysis results are shown below.

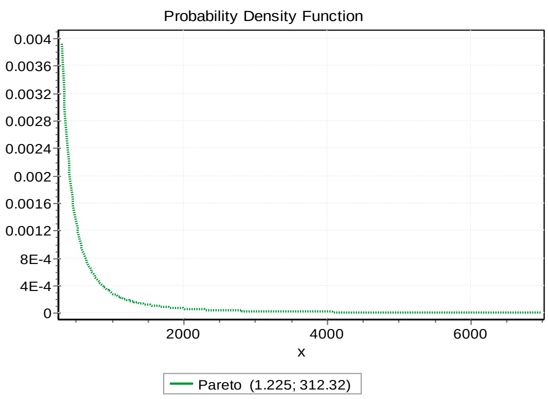

Figure 3.4. Pareto´s Density Function Plot for Simulation outcomes of Table 3.1.

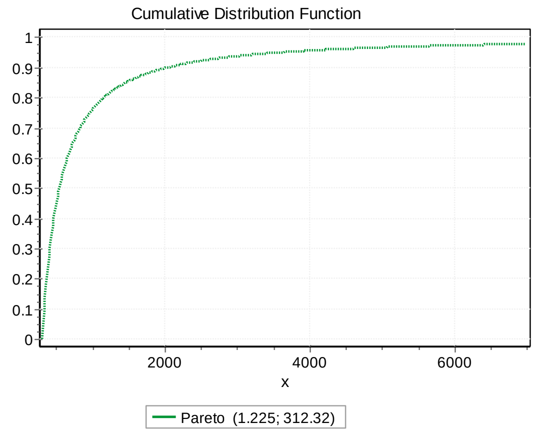

Figure 3.5. Pareto´s Cumulative Distribution Function Plot for Simulation outcomes of Table 3.1.

Tabla 3.3. Summary of Pareto' s Distribution fitting parameters for simulation outcomes

μ = 1.225		$\mathcal{W}$ = 321.32			
Kolmogorov-Smirnov					
Sample Size	10				
Statistic	0.25957				
P-Value	0.43717				
Rank	25				
α	**0.2**	**0.1**	**0.05**	**0.02**	**0.01**
Critical Value	0.322	0.3687	0.40925	0.45662	0.48893
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size	10				
Statistic	2.096				
Rank	35				
α	**0.2**	**0.1**	**0.05**	**0.02**	**0.01**
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	No	No	No

Observed analysis show a good fitting with Pareto Distribution Function (equations 1 and 2). With the shown validation analysis, the model can be considered for simulation process of Mexico´s income distributions analysis. Once the model is significantly accepted, it should be considered repercussions model parameters, (rewiring-probability, taxes and exchange-chance) would have in system´s dynamic. Experiments are designed to get insight on this question. In next section this experiments are described.

4. Experiments Performance

Gini coefficient is used to measure, in general, incomes and wealth distribution. Gini coefficient is a statistical measurement of dispersion and variability of income or wealth distribution of a nation. It is and indicator of inequality (Gini, 1909). Its values range from 0, which

ISBN 978-88-97999-85-0; Affenzeller, Bruzzone, Jiménez, Longo and Piera Eds.

represents a perfect distribution, that means that all residents would have the same wealth or income. On the other side if its value is 100 % that means that just one person concentrates total wealth. Parameters modeled in this project represent social dynamics by rewiring probability and exchange rate, and the influence that taxes have in money flow between agents or population. By a design of experiment will get insight on effects of these parameters on income distribution. The DoE performed is a full factorial design with 3 factors and different levels for each parameter. See Table 4.1. for experiment's description.

Table 4.1.Full Factorial Design of Experiment Parameters

Factors: 3	Replicates: 4
Basic runs: 48	Total runs: 192
Basic Blocks:1	Total Blocks: 4
Rewiring-Probability Factor	4 levels: 20%, 40%, 60% and 80%
Rate-exchange Factor	4 levels: 20%, 40%, 60%, 80%
Tax-on-exchange factor	3 levels: 15%, 25%, 35%
Response value:	Gini coefficient

After running all replicas, results obtained are analyzed by an AnoVa tool. Results are then analyzed and discussed in section 5.

5. RESULTS ANALYSIS AND DISCUSSION

5.1 Results analysis

After runs were carried out, and AnoVa tool is applied to simulation outcomes as described below

Source	DF	Adj SS	Adj MS	F-Value	P-Value
taxes	2	0.1618	0.08090	2.96	0.054
Exchange-rate	3	0.2474	0.08246	3.02	0.031
rewiring-probability	3	0.9547	0.31822	11.64	0.000
Error	183	5.0017	0.02733		
Lack-of-Fit	39	0.6822	0.01749	0.58	0.974
Pure Error	144	4.3196	0.03000		
Total	191	6.3656			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.165324	21.43%	17.99%	13.51%

Figure 5.1. ANOVA results from performed experiment

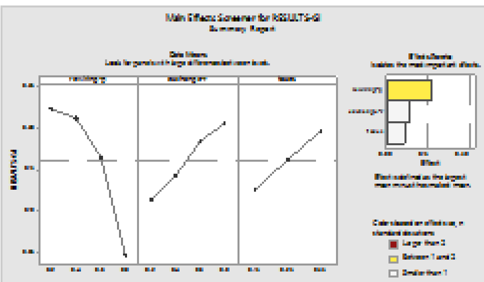

Figure 5.2. Main effect Plot for the three factors. At the upper right corner it is indicate which factor affects more than others

Figure 5.2. indicates the significantly effects a factor has over response. Rewiring-Probability (***P***) affects more than other two factors, but at higher levels of P results on negative Gini-coefficient ***G***, which are considered as outlier data. It is suggested that for higher levels of P and lower levels of exchange-rate (***R***) as well as for the lower levels of taxes ***(T)***.

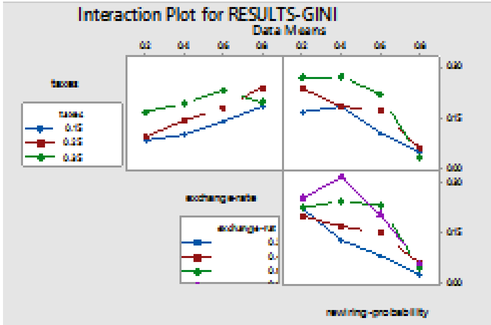

Figure 5.3. Interaction Plots for three factors. At top left interaction between exchange-rate and taxes are compared. Top right interaction between rewiring-probability and taxes are compared. Bottom left, interaction between rewiring probability and exchange rate is observed.

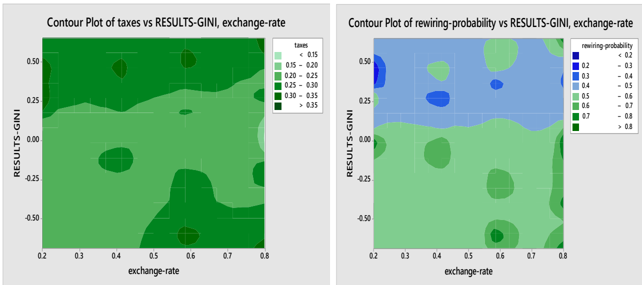

Figure 5.3. Contour Plots, left interaction between T, R and Gini coefficient G response is shown. Right interaction between R, P and G is shown

In figure 5.3 graphs of interaction between factors is explored. There appears to be a moderate interaction between P and R, but there seems to be more significant interaction between T and P factors. With contour plots effects between factors is more distinguishable, what is observable in both graphs is that are outlier data with negative values for G, and are produced mostly by higher values of P. An individual value Plot is used to identify this values.

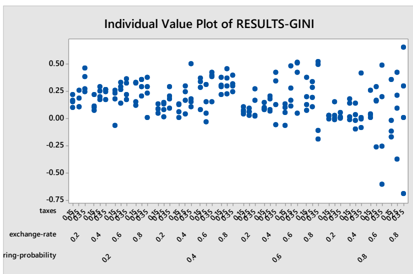

Figure 5.4. Individual Value Plot for Gini Coefficient. Negative data outcomes are observable with this plot.

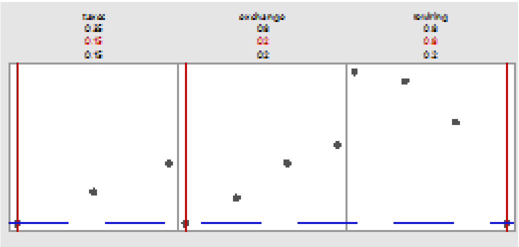

Figure 5.7. Optimal Plot. Red labels indicate optimal values to minimize Gini Coefficient. Blue Dot line indicates lower value of G.

As seen in graph 5.6 for individual values, there are many outlier data that can affect more significantly to general outcomes. Design of experiment applied should be reviewed. Figure 5.7 indicates the parameters value to obtain the lowest Gini coefficient value using this design.

5.2. Discussion

As P parameter represent social contacts in a real system, it is observed for high levels of P with lower level of R , G values tend to negative values, which are not possible, a review on this affection should be done with an one factor experiment design. Also, it is noted that interaction with taxes does not affect significantly to response since tax imposed affects equally to all agents. A remarkable overview is the optimal combination of factor levels that indicates that for the highest level of P and with the lowest levels of R and T optimal is obtained. This results indicates that with social interaction is important to diverse with whom to make trades, and exchange rate if it is low there are less opportunities for agents to lose its money. And more interesting analysis is that low taxes increase social equality.

CONCLUSIONS

This project is part of a doctoral research which main scope is to get insight on more significant factors that affect wealth and income distribution, and as a first insight a network model should be designed, with an evolutionary game interaction rule. But as there is limited space that part is left for a further research. Although with the present work it is possible to gain insight on some important factors and its interactions. And a more interesting result is taxes influence in income equality. Results on this project should be considered by politicians and stakeholders to design more adequate economic policies.

REFERENCES

Adami, C., Schossau, J. &Hintze, A., 2016. The reasonable effectiveness of agent-based simulations in evolutionary game theory: Reply to comments on “Evolutionary game theory using agent-based methods”. Physics of Life Reviews, Volume 19, pp. 38-42.

Anglin P., 2005, “Econophysics of wealth distributions: a comment”, in Eds. Chatterjee, Yarlagadda and Chakrabarti, pp.229-238 (2005).

Atkinson B. and T. Piketty, 2007, “Top Incomes over the Twentieth Century”, Eds.(2007) (Oxford University Press, Oxford).

Bouchaud J. P. and Mézard M., 2000, “Wealth condensation in a simple model of economy”, Physica A 282, (2000) 536-545.

Chatterjee A., 2009, “Kinetic models for wealth exchange on directed networks”, The European Physical Journal B 67, (2009) 593-598.

Chatterjee, B. K. Chakrabarti and R. B. Stinchcombe, 2005, “Master equation for a kinetic model of a trading market and its analytic solution”, Physical Review E 72, (2005) 026126.

Chatterjee, S. Yarlagadda and B. K. Chakrabarti, 2005, “Econophysics of Wealth Distributions”, Eds. (2005) (Springer, Milan).

del Castillo Negrete M., 2016, “La magnitud de la desigualdad en el ingreso y la riqueza en México, una propuesta de cálculo”, CEPAL, Eds. UNO. 215.

Di Matteo T., Aste T. and Hyde S. T., 2004, “Exchanges in complex networks: income and wealth distributions”, in The Physics of Complex Systems (New Advances and Perspectives) edited by F.Mallamace and H. E. Stanley (IOS Press, Amsterdam) (2004) p. 435

Drâgulescu A. and V. M. Yakovenko, 2003, “Statistical mechanics of money, income, and wealth: a short survey”, in Modeling of Complex Systems: Seventh Granada Lectures, edited by P.L. Garrido and J. Marro, American Institute of Physics (AIP) Conference Proceedings 661, (2003) pp. 180-183.

Gaffeo E., DelliGatti D., Desiderio S. and Gallegati M., 2008, “Adaptive microfoundations for emergent macroeconomics”, Eastern Economic Journal 34, (2008) 441-463.

Garlaschelli Diego and Loffred Maria I., 2004, “Wealth Dynamics on Complex Networks”, Physica, feb 2004.

Hua Mao-Bin, Jianga Rui, Wua Qing-Song, Wub Yong-Hong, 2008, “Simulating the wealth distribution with a Richest-Following strategy on scale-free network”, Physica A 387 (2008) 5862–5867.

Ichinomiya Takashi, 2012, “Wealth distribution on complex networks”, Physical Review E, 86, 066115(2012).

Iglesias J. R., Goncalves S., Pianegonda S., Vega J. L. and Abramson G., 2003 “Wealth redistribution in our small world”, Physica A 327, (2003) 12-17.

Nishino, N., Okazaki, M. & Akai, K., 2017. Effects of ability difference and strategy imitation on cooperation network formation: A study with game theoretic modeling and multi-agent simulation. Technological Forecasting and Social Change, pp. 1-12.

Piketty Thomas, “Capital in the Twenty-first Century”, Harvard University Press, 2014.

Souma Wataru y Fujiwara Yoshi, Aoyama Hideaki,2001, “Small-World Effects in Wealth

Distribution”, Statistical Mechanics (cond-mat.stat-mech), KUCP0189, 2001.
Tesfatsion Leigh, 2003, “Agent-based computational economics: modeling economies as complex adap8tive systems”, Information Sciences 149 (2003) 263–269.
Watts D. J. and Strogatz S. H., 1998“Collective dynamics of ‘small-world’ networks.” Nature, 393 (6684): 440–442,1998.
Gini, C., 1909. "Concentration and dependency ratios" (in Italian). English translation in *Rivista di Politica Economica*, **87** (1997), 769–789.

Acknowledgments
To CONACYT scholarship for the first author, and DGAPA-PAPIIT project ITl 02117 Accesibilidad y movilidad def transport e publico urbano en la ciudad de Mexico, el caso de la delegación Tlalpan

AUTHORS BIOGRAPHY

Javier Lara de Paz studied Physics at the Faculty of Science of National Autonomous University of Mexico (UNAM), a Specialization in Applied Statistics at the UNAM. He studied his Master degree in Operation Research at the Faculty of Engineering of the UNAM. He has been professor of Science at the and Operation Research professor at the Engineering School at the UNAM. Currently he is studying a Ph.D. in Operational Research at UNAM.

Idalia Flores de la Mota She received her Ph.D. in Operations Research at the Faculty of Engineering of the UNAM. She graduated Master with honors and received the Gabino Barreda Medal for the best average of her generation. She has been a referee and a member of various Academic Committees at CONACYT. She has been a referee for journals such as Journal of Applied Research and Technology, the Center of Applied Sciences and Technological Development, UNAM and the Transactions of the Society for Modeling and Simulation International. Her research interests are in simulation and optimization of production and service systems. She is a full time professor at the Postgraduate Program at UNAM.

Gabriel Policroniades Chipuli He studied Industrial Engineering at Technological Institute and Superior Studies of Monterrey , Campus Toluca (ITESM, Campus Toluca), he studied an engineering mastery with operation research orientation and ha advanced metrology studies for the calculation of uncertainty, and knowledge of Lean Six Sigma Black Belt and Logistics and supply chain Currently he is studying a Ph.D. in Operation Research at UNAM.