ALTERNATIVES AGGREGATION PETRI NETS APPLIED TO MODULAR MODELS OF DISCRETE EVENT SYSTEMS

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ABSTRACT
The use of modelling formalisms for the design of discrete event systems presents many advantages, such as the posibility of structural analysis of the model or performance evaluation. However, the difficulty of the process to obtain an appropriate model of the system require the use of methodologies to ease the work of the designers. In this paper, two main subjects are discussed. On the one hand, the modular construction of Petri nets, alleviate the design process by the use of blocks that can be assembled to build up a complete Petri net model. On the other hand, the development of decision support systems may require the assessment of the performance and properties of complete models obtained from different combinations of modular blocks. The formalism of the alternatives aggregation Petri net may help in the development of compact and efficient models that may reduce the use of scarce computer resources.

Keywords: modular Petri nets, alternatives aggregation Petri nets, decision support systems, performance evaluation

1. INTRODUCTION

The application of modelling and simulation methodologies as base for the development of decision support systems, requires in many cases making suppositions on the structure of the real system itself. However, it is usual that the structure of the real system is not completelly defined, but it should be clarified after making the subsequent decisions.

For this reason, the modular construction of models, allows the designer to use encapsulated blocks to construct the model of the system, easing the process of modelling. Moreover, it is common that the designer does not know, which is the best combination of blocks for the purposes of the system in process of being designed. For this reason, an automatic testing of the different possible combinations of the blocks for build up complete models would alleviate the modelling process.

Furthermore, as this problem is intensive in the use of computer resources, the development of adequate methodologies for obtaining compact and efficient models is a crucial issue in the development of decision support systems based on modelling and simulation.

The development of decision support systems based on modelling and simulation has been discussed by (Bruzzone and Longo, 2010), (Longo et al. 2013). The range of application of these decision support systems is briad, including the food industry (Latorre et al. 2014b), (Latorre et al. 2013b).

The use of the Petri nets as a versatile paradigm for modelling discrete event systems is considered in (Silva et al., 1993), (David and Alla 2005), and (Jensen and Kristensen 2009) In particular (Piera et al. 2004), (Latorre et al. 2013a) describe Petri net models for simulation. Moreover, (Mújica et al. 2010) is oriented to the application of simulation for quantifying a performance evaluation in the context of an optimization process.

A particularly difficult problem consists of designing a discrete evnet system, whose model should be chosen among a set of alternatives. In this case, it is convenient that the model of the system includes a set of exclusive entities (Latorre et al. 2010).

Moreover, an optimization process may be based on the simulation of a set of selected feasible decisions, chosen from a solution pool. In this case, the choice of the most promising decisions may be performed, for instance, by means of a search methodology guided by a metaheuristic (Latorre et al. 2013c).

In the following section, the topic of the modular Petri nets is discussed. Moreover, in section 3 brief comments on the concept of alternatives aggregation Petri net are provided, while in section 4, a discussion of the application of this formalism to represent a modular Petri net constructed as a sequence from combinations of four Petri subnets is given.

The next section is focussed on the conclusions and the future research work, while the last one is devoted to the bibliography.

2. MODULAR PETRI NETS

The construction process of models of discrete event systems that approximate complex real systems may be considered more an art than a precise and algorithmic procedure.

One of the most common methodology for coping with the modelling process of a complex discrete event system is the so called bottom-up approach. According to this idea a model is developed for every one of the subsystems in which the complete system can be divided. The level of detail required for every model depends its purpose and application.

This methodology derives naturally to the concept of modular construction of the Petri net model of a discrete event system. This idea implies the definition of a set of Petri net modules, ready to be assembled for the construction of complex models of real systems.

As an example, let us consider a set of four Petri subnets called R_A, R_B, R_C, and R_D. These Petri nets should present a compatible interfase to be connected to other subnets. In the subnets considered in this paper, a single inpunt link transition and a single output transition appear. Moreover, the input link transition presents a single output place, belonging to the considered subnet. Similarly, the only output link transition presents a single input place, also belonging to the considered subnet.

See *figure* 1 for a simplified representation of three of the mentioned Petri subnets, as well as some of their constituent elements, such as the input and output transitions and their output and input places.

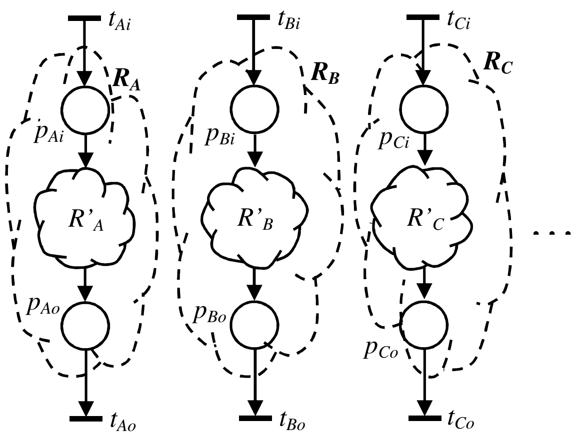

Figure 1: Three Petri subnets defined for the construction of a modular Petri net

In *Figure* 1, the input link transitions of the Petri subnets R_A, R_B, and R_C are, respectively, t_{Ai}, t_{Bi}, and t_{Ci}. On the other hand, the output link transitions of these same Petri subnets, in the same order, are t_{Aoh}, t_{Bo}, and t_{Co}.

These subnets might be combined in different ways to build up the complete model of a real system. Prior to a detailed analysis of every resulting model, it may be difficult to foresee the performance of any of them.

For this reason, a procedure can be defined in order to construct a set of feasible solutions for the complete model of the system by combining the subnets in appropriate ways. A second step in this procedure would be to develop a performance analysis of every complete model and, eventually, to compare the desired performance parameters calculated for every complete model in order to decide the best combination of subnets. *Figure* 2 and *figure* 3 show examples of combinations of the Petri subnets R_A, R_B, R_C, and R_D leading to complete models for a real system.

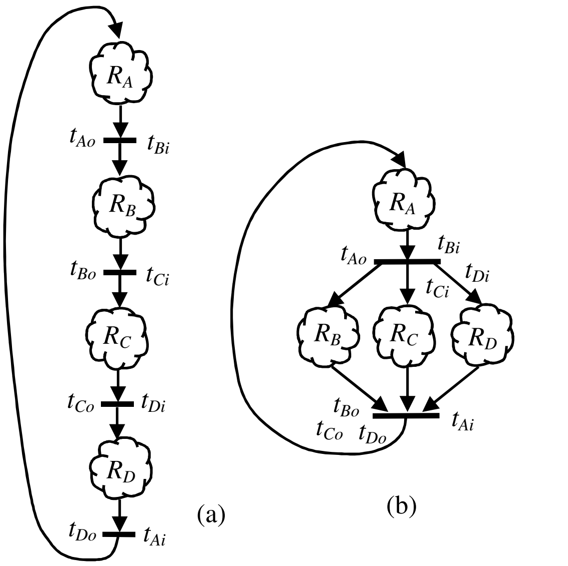

Figure 2: Different feasible combinations of 4 subnets

A Petri net model of a discrete event system can be developed for different purposes, such as performing structural analysis, calculating a certain subset of the set of reachable states, or for performance evaluation.

Specially in this last case, it is crucial for the success of the operation, to use an efficient algorithm, able to cope with the, sometimes, very costly process in terms of computer resources and time. One methodology, broadly used, that can virtually cope with every model, no matter how complex it is, is simulation.

Regarding the previous considerations, an important goal in the process of modeling a Petri net for performance evaluation is to obtain a formal description of the original system, as simple and reduced as it is possible. Hence, the costly process of simulation might be developed in affordable time and computer resources.

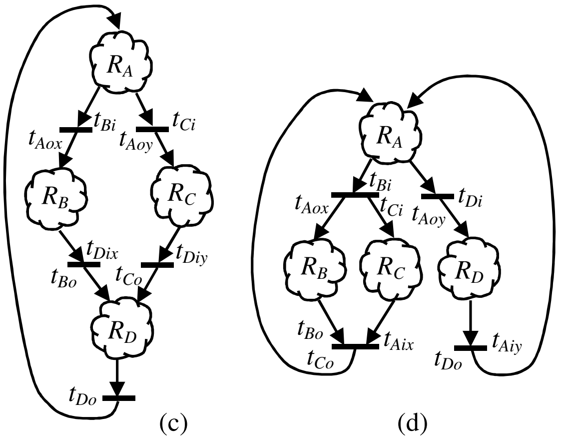

Figure 3: More feasible combinations of 4 subnets

It may be noticed the possibility of having different input or output link transitions for a giben Petri subnet in a certain complete model. Also, consider that these multiple input or output link transitions present, respectively, a single output or input place. This possibility is illustrated in both Petri net models depicted in *figure* 3.

One important application for performance evaluation of Petri net models using simulation, consists in decision-making support with the puropose of designomg a real system. The feasible models of the system in process of being designed can be compared by means of the result of a performance evaluation of every candidate model.

In the modular construction of a Petri net model, it may be interesting to test different or even all the feasible combinations of subnets that can be obtained. Every feasible solution is a candidate for bein selected as the final model of the system in a design process; hence, every solution is an alternative model for the system. For this reason, a Petri net formalism able to represent alternative Petri nets, such as one containing a set of exclusive entities, should be considered.

3. ALTERNATIVES AGGREGATION PETRI NETS

The existence of alternative models for the development of a given discrete event system, require the use of particular formalisms, able to cope with the particularities of this kind of design problems.

A family of formalims, based on the Petri net paradignm, specially developed for this purpose are the ones based on the idea of exclusive entities, deriving in formalisms such the set of alternative Petri nets or the alternatives aggregation Petri nets (Latorre et al., 2014a, 2014c, 2012,).

Both formalisms will be extensively used in this paper.

4. SEQUENCE OF FOUR SUBNETS

In this section, it will be considered the modular Petri net model constructed as a strict sequence from the four Petri subnets called R_A, R_B, R_C, and R_D as they were mentioned in section 2.

All the possible complete Petri net models, built up from different combinations of the four Petri subnets in a sequence will be considered, as feasible solutions for the design process of a real system. One of the feasible solutions is presented in *figure* 2.(a), called R_{a1}, while most of the rest of them, R_{a2} to R_{a5} are presented in *figure* 4. The only remaining modular Petri net, not represented in a figure and called R_{a6}, presents the sequence of Petri subnets R_A, R_D, R_C, and R_B and its representation is less interesting than the remaining five feasible modular Petri nets, as it will be shown when constructing the alternatives aggregation Petri net.

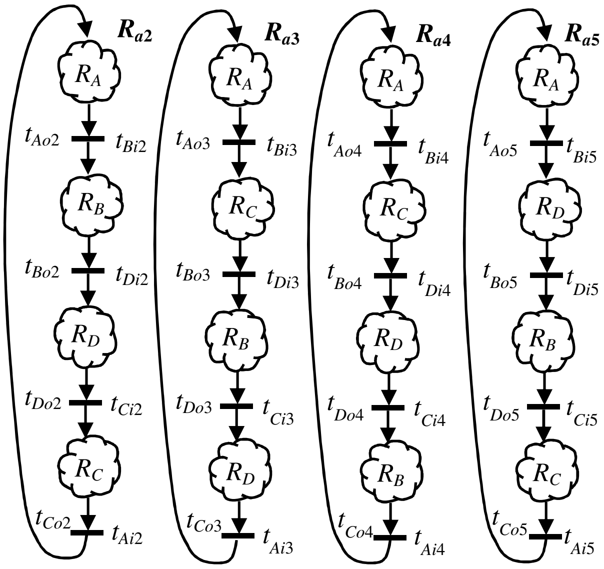

Figure 4: Different combinations of four Petri subnets for constructing modular Petri nets.

The six modular Petri nets may be feasible models of a real system in process of being designed. In this design process, it should be made a decision regarding the bmodular Petri net that best complies with the objectives of the real system, usualy measured or quantified by means of performance parameters.

In fact, the modular Petri nets are alternative Petri nets; hence, the model of the real system in process of being designed can be represented by a formalism containing a set of exclusive entities. Furthermore, using the appropriate formalism may reduce considerably the computational resources required to solve the associated decision-making problem.

Regarding previous results in other case-studies, the formalism of the alternatives aggregation Petri nets is chosen for modelling the real system in process of being designed, that is to say, to represent in a single model the six alternative Petri nets, removing from the model the redundant information.

One of the algorithms for obtaining an alternatives aggregation Petri nets from a set of alternative Petri nets (Latorre et al., 2013c) states that any of the alternative Petri nets may be chosen as the seed for the resulting the alternatives aggregation Petri net model. See *Figure* 2(a), where R_{a1} has been selected for this purpose. In this seed, every link transition should be associated to a choice variable a_1 as a guard of the transition itself.

The next steps of the algorithm for the construction of an alternatives aggregation Petri net from a set of alternative Petri nets belong to an iterative procedure, where every new alternative Petri net is added to the seed of the alternatives aggregation Petri net by including the new subnets (in this case-study there is not any of them) and all the link transitions associated to a guard function, given by the choice variable a_i that corresponds to the alternative Petri net R_{ai} containing the link transitions.

Following this algorithm, its second step consists of adding to the seed of the alternatives aggregation Petri net the alternative Petri net called $\boldsymbol{R_{a2}}$ (see *figure* 4). Due to the fact that this alternative Petri net does not present any subnet that is not already included in $\boldsymbol{R_{a1}}$, then the only modification of the seed of the alternatives aggregation Petri net introduced by the alternative Petri net $\boldsymbol{R_{a2}}$ is the addition of the link transitions. In the case of $\boldsymbol{R_{a2}}$, the link transitions are (see *figure* 4) t_{Ao2}, t_{Bo2}, t_{Do2}, and t_{Co2}, or, what is the same, the transitions called t_{Bi2}, t_{Di2}, t_{Ci2}, and t_{Ai2}. This transitions are added to the seed of the alternativs aggregation Petri net associated to the choice variable a_2.

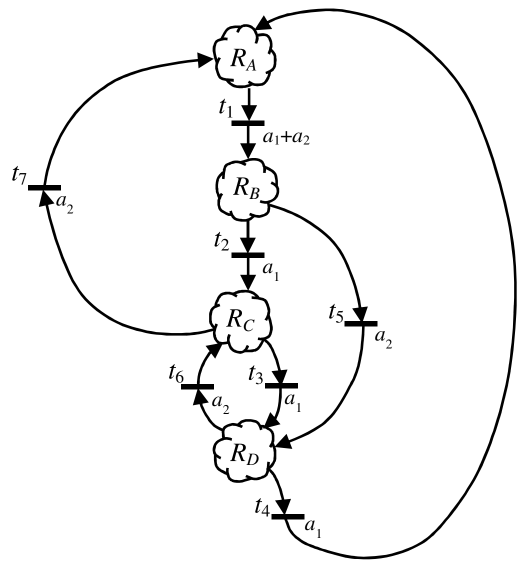

Figure 5: Second step in the construction of the alternatives aggregation Petri net

Once the new link transitions have been included in the seed of the alternatives aggregation Petri net, it is possible to apply a reduction rule, which groups together the quasi-identical transitions, modifying the associated function of choice variables. If it is possible to apply this reduction rule to a given operation, then the resulting model will be more simple, since it contains a lower number of transitions. In fact, a couple of quasi-identical transitions verify, having the same set of input and output places, as well as the same weight in the input and output arcs. Moreover, the transitions should be associated to different functions of choice variables, otherwise the transitions would be identical instead of quasi-identical ones.

In the example of this second step of the algorithm, a link transition of $\boldsymbol{R_{a2}}$ has been merged with t_1, just by constructing an associated function of choice variables with the logic operator OR applied to both choice variables a_1 and a_2 (see *figure* 5). The link transitions that have been added in this second step of the algorithm appear in *figure* 5 and are named t_5, t_6, and t_7.

In *figure* 5, the second step of the algorithm can be seen, while the complete alternatives aggregation Petri net is shown in *figure* 6.

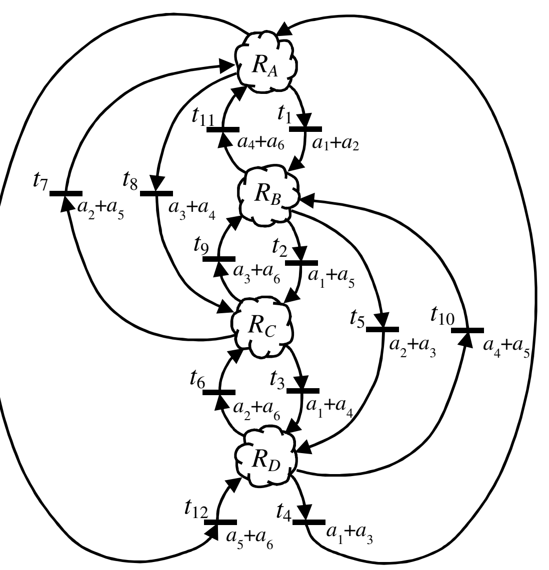

Figure 6: Complete alternatives aggregation Petri net

In order to complete the alternatives aggregation Petri net depicted in *figure* 7, it has been necessary to include five more link transitions, in addition to the four ones introduced by $\boldsymbol{R_{a1}}$ and to the three new transitions delivered by $\boldsymbol{R_{a2}}$.

In the following paragraphs, it will be discussed the compacity in the matricial representation of both models of a system in process of being designed by means of a set of alternative Petri nets (see *figure* 4). The compacity of the alternatives aggregation Petri net is based in the fact that a large amount of redundant information, present in the set of alternative Petri nets, has been removed from the model: the Petri subnets, which appear in every alternative Petri net.

The incidence matrix of any of the six alternative Petri nets has a dimension that can be calculated as follows:

Let us consider that the dimension of a Petri net is given by the multiplication of the number of rows and the number of columns of the associated incidence matrix.

According to this idea, the dimensions of the four Petri subnets of this case-study are:

$M(R_A) \in M_{A_r \times A_c}$, where A_r is the number of rows of the incidence matrix of R_A and A_c is its number of columns. Analogously:

$M(R_B) \in M_{B_r \times B_c}$; $M(R_C) \in M_{C_r \times C_c}$; $M(R_D) \in M_{D_r \times D_c}$

On the other hand, any of the alternative Petri nets $\boldsymbol{R_{a1}}$, $\boldsymbol{R_{a2}}$, $\boldsymbol{R_{a3}}$, $\boldsymbol{R_{a4}}$, $\boldsymbol{R_{a5}}$, and $\boldsymbol{R_{a6}}$ present the same dimension, which can be calculated as it is described below.

$M(R_{a1}), M(R_{a2}), M(R_{a3}), M(R_{a4}), M(R_{a5}), M(R_{a6}) \in M_{r \times c}$ where

$$r = A_r + B_r + C_r + D_r \tag{1}$$

$$c = A_c + B_c + C_c + D_c + 4 \tag{2}$$

It should be considered that r is the number of rows of the incidence matrix of R_{ai}, with $i = 1,\ldots,6$. On the other hand, c is the number of columns of the incidence matrix of R_{ai}, with $i = 1,\ldots,6$.

$$\begin{array}{cc} & \begin{array}{ccccc} & t_1 & t_2 & t_3 & t_4 \end{array} \\ M(R_{a1}) = & \left(\begin{array}{cccc|c|c|c|c} \boxed{M(R_A)} & 0\;0 & \cdots & 0\;\;0 & & & & \\ 0\;\;0 & \boxed{M(R_B)} & & \cdots & & & & \\ \cdots & & \boxed{M(R_C)} & 0 & & & & \\ 0\;\;0 & \cdots & 0 & \boxed{M(R_D)} & & & & \end{array}\right) \end{array}$$

Figure 7: Incidence matrix of an alternative Petri net

The number 4 that appears in the expression (2) is originated by the four link transitions included in every R_{ai}, with $i = 1,\ldots,6$.

Figure 7 shows a representation of the incidence matrix of any of the alternative Petri nets

The calculation of the resulting alternatives aggregation Petri net, R_{AA}, can be developed in a similar way.

$M(R_{AA}) \in M_{r' \times c'}$, where

$$r' = A_r + B_r + C_r + D_r \tag{3}$$

$$c' = A_c + B_c + C_c + D_c + 4 + 8 \tag{4}$$

$$\begin{array}{cc} & \begin{array}{cccccccc} & t_1 & t_2 & t_3 & t_4 & t_5 & \ldots & t_{12} \end{array} \\ M(R_{AA}) = & \left(\begin{array}{cccc|c|c|c|c|c|c|c|} \boxed{M(R_A)} & 0\;0 & \cdots & 0\;\;0 & & & & & & & \\ 0\;\;0 & \boxed{M(R_B)} & & \cdots & & & & & & \cdots & \\ \cdots & & \boxed{M(R_C)} & 0 & & & & & & & \\ 0\;\;0 & \cdots & 0 & \boxed{M(R_D)} & & & & & & & \end{array}\right) \end{array}$$

Figure 8: Incidence matrix of the alternatives aggregation Petri net

The comments on the previous expressions (3) and (4) are the same as done before. However, the number 8 in (4) is a consequence of the fact that the aggregation of alternative Petri nets to the seed of the alternatives aggregation Petri net introduces 8 new link transitions.

Figure 8 shows the incidence matrix of the alternatives aggregation Petri net.

In order to compare the convenience of using one of both models of a modular Petri net for developing a decision support system, the computer resources required to execute an optimisation algorithm using the simulation of one of both models can be compared.

In fact, some important computer requirements depend on the size of the model itself. Hence, the comparison between the set of alternative Petri nets and the alternatives aggregation Petri nets can be performed calculating a size ratio, defined in the following way:

$$size\ ratio = \frac{size\ of\ the\ alternatives\ aggregation\ Petri\ net}{size\ of\ the\ set\ of\ six\ alternative\ Petri\ nets}$$

$$size\ ratio = \frac{r' \times c'}{6 \times r \times c} \tag{5}$$

$$size\ ratio = \frac{(A_r + B_r + C_r + D_r) \times (A_c + B_c + C_c + D_c + 12)}{6 \times (A_r + B_r + C_r + D_r) \times (A_c + B_c + C_c + D_c + 4)}$$

Due to the fact that the number of rows is the same in the alternatives aggregation Petri net and in the alternatives aggregation Petri net, it is possible to cancel this number in the numerator and the denominator of the expression. As a result, it is possible to see that the size ratio does not depend on the number of places of the Petri nets.

$$size\ ratio = \frac{(A_c + B_c + C_c + D_c + 12)}{6 \times (A_c + B_c + C_c + D_c + 4)}$$

As an example, if every subnet presents 5 internal transitions, the size ratio has the value:

$$size\ ratio = \frac{(5 + 5 + 5 + 5 + 12)}{6 \times (5 + 5 + 5 + 5 + 4)} = \frac{1}{4.5}$$

In other words, for a small size of the Petri subnets, 5 internal transitions, the alternatives aggregation Petri net is 4.5 times smaller than the equivalent set of alternative Petri nets.

The calculation of the amount of redundant information removed from the set of alternative Petri nets is:

$$100 \times (1 - size\ \ ratio) = 100 \times (1 - \frac{1}{4.5}) \cong 77.8\%$$

Finally, it is possible to calculate the upper bound in both parameters: the size ratio and the amount of removed redundant information in the model of the system:

Let us call x_A, x_B, x_C, and x_D the number of columns of the incidence matrices of the Petri subnets R_A, R_B, R_C, and R_D respectively. Let us call $x = x_A + x_B + x_C + x_D$.

$$\lim_{x \to \infty}(size\ ratio) = \lim_{x \to \infty}\left(\frac{(x + 12)}{6 \times (x + 4)}\right) = \frac{1}{6}$$

On the other hand, the upper bound of the percentage of redundant information removed from the set of alternative Petri nets is:

$$100 \times (1 - size\ ratio) = 100 \times (1 - \frac{1}{6}) \cong 83.3\%$$

As it can be seen, the alternatives aggregation Petri net outperforms the complete set of alternative Petri nets, while this last formalism is more intuitive and easy to apply for the modelling of discrete event systems.

It is also interesting to point out, that the model based on the alternatives aggregation Petri net should add some additional information to the model itself: the functions of choice variables associated to every link transition.

5. CONCLUSIONS AND FUTURE RESEARCH LINES

In this paper, some considerations on the modular construction of Petri net models have been introduced. Furthermore, the application of these models to decision-support systems based on simulation requires the development of exigent algorithms in terms of computer resources.

In order to overcome or at list palliate this problem, a transformation of a non-efficient model based on a set of alternative Petri nets into an alternatives aggregation Petri net is discussed. Two parameters have been defined and calculated: the size ratio, to quantify the relative size between both models and the percentage of redundant information that has been removed in the alternatives aggregation Petri net but not in the original set of alternative Petri nets.

As conclusions, it can be stated that the modular construction of Petri net models is a promising research line to develop decision support systems to construct models of discrete event systems. On the other hand, there are formalisms, such as the alternatives aggregation Petri nets, able to reduce significantly the size of a model in the case of models composed of sequences of four Petri subnets.

As future research actions, it can be considered to extend the discussion of these methodologies and results to other layouts in the modular Petri nets, as well as considering a larger number of subnets and different constitutions of the subnets.

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