MODELLING A CLUSTERED GENERALIZED QUADRATIC ASSIGNMENT PROBLEM

Judith Fechter[a], Andreas Beham[b], Stefan Wagner[c], Michael Affenzeller[d]

[a],[b],[c],[d] Heuristic and Evolutionary Algorithms Laboratory, School of Informatics, Communications and Media,
University of Applied Sciences Upper Austria, Hagenberg Campus
Softwarepark 11, 4232 Hagenberg, Austria
[a],[b],[d] Institute for Formal Models and Verification, Johannes Kepler University Linz,
Altenbergerstr. 69, 4040 Linz, Austria

[a]judith.fechter@fh-hagenberg.at, [b]andreas.beham@fh-hagenberg.at, [c]stefan.wagner@fh-ooe.at
[d]michael.affenzeller@fh-hagenberg.at

ABSTRACT
This paper is about the modelling of an assignment problem motivated by a real world problem instance. We consider multiple pieces of equipment which need to be assigned to several locations taking into account capacities as well as relations between equipment and distances between locations. Additionally, a clustering of locations is taken into account that groups locations into areas or fields. It is forbidden to assign the same equipment to locations in different fields. The problem arises in many real world applications such as facility layout or location problems. We discuss the complexity of the problem and prove its NP-hardness. Further two linearization approaches are presented as well as computational studies of the original and the linearized models are conducted. Experimental tests are carried out using CPLEX.

Keywords: generalized quadratic assignment problem, location problem, logistics, linearization

1. INTRODUCTION

This research work is motivated by a real world problem instance. A production company should store multiple equipment to several storage locations which are grouped into areas, here groups. It is necessary to assign every equipment, but allow a single equipment to be assigned to more than one location, since the required storage can exceed the capacity of single locations. Thus, the overall number of locations must exceed the amount of equipment to be assigned. One location may only contain one equipment due to the nature of the storage order and transport vehicles.

The transportation costs which arise when equipment are demanded for further processing should be minimized. Those costs capture the costs of a vehicle moving between groups or locations collecting equipment that are required at the same time. The costs rely on a matrix which assigns a probability of collaborative further processing, as well as on a matrix which indicates the distances between locations.

1.1. .Related Problems

This problem class is strongly related to the quadratic assignment problem (QAP) (Koopman and Beckmann 1957) as well as to the generalized quadratic assignment problem (GQAP) (Lee and Ma 2004). The QAP usually assigns one equipment to one location. However, some researchers considered the QAP in a more general way (Garfinkel and Nemhauser 1972) (Sahni and Gonzalez 1976), where the number of equipment is less than the number of locations but without regarding demanded storage of equipment or capacities of locations. The GQAP assigns several equipment to one location but one equipment only to a single location, while considering distances, relations, storage demands and capacities. A generalization of the GQAP is the multi-resource generalized quadratic assignment problem (MRGQAP) (Yagiura et. al. 2007) that addresses an assignment of tasks to agents whereas an agent can handle several tasks but one task may only be handled by one agent. Further, a task may require various resources and one agent has a capacity of each resource, which represents the difference to the GQAP.

1.2. Computational Research and Complexity

Methods for solving the QAP have been researched extensively. (Burkard, Çela, Pardalos, and Pitsoulis 1998) present an overview of studies of the research community. Approaches for solving the GQAP are very limited. The authors of the GQAP (Lee and Ma 2004) presented three linearization approaches of the problem and a branch and bound algorithm for solving those. They employ a single-assignment method as a branching rule. By solving a heuristic greedy algorithm an upper bound is provided, whereas the lower bound at each node of the branch and bound tree is found by solving the generalized linear assignment problem (GLAP) (Ross and Soland 1975). They present an empirical study of 27 test instances varying from 5–16 equipment for 5–30 locations for all three linearization approaches. (Hahn, Kim, Guignard, Smith, and Zhu 2008) propose another exact solution method for the GQAP. A branch and bound algorithm is used whose bound is based on a Lagrangean dual. The dual is derived using the

Reformulation Linearization Technique (RLT) (Sherali and Adams 1996). The basis of the algorithm is a dual ascent procedure using first-level or Level-1 RLT. (Kim 2006) also propose an exact algorithm partially based on a reformulation linearization technique enhanced by adding a subgradient optimization step.

(Kim 2006) also developed a heuristic technique using new evaluations of neighboring solutions for various strategies for exploring the neighborhood. A memetic heuristic approach is presented by (Cordeau, Gaudioso, Laporte, and Moccia 2006) obtaining good results in reasonable computational time. In (Ahlatcioglu, et al. 2012) a combined method of convex hull relaxation (CHR) (Ahlatcioglu and Guignard 2007), which is a special case of the primal relaxation, and quadratic convex reformulation (QCR) (Plateau 2006), which aims to convert non-convex quadratic functions into convex quadratic functions, is proposed. The QAP and generalizations of it remain one of the hardest optimization problems. It is a NP-hard problem. Even finding an approximate solution cannot be done in polynomial time unless P=NP (Sahni and Gonzalez 1976). Due to this fact heuristic methods have become widely used. Most efficient heuristics are construction methods, enumeration or improvement methods or metaheuristics, such as simulated annealing, tabu search, genetic algorithms or ant systems (Burkard, Çela, Pardalos, and Pitsoulis 1998).

1.3. Our Contribution

So far the research community has done a lot of essential work regarding the development of various solution techniques for already existing models, which shall not be within the scope of this project. The focus of this work is the modelling of a new problem class. So far, problem classes considering a 1:1 or a n:1 assignment have been developed, some regarding further constraints like limited resources or capacities. Even so, no problem class has been defined regarding the splitting of equipment to several locations, speaking of a 1:n assignment. Further we take into account that locations belong to a certain area, while regarding distances between locations and relations between equipment. This problem class is important when the storage demands of equipment exceed the storage capacities of single locations as well as when collateral storage is needed due to collaborative further processing. Due to considering locations as part of a group, we name the developed model a clustered generalized quadratic assignment problem. The aim of this paper is to formulate the model and to perform a comprehensive computational study on it.

In section 2 the problem formulation is outlined in a descriptive and mathematical way. Properties like complexity and linearizations will be discussed. Computational studies are conducted in section 3. Tests are carried out on academic instances and show results regarding the solution quality and runtime and draw a comparison between the linearization approaches and the original problem.

2. PROBLEM DESCRIPTION

The clustered generalized quadratic assignment problem (CGQAP) considers the assignment of multiple and various equipment to one or several storage locations depending on the storage demands and location capacities. Storage locations are clustered and hence belong to a single group. When assigning equipment to locations we consider a probability of further collaborative processing between equipment as well as distances between locations inducing that two pieces of equipment having a high common probability shall be stored close to each other. Additionally, the problem takes into account that a single equipment may not be placed in more than one group. The objective is to minimize the transportation costs arising when articles are demanded for further processing.

The space requirement of the equipment has to be met by the space capacities of the locations. The CGQAP is a generalization of the GQAP in that a 1:n assignment is considered, meaning 1 equipment may be assigned to n locations which is an inverted assumption of the GQAP. Additionally, each location belongs to exactly one group. For the purpose of an optimal assignment probabilities of further collaborative processing as well as distances between locations are regarded. The objective is to optimally assign equipment to storage locations whereas an equipment may be located on several locations but only within one group, while regarding space limitations.

2.1. The Model

For the formulation of the mathematical model we use the following notation:

$M = \{1, \dots, m\}$: Set of equipment
$N = \{1, \dots, n\}$: Set of locations
$G = \{1, \dots, g\}$: Set of groups
$r_{i \in M} \in \mathbb{R}^+$: Space requirements per equipment
$c_{k \in N} \in \mathbb{R}^+$: Space capacity per location

Let α be a surjective mapping, such that each location is mapped to a group and G_l be the set of locations belonging to a group:

$$\alpha: N \to G: \alpha(k) \in G, \forall_{k \in N}$$

$$G_l = \{n | \alpha(n) = l\} \text{ with } G_l \cap G_o = \emptyset \;\; \forall l \neq o.$$

The function $a: N \times N \to \{1,0\}$ is equal to the identity in the case that two locations belong to the same group.

$$a(k,h) = \begin{cases} 1 & \alpha(k) = \alpha(h) \\ 0 & \text{otherwise} \end{cases} \quad k,h \in N$$

Between equipment there is a certain probability of how likely two pieces of equipment have a further collaborative processing. Function $w: M \times M \to \mathbb{R}_0^+$ assigns a probability, also called weights, to each possible pair of equipment.

$$w_{ij} \in \mathbb{R}_0^+ \quad \forall i,j \in M.$$

Between locations, function $d: N \times N \to \mathbb{R}_0^+$ determines the distance between each pair of locations.

$$d_{kh} \in \mathbb{R}_0^+ \quad \forall k, h \in N.$$

The costs arising when equipment are demanded for further processing shall be minimized. Those costs capture the distance the transport vehicles have to travel in order to pick up the demanded articles.
The binary decision variable determines the assignment of equipment to locations and can be written as

$$x_{ik} \to \begin{cases} 1 & \text{equipment } i \text{ is assigned to location } k \\ 0 & \text{otherwise} \end{cases}$$

We also define a binary decision expression denoting if at least one location in a group contains an article

$$z_l \to \begin{cases} 1 & \text{if } \sum_i^m \sum_{k \in G_l} x_{ik} \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

We can finally write the objective

$$\min \delta * \sum_l^g z_l + \gamma * \sum_i^m \sum_k^n \sum_j^m \sum_h^n x_{ik} * x_{jh} * w_{ij} * d_{kh} \qquad \text{(CGQAP)}$$

subject to

$$\sum_i^m x_{ik} \leq 1 \qquad \forall k \in N \qquad (1)$$

$$\sum_k^n x_{ik} * c_k \geq r_i \qquad \forall i \in M \qquad (2)$$

$$x_{ik} x_{ih} \leq a(k, h) \qquad \forall i \in M,\ k, h \in N \qquad (3)$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in M, k \in N \qquad (4)$$

$$\delta, \gamma \in \mathbb{R}^+$$

Constraint (1) ensures that each location is assigned only once. Constraint (2) ensures that the space requirements are met by the assigned locations. Constraint (3) ensures that the equipment is not assigned to locations in different groups.

2.1.1. Complexity

The formulation above is a generalization of the formulation of Koopman and Beckman (Koopman and Beckmann 1957) with further constraints. Assuming that one piece of equipment can be split into several pieces, we can prove that the CGQAP is NP-hard by reducing it to a QAP, which is known to be NP-hard (Sahni and Gonzalez 1976).

Theorem 1: The CGQAP is strongly NP-hard.

Proof: Assuming that equipment can be split into several pieces of the same equipment with lower demand, solving a CGQAP can be considered as solving a QAP, with additional constraints on clustered areas, for each possible way of splitting an equipment. Hence the CGQAP is at least as complex as the QAP which is known to be strongly NP-hard (Sahni and Gonzalez 1976). ■

Following the thought of the proof, a CGQAP becomes more complex as the storage capacity decreases since the equipment need to be split more often which results in solving more QAPs. The maximum number of QAPs to be solved is the number of multiples of the minimum storage unit within the e demand.

2.1.2. Linearization

Due to the high complexity of the QAP, there have been many linearization approaches (Xia and Yuan 2006) (Burkard, Çela, Pardalos, and Pitsoulis 1998) (Erdogan, 2006) (Punnen and Kabadi 2013). (Lee and Ma 2004) have conducted three approaches for the GQAP, namely following Frieze and Yadegar, Kaufman and Broeckx and a new linearization. In this paper, following (Lee and Ma 2004), we introduce Frieze and Yadegar linearization (Frieze and Yadegar 1983) and Kaufman and Broeckx (Kaufman and Broeckx 1978) linearization for the CGQAP.

Frieze and Yadegar

The products $x_{ik}x_{jh}$ of the binary variables are replaced by continuous variables $y_{ikjh} := x_{ik}x_{jh}$. Applying this approach to the CGQAP formulation we achieve the following mixed integer linear programming formulation.

$$\min \delta * \sum_l^g z_l + \gamma * \sum_i^m \sum_k^n \sum_j^m \sum_h^n y_{ikjh} * w_{ij} * d_{kh} \qquad \text{(FYL)}$$

$$\sum_{k=1}^N y_{ikjh} = x_{jh} \qquad \forall i, j \in M, h \in N \qquad (1a)$$

$$\sum_{h=1}^N y_{ikjh} = x_{ik} \qquad \forall i, j \in M\ k \in N \qquad (2a)$$

$$\sum_i^m x_{ik} \leq 1 \qquad \forall k \in N \qquad (3a)$$

$$\sum_k^n x_{ik} * c_k \geq r_i \qquad \forall i \in M \qquad (4a)$$

$$x_{ik} + x_{ih} - 2 * a(k, h) \leq 1 \quad \forall i \in M, k, h \in N \qquad (5a)$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in M, k \in N \qquad (6a)$$

$$y_{ikjh} \in [0; 1] \qquad \forall i, j \in M, k, h \in N \qquad (7a)$$

$$\delta, \gamma \in \mathbb{R}^+$$

The problem (FYL) has m^2n^2 continuous variables, mn binary variables and $n^2m + 2m^2n + mn + m + 3n$ constraints.

Theorem 2: The FYL is equivalent to the CGQAP.

Proof: We will only show one direction of the proof namely that a feasible solution of FYL is a feasible

solution to the CGQAP since the other direction is trivial to show.
Let (x, y) be a feasible solution to FYL. First we show:

$$x_{ik} = 0 \Rightarrow y_{ikjh} = 0, \exists i \in M, k \in N, \forall j \in M, h \in N$$

Considering (1a), we obtain:

$$\sum_{i=1}^{M} \sum_{k=1}^{N} y_{ikjh} = \sum_{i=1}^{M} x_{jh} = Mx_{jh}, \forall j \in M, h \in N$$

In case $x_{jh} = 0$ we have:

$$\sum_{i=1}^{M} \sum_{k=1}^{N} y_{ikjh} = 0 \Rightarrow y_{ikjh} = 0, \forall i \in M, k \in N.$$

Considering (2a), we obtain:

$$\sum_{j=1}^{M} \sum_{h=1}^{N} y_{ikjh} = \sum_{j=1}^{M} x_{ik} = Mx_{ik}, \forall i \in M, k \in N.$$

In case $x_{ik} = 0$ we have:

$$\sum_{j=1}^{M} \sum_{h=1}^{N} y_{ikjh} = 0 \Rightarrow y_{ikjh} = 0, \forall j \in M, h \in N$$

Further we show:

$$(x_{ik} = 1 \wedge x_{jh} = 1) \Rightarrow (y_{ikjh} = 1).$$

For this purpose we define a function ϕ, denoting the assignment of equipment to locations. In our case, it is an injective function (without proof).

$\phi: M \to N: \phi(i) = k$, such that $x_{i\phi(i)} = 1, \forall i \in M.$

We need to show:

$$y_{i\phi(i)j\phi(j)} = 1, \forall i, j \in M.$$

We write

$$\sum_{k=1}^{N} y_{ikjh} = y_{i\phi(i)jh}, \ \forall i, j \in M, h \in N.$$

We have

$$\sum_{i=1}^{M} \sum_{k=1}^{N} y_{ikjh} = \sum_{i=1}^{M} y_{i\phi(i)jh} = Mx_{jh}, \forall j \in M, h \in N$$

For $\phi(j) = h \Leftrightarrow x_{jh} = 1$, we achieve

$$\sum_{i=1}^{M} y_{i\phi(i)j\phi(j)} = M, \forall j \in M$$

Hence $(x_{ik} = 1 \wedge x_{jh} = 1) \Rightarrow (y_{ikjh} = 1)$. ■

Kaufman and Broeckx
The products $x_{ik}x_{jh}$ of the binary variables are replaced by continuous variables y_{ik} determining the product of relations and distances to all located equipment of a fixed located equipment. Applying this approach to the CGQAP formulation we achieve the following mixed integer linear programming formulation.

$$\min \delta * \sum_{l}^{g} z_l + \gamma * \sum_{i}^{m} \sum_{k}^{n} y_{ik} \quad \text{(KB)}$$

$$\sum_{i}^{m} x_{ik} \leq 1 \qquad \forall k \in N \quad \text{(1b)}$$

$$\sum_{k}^{n} x_{ik} * c_k \geq r_i \qquad \forall i \in M \quad \text{(2b)}$$

$$x_{ik} + x_{ih} - 2 * a(k, h) \leq 1 \ \forall i \in M, k, h \in N \quad \text{(3b)}$$

$$\sum_{j=1}^{M} \sum_{h=1}^{N} w_{ij} * d_{kh} * x_{jh} + v_{ik} * x_{ik} - y_{ik} \leq v_{ik}, \forall i \in M, k \in N \quad \text{(4b)}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in M, k \in N \quad \text{(5b)}$$

$$y_{ik} \geq 0 \ \ \forall i \in M, k \in N \quad \text{(6b)}$$

$$\delta, \gamma \in \mathbb{R}^+$$

We define $\forall i \in M, k \in N$:

$$y_{ik} := x_{ik} * \sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} * d_{kh} * x_{jh}$$

$$v_{ik} := \sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} d_{kh}$$

The problem (KB) has mn continuous variables, mn binary variables and $m + mn + 3n$ constraints.

Theorem 3: The KB is equivalent to the CGQAP.

Proof: We will only show one direction of the proof namely that a feasible solution (x, y) of KB is a feasible solution to CGQAP since the other direction is trivial to show.
Let (x, y) be a feasible solution to FYL. First we show that (4b) is satisfied for $x_{jh} = 0$ and/or $x_{ik} = 0$.

$$(x_{jh} = 0) \ \Rightarrow v_{ik} * (x_{ik} - 1) \leq y_{ik}, \forall j \in M, k \in N$$

$$(x_{ik} = 0) \Rightarrow -w_{ik} + \sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} * d_{kh} * x_{jh} \leq y_{ik}, \forall i \in M, k \in N$$

Obviously $v_{ik} * (x_{ik} - 1)$ and

$$-\sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} d_{kh} + \sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} * d_{kh} * x_{jh}$$

are both non-positive. Since $y_{ik} \geq 0$ and y_{ik} shall be minimized, y_{ik} must be zero.

Further we need to prove (7b) for all $i \in M$ and $k \in N$.
For this purpose we use the function ϕ, defined in 2.1.2: Frieze and Yadegar.
Since $w_{ik} * (x_{ik} - 1)$ is non-positive, we know:

$$y_{ik} = y_{i\phi(i)}$$
$$\geq (\sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} * d_{\phi(i)h} * x_{jh}) + v_{i\phi(i)} * (x_{i\phi(i)} - 1)$$

$$= \sum_{j=1}^{m} w_{ij} * d_{\phi(i)\phi(j)} * x_{j\phi(j)} + v_{i\phi(i)} * (x_{i\phi(i)} - 1)$$

$$= \sum_{j=1}^{m} w_{ij} * d_{\phi(i)\phi(j)}$$

In order to minimize the sum of y_{ik}, we can conclude:

$$y_{ik} = x_{ik} * \sum_{j=1}^{m} \sum_{h=1}^{n} w_{ij} * d_{kh} * x_{jh}, \forall i \in M, k \in N.$$

■

3. EXPERIMENTAL STUDIES

In this section computational studies on both linearization approaches and on the original problem are presented. The studies are conducted on a variety of GQAP problem instances (see 3.1). We draw a comparison of results achieved by CPLEX (IBM C. , 2015) regarding solution quality and runtime.
All tests were calculated on a laptop with an Intel(R) Core(TM) i7-4600U CPU @2.10GHz 2.70GHz processor.

3.1. Problem Instances

The problem instances used are described in (GitHub, 2015). However, they had to be adapted to this problem formulation due to the inversed assignment order (1:n) and the constraint on locations belonging to certain groups. The instances are turned around such that the original locations become equipment and the original equipment become locations and also distances and weights have been interchanged. The capacities of locations are varied in order to obtain instances of various complexity. The instances are named in the way "N-M-p" meaning that N equipment shall be assigned to M locations. The total storage requirement of equipment is p percentage of the total capacity of locations. Obviously the higher the percentage the more complex is the problem instance. The problem instances are published on the HeuristicLab website (HEAL, 2015).

3.2. Experimental Results

Due to the complexity of the problem we set a timelimit of 10800 seconds in CPLEX. The reason for choosing three hours is that preliminary tests show good results within that time.

Table 1 summarizes results regarding the best found solution and runtime to that solution. What can be seen is that the KB-formulation obtains the best solution quality for most instances. The FYL-linearization shows best results for the instances of lowest complexity (10-50-38, 15-35-45, 20-30-45) and also for all instances of the 10-50-class.

Note that two of the most complex instances (15-35-91, 20-30-91) could not be solved by the FYL-model within the timelimit (marked with an "x"), which in turn is due to the high number of variables and constraints. Here it is worth to mention that the original problem formulation could be solved to its best found solution within 257 seconds for one of those problems (15-35-91), which is way better than the solving time of the KB-model for that instance.

Having a closer look at the runtime until the best solution is found, again the KB-linearization performs best for most problem instances. This is a reasonable fact due to the little number of variables and constraints. Nevertheless, it is noticeable that the runtime to the best found solution of KB is significantly higher for the instances of highest complexity (10-50-77, 15-35-91, 20-30-91). This characteristic is not that distinct for FYL and the original formulation. However, in table 2 we take a look at the time until a first solution is found, where the KB-linearization is the most efficient one for each problem instance. At this point we can conclude that the KB-model is very efficient in finding a first solution which is also further improved during the solving process. It takes considerable time to find the optimal solution for those instances.

Table1: Best solution found within 10800 sec. and time to this solution

	Original		FYL		KB	
Instance	**Best Solution**	**Time to Best**	**Best Solution**	**Time to Best**	**Best Solution**	**Time to Best**
10-50-77	1,0037	6820	**1,0006**	10561	1,003	**10379**
10-50-51	0,669	7398	**0,6673**	1601	0,6681	**250**
10-50-38	0,6675	5202	**0,6671**	947	0,6674	**214**
15-35-91	1,0036	**257**	x	x	**1,0026**	10777
15-35-61	0,6678	6827	1,0013	3322	**0,6677**	**16**
15-35-45	0,6675	105	**0,6673**	3092	**0,6673**	**78**
20-30-91	1,0022	1611	x	x	**1,0019**	**1065**
20-30-61	1,0011	579	1,001	4403	**1,0009**	**60**
20-30-45	**1,0009**	4502	**1,0009**	10569	**1,0009**	**17**

Table 2: First solution found

	Original		FYL		KB	
Instance	**First Solution**	**Time to First**	**First Solution**	**Time to First**	**First Solution**	**Time to First**
10-50-77	1,0053	121	1,001	1200	**1,0038**	**16**
10-50-51	1,0024	143	**0,6673**	1601	0,6687	**12**
10-50-38	1,0013	66	**0,6671**	947	0,6676	**11**
15-35-91	1,2934	144	x	x	**1,0043**	**19**
15-35-61	1,0013	166	1,0013	3322	**0,6679**	**9**
15-35-45	0,6675	105	**0,6674**	1811	0,6675	**12**
20-30-91	1,2916	1144	x	x	**1,0023**	**62**

The FYL-formulation also obtains the best solution quality for some instances, the runtime to the first solution found is significantly longer though. This fact can also be seen in the performance of all three formulations for the problem instances with lowest complexity. Having a look at figure 1, it can be seen that the FYL-model has a long initialization time for each instance while the KB model reaches a first feasible solution very fast. However, it is worth to mention that no definite preference can be identified between the original model and FYL regarding the solution quality.

4. CONCLUSION

In this paper we define a new problem class namely the clustered generalized quadratic assignment problem (CGQAP). It is a generalization of the quadratic assignment problem (QAP) and the generalized quadratic assignment problem (GQAP). Based on the complexity of the QAP, which is known to be NP-hard, we show the NP-hardness of the CGQAP. Further we present two linearization approaches and prove its equivalence to the original problem. Computational studies are conducted regarding runtime and solution quality on adapted problem instances of the GQAP. The presented problem class has numerous applications whenever multiple equipment need to be assigned to several locations which again can be grouped into certain areas where also capacities and storage amount need to be taken into account. The problem is also very interesting from a theoretical point of view since it is based on the QAP which is one of the most challenging combinatorial optimization problems.

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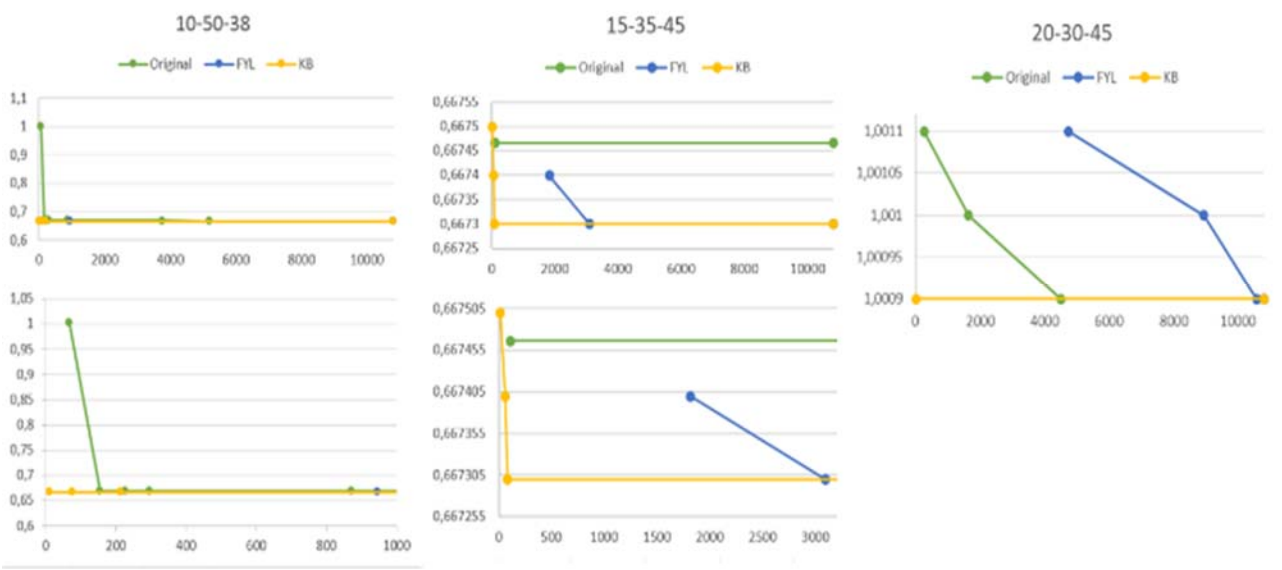
Figure 1: Performance of the original model, FYL and KB



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AUTHORS BIOGRAPHY

JUDITH FECHTER received her Master in Mathematics in 2014 from TU Vienna, Austria, and is a research associate at the Research Center Hagenberg. Her research interests include metaheuristic methods applied to simulation-based optimization problems.

ANDREAS BEHAM received his Master in computer science in 2007 from JKU Linz, Austria, and is a research associate at the Research Center Hagenberg. His research interests include metaheuristic methods applied to combinatorial and simulation-based problems. He is a member of the HeuristicLab architects team.

STEFAN WAGNER received his PhD in technical sciences in 2009 from the Johannes Kepler University Linz, Austria. He is a professor at the University of Applied Sciences Upper Austria, Campus Hagenberg. He is the project manager and head developer of the HeuristicLab optimization environment.

MICHAEL AFFENZELLER has published several papers, journal articles and books dealing with theoretical and practical aspects of evolutionary computation, genetic algorithms, and meta-heuristics in general. In 2001 he received his PhD in engineering sciences and in 2004 he received his habilitation in applied systems engineering, both from the Johannes Kepler University of Linz, Austria. Michael Affenzeller is professor at UAS, Campus Hagenberg.