DISYUNTIVE COLOURED PETRI NETS: A FORMALISM FOR IMPROVING THE APPLICABILITY OF CPN TO THE MODELING OF DES WITH ALTERNATIVE STRUCTURAL CONFIGURATIONS

Latorre-Biel JI[(a)], Pérez-Parte M[(b)], Jiménez-Macías E[(c)]

[(a,b)] Department of Mechanical Engineering. University of La Rioja. Logroño. Spain.
[(c)] Department of Electrical Engineering. University of La Rioja. Logroño. Spain.

[(a)] juan-ignacio.latorre@unirioja.es, [(b)] mercedes.perez@unirioja.es, [(c)] emilio.jimenez@unirioja.es

ABSTRACT

Colored Petri nets is a well-known formalism for constructing models of discrete event systems with subsystems presenting structural similarities. The folding of these common structures, described by means of ordinary or generalized Petri nets leads to compact and easy-to-understand models. The disjunctive colored Petri nets, can be considered as an extension of the colored Petri nets, making this formalism able to cope with the modeling of a discrete event system with alternative structural configurations. This modeling may be very useful for the task of designing a discrete event system, where some freedom degrees in the structure of the system in process of being designed lead to a set of alternative configurations for the system. This paper presents the disjuntive colored Petri nets, provides with some of their characteristics, as well as an algorithm for constructing models, and explains case study for illustrating its applicability.

Keywords: disjunctive colored petri nets, manufacturing facility, design, decision support system, discrete event system.

1. INTRODUCTION

Colored Petri nets is a well-known formalism, based on the paradigm of the Petri nets, (Silva 1993), for the development of models of discrete event systems with subsystems presenting structural similarities (Jensen and Kristensen 2009). The folding of these common structures, described by means of ordinary or generalized Petri nets leads to compact and easy-to-understand models (Macias and Perez, 2004). This is a main advantage of colored Petri nets, which make them very popular among practitioners (Xiao and Ming 2011), (Zaitsev and Shmeleva 2011), (Piera et al. 2004).

The folding process mentioned in the previous paragraph transfers the redundant information of repeated subsystems into attributes of the tokens. The values of the attributes of a given colored token allow knowing through which one of the original subsystems it belongs, despite in the colored model there is a single subsystem, equivalent to a set of them in the original model (David and Alla 2005).

Dealing with models of discrete event systems, such as those attainable by means of the formalism of the colored Petri nets, allow developing processes of structural analysis, and specially performance evaluation, simulation, and optimization for decision making (Mújica et al. 2010). Petri nets are not the only formalism able to cope with these tasks, as it can be seen in (Bruzzone and Longo 2010) and (Longo et al. 2013), however, they have been applied with success in many applications: generalized Petri nets (Latorre and Jiménez 2013a), (Latorre et al. 2013c), as well as colored Petri nets (Piera and Mušič 2011), (Zaitsev and Shmeleva 2011), (Piera et al. 2004).

The disjunctive colored Petri nets can be considered as an extension of the colored Petri nets, which make them able to cope with the modeling of a discrete event system with alternative structural configurations (Latorre et al. 2014a), (Latorre et al. 2010). This modeling may be very useful for the task of designing a discrete event system, where some freedom degrees in the structure of the system in process of being designed lead to a set of alternative configurations for the structural freedom degrees (Latorre et al. 2012). This formalism has been proven very useful for obtaining compact models for decision making (Latorre et al. 2014b)(Jimenez et al, 2014).

This paper presents the disjunctive colored Petri nets, provides with some of their characteristics, as well as an algorithm to construct a model of a discrete event system based on the mentioned formalism, and explains an example for illustrating its applicability.

2. DISJUNCTIVE COLORED PETRI NETS

There are several ways to define a disjunctive colored Petri nets. In all the possible definitions of disjunctive colored Petri nets there should be a mechanism to model a set of exclusive entities, modeling the associated set of alternative structural configurations for the modeled discrete event system.

Definition 1. Set of boolean choice colours.

$S_C = \{c_1, c_2, \ldots, c_n \mid c_i$ is boolean and $\exists!\ c_i =$ true, $i \in \mathbb{N}^*$, $1 \leq i \leq n \wedge c_j =$ false $\forall j \neq i, j \in \mathbb{N}^*$, $1 \leq j \leq n \}$, and the assignment $c_i =$ true is the result of a decision.

□

In CPM ML language, the set of boolean choice variables would lead to the following sentences.

First, the color set for a single boolean choice variable is defined:

```
colset CHOICE = bool;
```

The choice color set of an token is a n-tuple of so many boolean values as the cardinality of the set of boolean choice colors S_C.

```
colset DECISION = product CHOICE * CHOICE * … *
CHOICE;
```

Where the color set `choice` appears $|S_C|$ times.

Of course, there are other ways to represent a set of excusive entities by means of the color of a token. However, the one presented here is enough to illustrate the concept.

The following definition will deal with the definition of a set of exclusive entities, as a way to represent in a Petri net model, a set of alternative structural configurations for a discrete event system.

Definition 2. Monotypic set of exclusive entities.
Given a discrete event system D, a monotypic set of exclusive entities associated to D is a set $S_x = \{ X_1, \ldots, X_n \}$, which verifies that

i) The elements of S_x are exclusive, that is to say, only one of them can be chosen as a consequence of a decision.

ii) $\forall\ i, j \in \mathbb{N}^*$, $i \neq j$ and $1 \leq i, j \leq n$ it is verified that $X_i \neq X_j$.

iii) The elements of S_x are of the same type.

iv) $\exists$ f: $S_x \rightarrow S_R$ such that

iv.a) $S_R = \{ R_1, \ldots, R_n \}$ is a set of alternative Petri nets, feasible models of D.

iv.b) f is a bijection $\Rightarrow \forall\ X_i \in S_x\ \exists!\ \mathrm{f}(X_i) = R_i \in S_R$ such that R_i is a feasible model for D and $\forall\ R_i \in S_R\ \exists!\ \mathrm{f}^{-1}(R_i) = X_i \in S_x$.

□

In (Latorre *et al*. 2014b) it has been proven that the conditions of the definition of monotypic set of exclusive entities are verified for any set of Boolean choice variables.

Definition 3. Monochrome choice marking.
Let $R = \langle N, \mathbf{m}_0 \rangle$ be a colored Petri net system.
Let us consider a feasible marking $\mathbf{m}$ of R, reached from the initial marking $\mathbf{m}_0$ when the sequence of transitions $\sigma(R)$ is fired.
Let S_C be a set of boolean choice colors such that $|S_C| = n$.

If every token of $\mathbf{m}$ verifies that $\forall\ c_i \in S_C$, c_i is constant, then the marking $\mathbf{m}$ of the Petri net system R is said to be a monochrome choice marking.

□

It might happen that in a certain application, a colored Petri net contains other colors, different from the choice colors. In that case, only the choice color has to be monochrome.

Furthermore, **definition 3** refers to a monochrome marking presenting constant color at a certain stage in the evolution of the Petri net. Nevertheless, there is not any restriction to the possible change of color when a transition is fired. This constraint is included in **definition 4**, which deals with the concept of a disjunctive colored Petri net. The choice marking should be constant for every marking since it will be associated to a certain decision and a decision constraints the complete evolution of a certain system.

Definition 4. Disjunctive colored Petri net
A disjunctive colored Petri net $R = \langle N, \mathbf{m}_0 \rangle$ is a twelve-tuple

$CPN = \langle P, T, F, \mathbf{m}_0, \Sigma, V,$ c, g, e, i, S_α , $S_{val\alpha} \rangle$, where:

1. P is a finite set of places.
2. T is a finite set of transitions T such that $P \cap T = \emptyset$.
3. $F \subseteq P \times T \cup T \times P$ is a set of directed arcs.
4. $\mathbf{m}_0$ is the initial marking that is a monochrome choice marking.
5. Σ is a finite set of non-empty color sets, such that verifies one of the following two conditions:
   5.a. $\exists\ S_C$ set of boolean choice variables such that $S_C \in \Sigma$.
   5.b. $\exists\ (c, C)$ a natural choice color such that $C \in \Sigma$.
6. V is a finite set of typed variables such that type[v] $\in \Sigma$ for all variables $v \in V$.
7. c : $P \rightarrow \Sigma$ is a color set function that assigns a color set to each place.
8. g : $T \rightarrow EXPR_V$ is a guard function that assigns a guard to each transition t such that type[g(t)] = Boolean.
9. e : $F \rightarrow EXPR_V$ is an arc expression function that assigns an arc expression to each arc a such that type[e(a)] = c(p)$_{MS}$, where p is the place connected to the arc a.
10. S_α is a set of undefined parameters.
11. $S_{val\alpha}$ is a set of feasible combination of values for the undefined parameters.

And it is verified that $\forall$ $\mathbf{m} \in \mathrm{rs}(N, \mathbf{m}_0)$, $\mathbf{m}$ is a monochrome choice marking and every token of $\mathbf{m}$ verifies that c is constant and $\forall$ $c_i \in S_C$, c_i is constant.

□

In the following section, it will be described an algorithm to construct a disjunctive colored Petri net model of a discrete event system with alternative structural configurations.

3. MODELING ALGORITHM

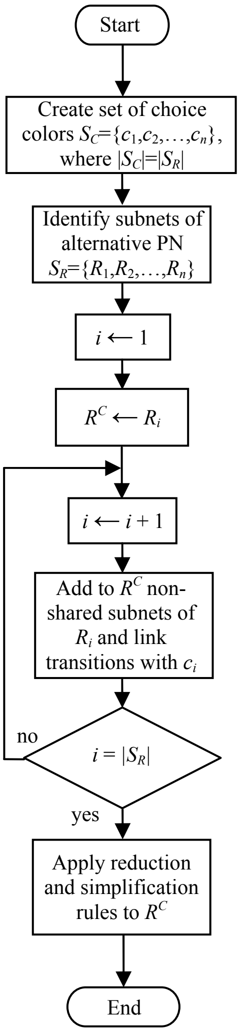

Figure 1: Algorithm for constructing a disjunctive colored Petri net from a set of alternative Petri nets.

In order to produce a disjunctive colored Petri net model for a discrete event system with alternative structural configurations, it is very convenient the use of a systematic methodology. The following algorithm is proposed for the case of a DES with alternative structural configurations, where every configuration is modeled by means of a low-level Petri net (non-colored) and the resulting set of alternative Petri nets presents a certain number of shared subnets.

In the aforementioned algorithm, an iterative procedure is described for constructing a single disjunctive colored Petri net.

Let us consider, $S_R = \{R_1, R_2, \ldots, R_n\}$, a set of n alternative Petri nets, where $n \in \mathrm{N}$, and N is the set of natural numbers, and let us create a set of choice colors $S_C = \{c_1, c_2, \ldots, c_n\}$, such as $|\ S_C\ | = |\ S_R\ |$. As a consequence of having the same cardinality both sets, it is possible to define a bijection between them and, hence, to associate one and only one choice color from S_C to every alternative Petri net from S_R. As a general rule it will be associated c_i to R_i, where $1 \le i \le n = |\ S_R\ | = |\ S_C\ |$.

The first of the stages consists of decomposing the different alternative Petri nets into subnets and link transitions. A fast and usually effective way of coping with this task consists in considering as subnets the Petri net models of physical subsystems present in the manufacturing facility.

As a second stage, it is envisaged assigning the first alternative Petri net R_1 to the disjunctive colored Petri net R^C. In this process, the link transitions are associated to a guard function that consists of the choice variable corresponding to the first alternative Petri net. Furthermore, the initial marking, conditioned by the mentioned choice color, will be composed exclusively by tokens of this choice color.

The following step in building up the disjunctive colored Petri net will be adding the subnets of the alternative Petri net R_2 not contained by R_1, also called subnets not shared by R_2. Afterwards, the link transitions, with guard functions consisting of the choice color associated to R_2 should be added to R^C.
This last step should be repeated so many times as alternative Petri nets have not been considered yet, in fact $|S_R|$-2.

The resulting Petri bet will be a disjunctive colored Petri net. The appellative “colored” is due to the fact that the tokens may have attributes or colors and the adjective “disjunctive” is explained because the set of colors includes a subset of choice colors, which is a set of exclusive entities.

In the following section an example of application of this algorithm will be applied to the process of design of a manufacturing facility.

4. EXAMPLE OF APPLICATION

In the case study described in this section, it will be illustrated how the modeling process of a discrete event system with alternative structural configurations can be developed by the use of a disjunctive colored Petri net.

The system that will be considered is a manufacturing line in process of being designed, which will be composed of three stages: the raw materials supply, the machining of the semifinished parts, and the assembly and packing of the resulting products.

The raw materials reception system has already been chosen by the decision makers involved in the design process and will be called subsystem “A”. Moreover, the machining process can be implemented by means of two alternative subsystems, offered by two different suppliers, which will be called “B” and “C” respectively. In the same way, the assembly and packing cell can be built up by means of other two alternative subsystems, called “D” and “E”.

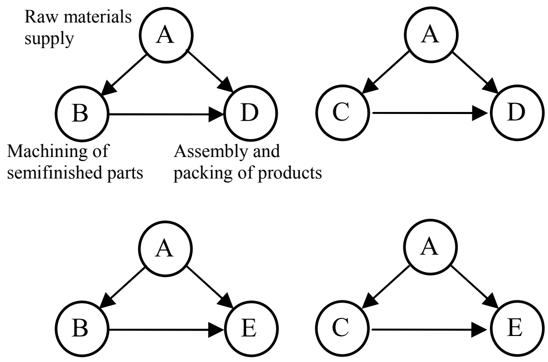

Figure 2: Alternative Configurations for the DES in Process of Being Designed.

The design of the resulting manufacturing facility will require choosing a single solution from the pool of four alternative systems obtained by the different combinations of the alternative subsystems.

The mentioned four solutions have been represented in a simplified way in figure 2, where the different subsystems are depicted by labeled circles, while the arrows inform about material flow in the manufacturing process.

A natural modeling process of the resulting discrete event system consists of obtaining a Petri net for every one of the alternative structural configurations of the system. This resulting system, modeled in the form of a set of alternative Petri nets, may be inefficient for tasks such as performance evaluation and optimization, since there is usually redundant information that can be removed.

As a result of this modeling process, it is possible to obtain four different alternative Petri nets, represented in a simplified way by means of subnets, depicted by clouds, and link transitions between some of them.

The four alternative Petri net models, $S_R = \{R_1, R_2, R_3, R_4\}$, have been represented in the figure 3.

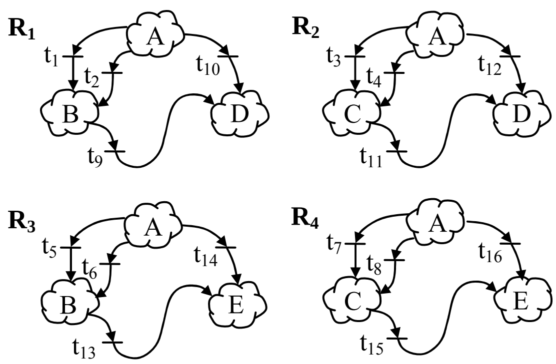

Figure 3: Simplified Representation of the Four Alternative Petri Nets decomposed into subnets and link transitions.

As it can be seen in figure 3, the subnets represented in the alternative Petri nets, which correspond to real subsystems in the DES in process of being designed, are shared by different nets. For example, subnet “A” is shared by the four alternative Petri nets, while subnet “B” is shared by R_1 and R_3.

For this reason, the set of four models represented in figure 3 include redundant information. This redundant information arises due to the subnets shared by several alternative Petri nets. This redundant information may reduce the computational performance of a decision making algorithm implemented to cope with the decision making in the design process of the manufacturing facility.

One way to remove the redundant information of the nets is by using CPN, where the attributes or colors of the tokens will avoid losing information when this removal is applied.

However, two considerations should be made before dealing with this modeling process. First of all, a conventional CPN is not appropriate for modeling a discrete event system with alternative structural configurations. For this use, a disjunctive colored Petri net is much more adequate, since in its definition it is included a subset of choice colors, which is a set of exclusive entities, as the set of alternative Petri nets is.

Secondly, a folding of shared subnets, while a certain number of them are present and also a given number of Petri nets are taken into account may be complicated without a clear and simple methodology.

This example will illustrate the application of a technique, described in the previous section, able to cope with this problem.

The starting point of the application of the algorithm for constructing a disjunctive colored Petri net from the original discrete event system is a decomposition of the alternative Petri nets into subnets and link transitions as it has been shown, in a simplified way, in figure 3. The criterion followed for achieving the mentioned decomposition is to consider as subnets, the models of the subsystems present in the manufacturing system: the raw materials supply, the machining centers, and the assembly and packing system.

The following step in the application of the algorithm is to consider, in a first iteration, the first alternative Petri net as the disjunctive colored Petri net that will be constructed. A guard function, which corresponds to the choice color related to the first alternative Petri net, c_1, is associated to the link transitions $\{t_1, t_2, t_9, t_{10}\}$ of R_1. In figure 4 it is possible to see the result.

As a second step in the construction of the disjunctive colored Petri net, as model of the discrete event system consists of including in R^c the subnets of R_2 that are not present in R^c.

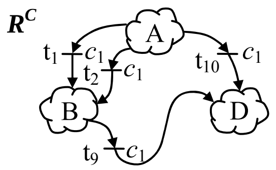

Figure 4: Simplified Representation of the disjunctive colored Petri net after the application of the first step of the algorithm.

At this early stage in the application of the construction algorithm is the same as saying that the subnets to be included in R^c should be present in R_2 but not in R_1. In particular, the subnets in which, by decision of the modeler, the alternative Petri net R_2 has been decomposed are {A, C, D}. It is a fact that {A, D} are shared by R_1 and R_2, however, {C} belong to R_2 but not to R_1; hence, it should be included in R^c as well as all the link transitions of R_2, which are {t_3, t_4, t_{11}, t_{12}}. In figure 5 it can be seen the result of the application of this step of the construction algorithm.

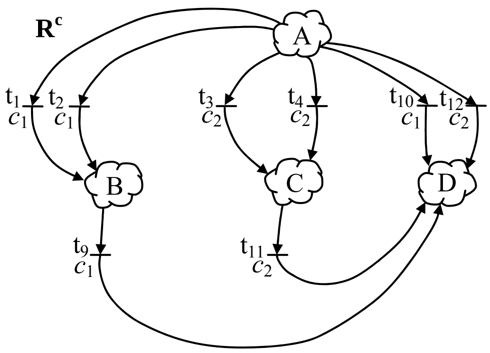

Figure 5: Simplified Representation of the disjunctive colored Petri net after the application of the second step of the algorithm.

The third step in the application of the algorithm consists in including in the disjunctive colored Petri net, the subnets of R_3 that do not belong to R^c so far, that is to say {E}. Moreover, all the link transitions of R_3 should also be included: {t_5, t_6, t_{13}, t_{14}}. The result of these operations can be found in figure 6.

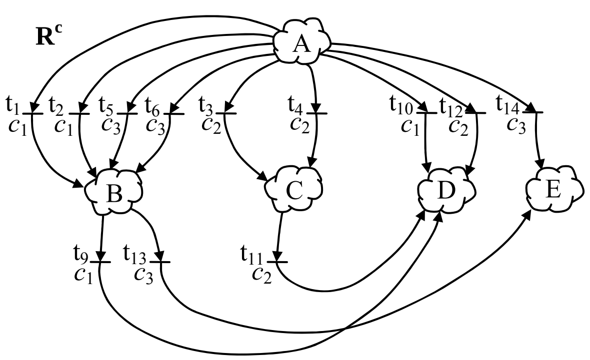

Figure 6: Simplified Representation of the disjunctive colored Petri net after the application of the third step of the algorithm.

As fourth step in the application of the algorithm, it has to be considered the alternative Petri net R_4. All the subnets in which R_4 has been decomposed already belong to R^c so far. For this reason, the application of this step only implies adding to R^c the link transitions of R_4; in other words, the transitions of R_4 that does not belong to any of the subnets in which this alternative Petri net has been decomposed.

The result of this fourth step of the algorithm has been represented in figure 7.

The last step in the application of the algorithm consists of simplifying the last model obtained from the development of the previous steps in order to try to limit the number of link transitions with the purpose of reduce the size of the model.

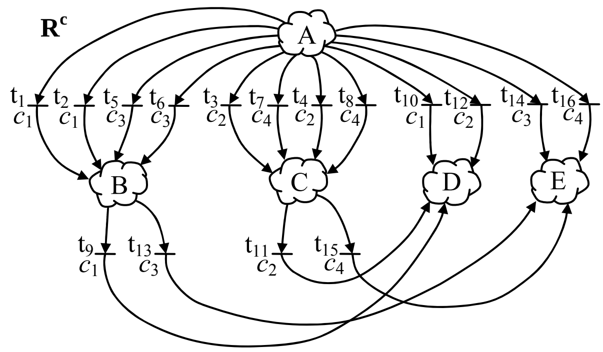

Figure 7: Simplified Representation of the disjunctive colored Petri net after the application of the fourth step of the algorithm.

Even though the details of every subnet of R^c are not shown in this paper, since they are not essential for illustrating the construction of a disjunctive colored Petri net, it is possible to state that in the example that has been considered the transitions {t_1, t_5} are quasi identical. This fact means that they present input and output arcs of the same weight from and to the same places. The only difference between quasi-identical transitions is the guard functions associated to them. In the case of t_1, the guard function is the choice color c_1, while in the case of t_5, the guard function is c_3.

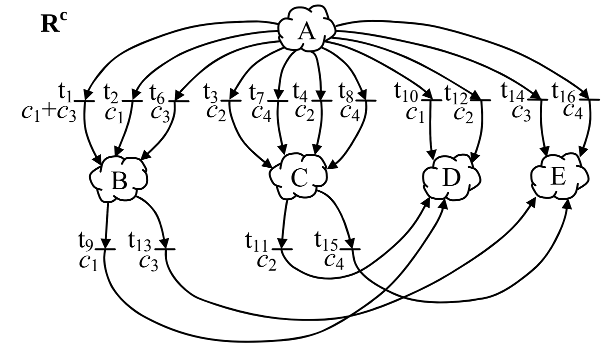

Figure 8: Simplified Representation of the disjunctive colored Petri net after the application of a reduction rule to the quasi-identical transitions {t_1, t_5}.

Two or more quasi-identical transitions can be reduced into a single one by creating a guard function that combines by means of the logic operator "or" the guard functions of the quasi-identical transitions. In the case of $\{t_1, t_5\}$, the new transition will be called t_1 and the associated guard function will be $c_1 + c_3$.

The result of the application of the reduction rule to the couple of quasi-identical transitions $\{t_1, t_5\}$ can be found in figure 8.

It is possible to continue applying the reduction rule mentioned in the previous paragraphs. In doing so, it is possible to identify the following couples of quasi-identical transitions: $\{t_2, t_6\}$, $\{t_3, t_7\}$, $\{t_4, t_8\}$, $\{t_1, t_5\}$, $\{t_{10}, t_{12}\}$, and $\{t_{14}, t_{16}\}$.

The result in the application of the reduction rule to the couples of quasi-identical transitions has been represented in figure 9. This resulting disjunctive colored Petri net can be compared with the one presented in figure 7, where it had not been applied any reduction rule yet.

Furthermore, it is also possible to compare the disjunctive colored Petri net of figure 9 with the set of four alternative Petri nets depicted in figure 3.

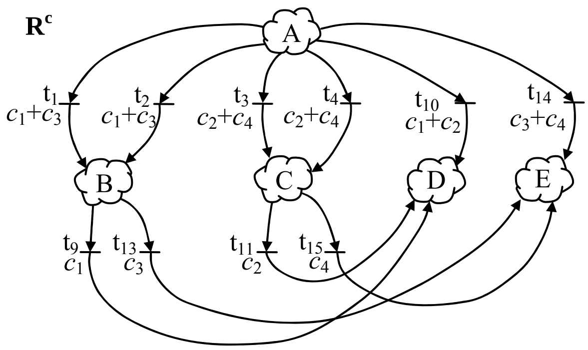

Figure 9: Simplified Representation of the disjunctive colored Petri net after the application of a reduction rule to all the quasi-identical transitions of R^c.

5. CONCLUSIONS

In this paper, an extension of the colored Petri nets, the disjunctive colored Petri nets have been introduced as a way to construct models of discrete event systems with alternative structural configurations.

Furthermore, an algorithm, describing the steps to be followed for constructing such model, is also detailed, as well as an application example, where a manufacturing facility in process of being designed has been modeled.

As a result, it can be said that the formalism of the disjunctive colored Petri nets is a very promising one for the description of discrete event systems with alternative structural configurations for diverse purposes, such as structural analysis, performance evaluation, or optimization, for example for the development of decision support systems.

As future lines of research, it can be envisaged the application of this formalism to a wider range of sectors and discrete event systems.

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