A MATHEMATICAL MODEL FOR DETERMINING TIMETABLES THAT MINIMIZES THE NUMBER OF STUDENTS WITH CONFLICTING SCHEDULES

Anibal Tavares de Azevedo[(a)], Alexander Kameyama[(b)], Joni A. Amorim[(c)] , Per M. Gustavsson[(d)]

[(a)]Universidade Estadual de Campinas – UNICAMP, Brazil
[(b)]Universidade Estadual Paulista – UNESP, Brazil
[(c)]Högskolan i Skövde – HiS, Sweden
[(d)]Saab Group, Sweden

[(a)]anibal.azevedo@fca.unicamp.br / atanibal@gmail.com, [(b)]alexkameyama@hotmail.com, [(c)]joni.amorim@his.se / joni.amorim@gmail.com, [(d)]per.m.gustavsson@saabgroup.com

ABSTRACT
With the increasing complexity of educational initiatives, several challenges arise as to appropriately allocate human and material resources or how to select alternative investments within a portfolio. Due to the complexity, solutions based on intuition are risky, which leads to a search for less intuitive and more reliable ways of solving educational problems. An engineering approach to the problem may lead to operations research mathematical modeling as a way to help on finding the solution for timetabling. In this case, a timetable is a schedule or, more precisely, a list or table of events arranged according to the time when they take place. This text features new and useful software to optimize the use of human resources. The software is based on a mathematical model for determining the timetable while minimizing the number of students with conflicting schedules.

Keywords: mathematical modeling, optimization, scheduling, timetable.

1. INTRODUCTION

Timetable is an event table used to specify who will participate, who will be held where and when such an event occurs. Generally, build an adequate timetable is not an easy task since some kind of constraints should be fulfilled in a manner there is no conflict in the schedule. That is it; Timetable is so hard that find a feasible solution is even difficult.

One category of Timetable problem is the Educational Timetabling. This problem can be classified in two sub-categories: exam and course timetabling (Al-Yakoob, Sherali, and Al-Jazzaf, 2010; Carter and Laporte, 1998).

To propose a course timetabling of a university is necessary to relate the different variables and interests related with students, teachers and classrooms. Some of examples of them are prerequisites established by the university for each discipline, individual preferences of teachers/students for certain disciplines to be taught/routed and, most often, the downtime between classes should be avoided. Added to this, there are risks of errors in the definition of the grids and these may be detected only when the classes have already begun (Al-Yakoob, Sherali, and Al-Jazzaf, 2010).

According to (Carter and Laporte, 1998) the course timetabling problem can be divided into five subproblems: teacher assignment, class-teacher timetabling, course scheduling, student scheduling and classroom assignment. The student scheduling problem only allocates students to courses without using the information about the courses allocation to time periods. Student scheduling problem often uses a given allocation of teachers and courses (Gunawan, Ng, and Poh, 2013).

This work will address the Problem of Assignment of Classes to Students (PACS) that combines student scheduling and courses allocation problem simultaneously within a university. A special feature had been considered in the model in order to consider student incompatibility to attend for more than one class. That is, as general purpose, students must attend the classes with the least possible incompatibilities and different from the one considered in recent literature (Al-Yakoob and Sherali, 2013).

PACS is part of the set of combinatorial optimization problems (Schaerf, 1999; Willenmen, 2002) which justify the development of heuristics and meta-heuristics. Another contribution of this work is to develop and apply an approach that enables solving of real large-scale problems.

This paper is structured as follows. Section 2 presents the mathematical model of PACS, while section 3 presents the proposed solution method and the developed software. Section 5 addresses conclusions and future work.

In this research report, in the section devoted to materials and methods, we present a mathematical model for determining the timetable while minimizing the number of students with conflicting schedules. This model is useful for determining the number of prospective students to be met for the subject matters of a university course that employs the system of credits.

Then, in the section with results, it is both presented and discussed software with features that aim to simplify the use of the method, especially when using an interface that allows a user without basis in advanced mathematics to enter the relevant information and perform the optimization of the use of resources. Finally, a section presents the main conclusions and suggests future work.

2. MATHEMATICAL MODEL

To determine the possible number of students to be served by the subjects of a college course that used the credit system, the mathematical model given below was developed.

Table 1: Materials in a course that employs the system of credits.

Number	Subject matter	Prerequisites
1	Calculus I	-
2	Analytic Geometry	-
3	Calculus II	Calculus I and Analytic Geometry
4	Calculus III	Calculus II

A constraint that must be met is to check whether a given student has the prerequisites required to perform a given subject matter. In other words, there are subject matters that can only be routed after verifying that the student has been approved in another one. For example, assume a course whose contents are described in Table 1 and whose prerequisites are given in Figure 1. Subject matter 3 can only be followed if the student has attended and successfully passed the subject matters 1 and 2.

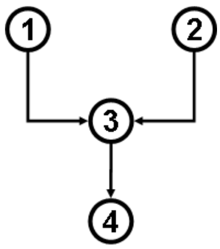

Figure 1: Indication of the relationship between prerequisites between subject matters of Table I.

From Figure 1 it is possible to determine whether a given student may or may not attend a subject matter based on the approval history. For example, if a student has been approved only in subject matter 1, then this one cannot attend the subject matters 3 or 4. The same statement can be made if the student has been approved only in the subject matter 2. It is important to note that Figure 1 is a graph where the nodes represent the materials and the relationship arcs of prerequisites between them. Figure 1 also can be used to show the evolution of the student during the course in each time period as given in Figure 2.

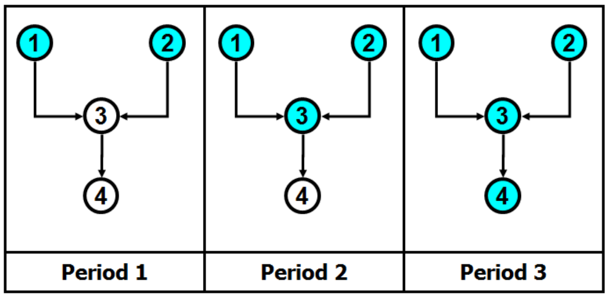

Figure 2: Illustration of a student's progress throughout the course considering data shown in Table 1.

Figure 2 illustrates the evolution of a student who has been approved in the subject matters 1 and 2, which allowed the same in Period 2 to attend the subject matter 3 and finally with the approval of the latter to attend subject matter 4 in period 3.

In order to facilitate the storage of information relating to the approval history of a variable number of students, y_{jkt}, a strategy that can be employed follows. The variable y_{jkt} is equal to 1 if the student *k* is approved in the subject matter *j* at the end of period *t*; y_{jkt} = 0, otherwise. The example of Figure 2 for one student corresponds to the values given in Figure 3.

	Subject matter 1	Subject matter 2	Subject matter 3	Subject matter 4
Period 1	1	1	0	0
	y_{111}	y_{211}	y_{311}	y_{411}
Period 2	1	1	1	0
	y_{112}	y_{212}	y_{312}	y_{412}
Period 3	1	1	1	1
	y_{113}	y_{213}	y_{313}	y_{413}

Figure 3: Representation of the evolution of a student, given in Figure 2, in the variable y_{jkt}.

From the variable y_{jkt} it is possible to express in mathematical terms whether a student may attend a subject matter based on his/her history of approvals and prerequisites between the subject matters. To do so, it must be also defined a new variable x_{ikt} indicating whether the student *k* in period *t* can attend the subject matter *i*. A matrix M_{ij} may be defined such that if M_{ij} = 1, then, the subject matter *j* is prerequisite for the subject matter *i*.

It is noticeable that the array of prerequisites is not only invariant with respect to the period, but that it is also supposedly unique. If there are students from different courses, then you need it is necessary to consider different matrices according to the courses of each student. The matrix M_{ij} of prerequisites associated with the example of Figure 2 is given in Figure 4.

	Subject matter 1	Subject matter 2	Subject matter 3	Subject matter 4
Subject matter 1	0	0	0	0
	$\mathbf{M_{11}}$	$\mathbf{M_{12}}$	$\mathbf{M_{13}}$	$\mathbf{M_{14}}$
Subject matter 2	0	0	0	0
	$\mathbf{M_{21}}$	$\mathbf{M_{22}}$	$\mathbf{M_{23}}$	$\mathbf{M_{24}}$
Subject matter 3	1	1	0	0
	$\mathbf{M_{31}}$	$\mathbf{M_{32}}$	$\mathbf{M_{33}}$	$\mathbf{M_{34}}$
Subject matter 4	0	0	1	0
	$\mathbf{M_{41}}$	$\mathbf{M_{42}}$	$\mathbf{M_{43}}$	$\mathbf{M_{44}}$

Figure 4: Matrix prerequisites associated with the example in Figure 2.

Notice that the third row of Figure 4 shows that, for the subject matter 3 to happen, it is necessary to have an approval in the subject matters 1 ($M_{31} = 1$) and 2 ($M_{32} = 1$). The value x_{ikt} can be obtained from y_{jkt} and M_{ij} by means of (1).

$$x_{ikt} = \prod_{j=1}^{J} 1 - \left| M_{ij} - y_{jkt} \right| \tag{1}$$

An additional constraint on the variable y_{jkt} is that this should be such that after a student *k* be approved in a matter *j* in period *t* this approval shall be considered in subsequent periods. This constraint can be represented by (2).

$$y_{jkt} \geq y_{jk(t+1)} \tag{2}$$

In addition to (1) and (2) it is necessary to consider, too, that every subject matter should be offered according to a given workload. Therefore, it is necessary to set a timetable such that the spaces in the grid, called slots, indicate whether in a given day and time a subject matter will be given. In order to facilitate the appropriate reference to these spaces in the grid hours, the slots are associated with a number pair *(r, c)*, where *r* indicates the time interval and *c* indicates the day as given in Figure 5.

	Monday	Tuesday	Wednesday	Thursday	Fryday	Saturday
07:30-08:20	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
08:20-09:10	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
09:30-10:20	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
10:20-12:10	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)

Figure 5: Numerical correspondence between the pair *(r, c)* and gaps (slots) of the timetable.

To represent that a particular slot *(r,c)* of the timetable is occupied by a subject matter *i* followed by student *k* in a period *t*, the variable $h_{ikt}(r,c)$ with a value of 1 indicates that it is occupied while the value of 0 indicates that the slot is empty. This new variable, however, must meet two constraints: (1) meet the workload of a subject matter, and (2) avoid the conflict between subject matters. The representation of these two constraints, in mathematical terms, is given below.

For constraint 1, which addresses the workload of a subject matter, we have that a particular subject matter *i* should answer a weekly workload CH_i. Mathematically, this is given by (3).

$$\sum_{r=1}^{R} \sum_{c=1}^{C} h_{ikt}(r,c) = CH_i \tag{3}$$

For the second constraint on the conflict zone between subject matters, we have a slot *(r, c)* occupied by a subject matter *i* followed by a student *k* in a period *t* that cannot be shared by other subject matter. Otherwise, there will be a conflict of time between subjects and the student must choose to take only one of them. In mathematical terms, this restriction is represented by (4).

$$\sum_{r=1}^{R} \sum_{c=1}^{C} \sum_{i=1}^{I} h_{ikt}(r,c) x_{ikt} \leq 1 \tag{4}$$

Finally, the objective function is such that it should minimize the number of students that despite the prerequisite to study a subject matter *i* cannot do so because the slots that such subject matter *i* occupies in the timetable are conflicting with one or more subject matters. A possible objective function is one such that it only counts the number of slots in which mismatch occurs between the *I* subject matters for all *K* students in all *T* periods as given by (5).

$$Min \sum_{k=1}^{K} \sum_{t=1}^{T} \sum_{r=1}^{R} \sum_{c=1}^{C} \sum_{i=1}^{I} \sum_{j=i+1}^{J} \begin{pmatrix} h_{ikt}(r,c) x_{ikt} \\ \bullet\, h_{jkt}(r,c) x_{jkt} \bullet M_j \end{pmatrix} \tag{5}$$

The complete mathematical model is given by (6). Note that the model given by (6) is a nonlinear, integer and stochastic problem. The stochasticity arises from the fact that it is not possible to know in advance the approval history of a student, ie, the value of y_{jkt}. To this end, there are two possible solutions: (a) assuming that the value y_{jkt} is known only to the period *t* and that the optimization process is performed only for the period immediately follows, or period *(t+1)*, and (b) assuming that the best estimate provided by a predictor or a process of random generation based on the history of previous students is employed.

$$\text{Min} \quad \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=i+1}^{J}\begin{pmatrix} h_{ikt}(r,c)x_{ikt} \\ \bullet\, h_{jkt}(r,c)x_{jkt} \\ \bullet\, M_j \end{pmatrix}$$

$$\text{S.a.:} \quad x_{ikt} = \prod_{j=1}^{J} 1 - \left| M_{ij} - y_{jkt} \right|$$

$$y_{jkt} \geq y_{jk(t+1)} \qquad (6)$$

$$\sum_{r=1}^{R}\sum_{c=1}^{C} h_{ikt}(r,c) = CH_i$$

$$\sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{i=1}^{I} h_{ikt}(r,c)x_{ikt} \leq 1$$

To solve this problem, in this work, we adopted the second hypothesis. Having the variable's value y_{jkt}, it is possible to determine the variable x_{ikt} by (1) and the problem can be reformulated as given by (7).

$$\text{Min} \quad \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{i=1}^{I}\sum_{j=i+1}^{J}\begin{pmatrix} h_{ikt}(r,c)x_{ikt} \\ \bullet\, h_{jkt}(r,c)x_{jkt} \\ \bullet\, M_j \end{pmatrix}$$

$$\text{S.a:} \quad \sum_{r=1}^{R}\sum_{c=1}^{C} h_{ikt}(r,c) = CH_i \qquad (7)$$

$$\sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{i=1}^{I} h_{ikt}(r,c)x_{ikt} \leq 1$$

Notice that the model (7) is a combinatorial problem whose complexity is to allocate schedules of materials in the timetable and requires exponential computational effort to find the optimal solution. Referring to the example of Figure 5, there are 8 time slots to be placed in one of 24 positions, namely 24^8, which is approximately 10^{11}, or almost one trillion possible solutions. The enumeration of all possible solutions with subsequent evaluation of the degree of incompatibility of each is not a suitable alternative to real problems in that the number of subjects and the timetable is greater.

To solve the model (7), it is proposed as an approach to apply heuristic methods in which there is no guarantee of obtaining the optimal solution. Despite this, good quality solutions can still be found in a suitable computational time.

However, even with the use of heuristics there is the problem of the amount of information needed to encode a given solution. Note that, for the case of Figure 7, it is necessary to employ H x C x T x I binary variables, or 4 x 6 x 2 x 1 = 48.

There is, however, an alternative in which the number of variables for each solution depends only on the number of periods T and, for Figure 5, it would result in employing only a single integer variable. This alternative is the representation of the solution by rules, which will not be detailed in this section. However, it employs two key concepts: (a) the representation of occupation timetable through a matrix and (b) this matrix modification may suffer depending on the application of a rule to fill the timetable. The most important aspect of the approach is that the set of subject matters provided for each period *i* is represented by a matrix B and that this array is filled to ensure obtaining a feasible timetable provided that all prerequisites are met. It is of interest to note that the proposed algorithm facilitates the application of heuristics in solving the model given by (7).

3. COMPUTATIONAL SYSTEM

Instead of direct solving the binary mathematical model given by Eq. (7), one option is to use a simulation procedure combined with rules of filling the slots of timetable in a unified framework. The complete procedure is as follows:

(1)Apply filling rules that can be based on some type of criteria like teacher assignment, or in some pedagogic feature.

(2)Evaluate how many students could not attend to a course, although he or she already the prerequisites. This may happen because two courses that a student could attend is sharing one or more slots in Timetable.

(3) Return to (1) and apply different rules.

This combination of simulation and rules framework is necessary to avoid increasing of computational burden for real large-scale problems and was successfully applied for other types of problems with more complex binary model for problems with matrix structure like Stowage Planning (Azevedo et al., 2012).

The developed program used this features that aim to simplify the use of solving the problem. This becomes possible by using an interface that allows a user without advanced mathematics knowledge to enter the relevant information and perform the optimization. Figure 6, below, shows the main screen of the software.

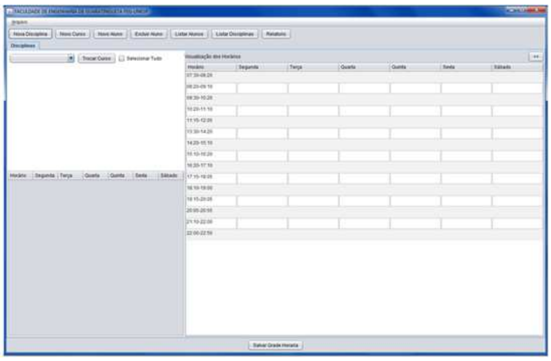

Figure 6: Software interface for determining the timetable.

The software was developed in Java (Deitel and Deitel, 2006). In the upper left corner of the window shown in Figure 6, on "File", we have the following: (a) "new", which allows you to open the window for registration of disciplines, opens the window for registration of students and allows you to open the window registration of each course; (b) "save", which lets you save the timetable of the selected course; and (c) "leave", which closes the program.

In Figure 7, the bar is highlighted with seven buttons perceived in Figure 6. An example is shown in Figure 7, with the window associated with a new discipline where the corresponding fields must be filled out with information such as the name of discipline, number of students, course load, course prerequisites.

Figure 7: Bar with buttons and window associated with the registration of a new discipline.

The system takes care of managing if a student has the prerequisites to attend a particular matter and presents a list of possible subjects that the student can attend next semester. This list is automatically generated for all registered students so that it is possible to evaluate each suggestion that, despite having the prerequisite to attend a course, cannot do it, because it is in conflict at that schedule with another.

4. CONCLUSIONS AND FUTURE WORKS

It can be concluded that the OR has great potential for application to problems of administrative nature, including those related to initiatives in education. However, mathematical modeling often inhibits the use of tools and optimization techniques given the need for deeper knowledge of the procedures and algorithms.

In this perspective, this research report presents software with various features that allows, through a direct understanding interface, perform the optimization of resource use, in this case having been focused on determining timetable minimizing the number of students with scheduling conflict. The determination timetable is a significant problem for different types of institution, but especially for those responsible for larger initiatives involving a larger number of students.

Future work will involve the survey and analysis of data on the use of the software here presented in real situations, with a view to improving procedures and algorithms in use. Since the algorithm developed to determine the timetable facilitates the application of heuristics through rules and simulation instead of direct solving the binary model explained by equations. Further investigations may also involve the exploitation of the potential of Beam Search (Della Croce and T'kindt, 2002; Ribeiro and Azevedo, 2009; Valente and Alves, 2005) for search the better combination of rules application, among other types of meta-heuristics combined with rules application with simulation.

ACKNOWLEDGMENTS

The authors would like to thank the following organizations for their support: Saab Group (www.saabgroup.com), University of Skövde (www.his.se), University of São Paulo (www.usp.br), University of Campinas (www.unicamp.br) and National Counsel of Technological and Scientific Development (www.cnpq.br).

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AUTHORS BIOGRAPHY

Anibal Tavares de Azevedo. PhD in Engineering. Dr. Azevedo teaches at Universidade Estadual de Campinas – UNICAMP (http://www.unicamp.br/), in Brazil. Previously, he worked as a researcher and as a teacher at Universidade Estadual Paulista – UNESP (http://www.unesp.br/), Brazil. Dr. Azevedo graduated in Applied and Computational Mathematics from UNICAMP (1999), holds a Master’s degree in Electrical Engineering from UNICAMP (2002) and received his Ph.D. degree in Electrical Engineering from UNICAMP (2006). He has experience in software development and in mathematical modeling for Production Engineering and Planning, for Scheduling of Power System Operation and for Education. His research has an emphasis on Linear Programming, Nonlinear Programming and Mixed Dynamics in the following topics: interior point methods, planning and production control of manufacturing, flows in networks, linear programming, graph generalized combinatorial optimization, allocation of cells to central offices, loading and unloading of containers on ships 2D and 3D, genetic algorithms, beam search and simulated annealing. < http://lattes.cnpq.br/9760457138748737 >.

Alexander Kameyama. Engineer. Previously worked as a researcher at Universidade Estadual Paulista – UNESP (http://www.unesp.br/), Brazil.

Joni A. Amorim. PhD in Engineering. Postdoctoral Fellow at the University of Skövde, or Högskolan i Skövde – HiS (http://www.his.se/english/), in Sweden, in collaboration with SAAB Training and Simulation (http://www.saabgroup.com/en/training-and-simulation/). The University of Skövde offers first-class programs and competitive research, which attracts research scientists and students from all over the world. The University of Skövde is one of the most specialized universities in Sweden and its research is focused on the development and use of advanced information technology systems and models. Dr. Amorim previously worked as a researcher and as a teacher at Universidade Estadual de Campinas – UNICAMP (http://www.unicamp.br/), in Brazil. Dr. Amorim collaborates with researchers at UNICAMP, a university with more than 3,600 original scientific publications published in 2009, 78% of which in journals indexed in the ISI/Web of Science. Dr. Amorim collaborates with researchers at Universidade de São Paulo – USP (http://www.usp.br/), the major institution of higher learning and research in Brazil. His research has an emphasis on multimedia production management, project portfolio management, distance education and training based on serious games and simulations. < http://lattes.cnpq.br/3278489088705449 >.

Per M. Gustavsson. PhD in Computer Science. Dr. Gustavsson works as a Research Scientist at Saab Group (http://www.saabgroup.com/). Saab serves the global market with products, services and solutions ranging from military defence to civil security. Dr. Gustavsson also works at the Swedish National Defence College – SNDC (http://www.fhs.se/en/), in Sweden. At SNDC, research is carried out in diverse, but inter-related subject areas and subsequently disseminated to other interested sectors of society both nationally and internationally; the College trains and educates military and civilian personnel in leading positions, both nationally and internationally as part of the contribution to the management of crisis situations and security issues. < http://se.linkedin.com/in/pergu >.