SOLVING SMALL TSP ACCORDING TO THE PRINCIPLE OF MINIMUM ACTION

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ABSTRACT

To solve the well-known Travelling Salesman Problem (TSP), many solutions based on combinatorial optimization, heuristic and meta-heuristic have been proposed. However, in managing business processes, a few times we attend to the real time optimization of picking routes either inside a warehouse or within materials or waste recovery distribution systems. This study proposes a new algorithm which is based on the analogy between TSP and conduction heat transfer; in particular, the application of the principle of minimum action to the heat transfer of a flat plate, which is coincident with the physical domain, over which the TSP points stress, helps identifying the order sought. The algorithm has been implemented in an Excel® spreadsheet; the quality of solutions which have been found is midway between the nearest neighbor algorithm and a genetic one; data processing time appears suitable for logistic processes management.

Keywords: TSP, principle of minimum action, space filling curves, unsteady state conductivity heat transfer.

1. INTRODUCTION

Given a space and a set of points to visit, the Travelling Salesman Problem (TSP) consists in finding the shortest path that enables to visit only once all the points and to return to the starting point. The minimum path has a more general meaning that comprehends the path at the least cost.

Formally, the TSP can be described as the search for the minimum of the function that represents the length of the route described above when varying the sequence in which the points are visited:

$$min(F(N,\pi(i))) = \sum_{i=1}^{N} d_{i,\pi(i)} \qquad (0)$$

where the elements of the matrix di, j (i, j = 1 .. N) are the mutual distances between N points to be visited and π(i) is the permutation of the sequence in which points are visited.

The TSP has applications in many fields: material handling, order picking, vehicle routing related to materials distribution systems, both direct and reverse logistics (the latter case is of particular and current interest in the waste management); although the optimization of these activities should always take into account constraints that can greatly limit the search for the minimum of the cost function (0) (binding capacity), management or organizational improvement, often, can significantly lead to the reduction of such restrictions. Similarly job scheduling and machinery sequencing can be solved by using the TSP methods of solution. Further applications of the problem can be counted as part of communication networks, statistics, psychology and biostatistics.

The extraordinary proliferation of studies on the TSP and its applications in science led to several methods of solution. Table 1 tries to summarize the most popular:

Table 1: Main TSP methods of solution

Method	Algorithm
Linear programming	Cutting plane (Dantzig, Fulkerson, and Johnson 1954)
Linear and integer programming	Branch and bound (Land and Doig 1960)
Local research (tour construction)	Sweep (Gillet and Miller 1974) Nearest neighbor (Rosenkrantz, Stearns, Philip and Lewis 1977) Nearest insertion Farthest insertion (Rosenkrantz, Stearns, Philip and Lewis 1977)
Local research (tour improvement)	K-opt (Rego and Glover 2002; Croes 1958; Lin 1965) Lin-Kernighan (Lin and Kernighan 1973)
Meta-heuristics (local research)	Simulated Annealing (Kirkpatrick, Gelatt and Vecchi 1983) Termodynamical Approach (Cerny 1985) Tabu search (Glover 1989) Genetic Algorithms (Grefenstette, Gopal, Rosimaita, and Gucht 1985); Homaifar, Guan and Liepins 1993)
Meta-heuristics (multi-agents)	Ant colonies (Dorigo and Di Caro 1999, Dorigo, Maniezzo and Colorni 1996, Dorigo and Gambardella 1997)
Approximated (graph based)	Minimum Spanning Tree (Pizlo, Stefanov, Saalweachter, Li, Haxhimusa and Kropatsch 2005) Spacefilling Curves (Platzman and Bartholdi 1989)

Such methods, however, are usually dedicated to solve large dimension TSP or very complicated problems that we can consider as belonging to the project management knowledge; some methods of resolution approach the TSP with hundreds of thousands of points. On the other hand, the same methods are often more difficult to apply in the management of business processes; for example picking from storage or vehicle routing in local distribution usually show routes whit a smaller number of points to visit (N<100) but with a high frequency of evaluation.

A set of techniques that makes constraint programming a technique of choice for solving small (up to 30 nodes) traveling salesman problems has been presented in literature for TSPs transportation problems that either come from "real" transportation problems (e.g., with trucks) or from moving mechanical parts (Caseau and Laburthe 1997).

We can assert that TSP is so fascinating that, in some cases, has become a game and a challenge rather than solving a real problem. Indeed much research focuses on finding most suitable operators for applications or on solving large-scale problems. However, rarely research addresses the performance of different operators in small- or medium-scale problems. In addition, the differences between small- and medium-scale TSPs on suitable GA design are studied (Liu and Kroll 2012).

This paper proposes, therefore, to implement a new method to solve the TSP in order to make it flexible when a change of boundary conditions is requested (i.e. the geometric domain, the number of points of the routes, the types of routes) and easy to use in management of logistics processes.

In the first part of the paper a brief literature review is reported; then the proposed model is described; it is based on the principle of the minimum action which is applied to heat transfer in unsteady state conditions. In the second part of the paper the model is applied to solve the TSP having to visit randomly generated points in the range [10 ... 30]; the results are finally compared with those obtained by the application of algorithms which are, at least in perspective, easily implementable in the same simulating environment (nearest neighbor algorithm and a genetic algorithm).

2. METHODOLOGY

The first principle of thermodynamics is applied to an elementary control volume under the following assumptions:

- the medium is composed of a fixed solid whose thermo-physical properties aren't time dependent;
- changes in volume, due to changes in temperature, are negligible if compared to the same volume;
- internal heat sources, described by $\dot{q}$(x,y,z) function, represent the energy generated per unit of volume and time.

Given the limited variation in volume, mechanics work exchanged by the elementary volume is negligible and the change of the internal energy is equal to the heat which is exchanged with the nearest neighbors in the unit of time: dU = dQ.

So the internal energy variation is only a function of temperature and internal sources:

$$dU = (\rho c \frac{\partial T}{\partial t} + \dot{q}(x,y,z))dxdydzdt \quad (1)$$

As regards the heat exchange, it is assumed to be only conductive; so the balance of heat flows along each direction allows writing the equation (Fig.1):

$$dQ=(q_x-q_{x+dx})dydzdt+(q_y-q_{y+dy})dxdzdt+(q_z-q_{z+dz})dxdydt \quad (2)$$

$$q_x = - k \frac{\partial T}{\partial x} \quad (3)$$

$$q_{x+dx} = - [k \frac{\partial T}{\partial x} + \frac{\partial\left(k\frac{\partial y}{\partial x}\right)dx}{\partial x}] \quad (4)$$

where dx, dy and dz are the elementary volume sizes, ρ is the density of the material, c the specific heat, T is the temperature, k is the thermal conductivity and t is the time.

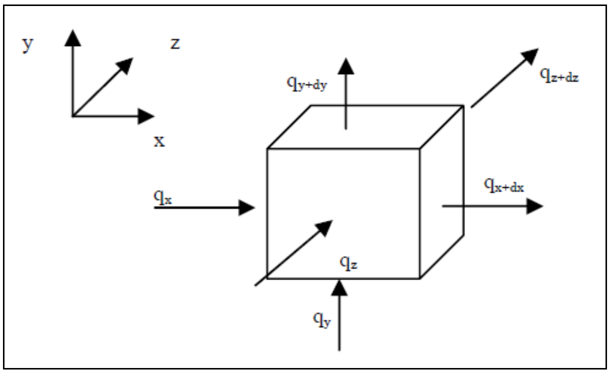

Figure 1: Balance of heat conduction flow inside the elementary volume

Assuming thermal conductivity k as a constant, the above mentioned heat balance, which is the application of the first principle of thermodynamics, allows deriving the general conduction equation (Ozisikin 1980):

$$a\nabla^2 T + \frac{q}{\rho c} = \frac{\partial T}{\partial t} \quad (5)$$

where: a = k/(ρc) is the so-called heat diffusivity of the material.

Let us consider, now, the thermodynamic system showed by figure 2; it is indefinitely along the z dimension, so it may be considered as a flat, square plate whose side measurers are both L.

Boundary conditions for the figure 2 system are now defined: temperature is fixed and equal to Ta along border edges; temperature is fixed and equal to T_{fix} for a given set S of points $P_i(x_i, y_i)$, which belong to that flat-square plate; the constancy of temperature at a point is equivalent to assume that heat exchanging from the same point to the outside world can happen with infinite intensity; finally if there are no internal heat sources, the general conduction equation applied to the thermodynamic system and its boundary conditions are the following:

$$\nabla^2 T = \frac{\partial T}{\partial t};\ \forall\, x, y$$
$$T(0,y) = T_a;$$
$$T(L,y) = T_a; \qquad (6)$$
$$T(x,0) = T_a;$$
$$T(x,L) = T_a;$$
$$T(P_i) = T_{fix}\ \forall P_i \in S.$$

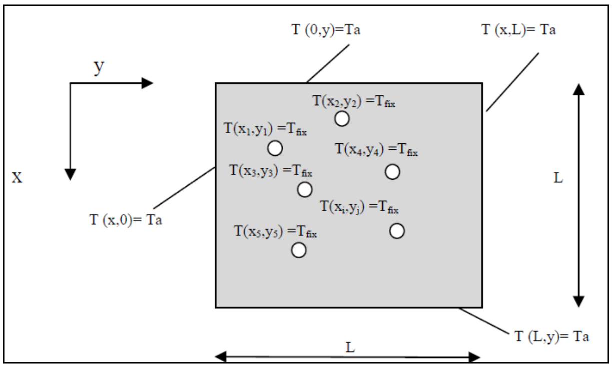

Figure 2: Boundary conditions of the flat-square plate system

If $T_{fix} \neq Ta$, the above thermodynamic system, after a thermal transient condition, reaches a state of thermal equilibrium; this occurs because heat flow which is transmitted through the border edges is virtually equal to that exchanged with the outside environment through points $Pi \in S$.

So the equation (5), after the thermal transient condition, turns into the Laplace equation.

The system of equations (6) can be numerically integrated by using the finite differences method.

The flat plate system can be discretized by using the finite element method; likewise heat conduction system of equation (6), which is composed of a partial differential equation, can be integrated by using finite differences formulae as below reported:

$$\frac{\partial T}{\partial t} \cong \frac{T_{i,j}^{k+1} - T_{i,j}^{k}}{\Delta t} \qquad (7)$$

$$\frac{\partial^2 T}{\partial x^2} \cong \frac{T_{i,j+1}^{k} - 2T_{i,j}^{k} + T_{i,j-1}^{k}}{\Delta x^2} \qquad (8)$$

$$\frac{\partial^2 T}{\partial y^2} \cong \frac{T_{i+1,j}^{k} - 2T_{i,j}^{k} + T_{i-1,j}^{k}}{\Delta y^2} \qquad (9)$$

Putting equations (7), (8) and (9) in (5), together with the $\Delta x = \Delta y$ condition, leads to the finite differences equation of conduction for the each finite control volume:

$$T_{i,j}^{k+1} = T_{i,j}^{k}\left(1 - 4a\frac{\Delta t}{\Delta x^2}\right) + a\frac{\Delta t}{\Delta x^2}\left(T_{i-1,j}^{k} + T_{i+1,j}^{k} + T_{i,j+1}^{k} + T_{i,j-1}^{k}\right) \qquad (10)$$

The equation (10) is recursive; the calculation process is stable if and only if it satisfies the following criterion (Ozisikin 1980):

$$0 \leq \left(1 - 4a\frac{\Delta t}{\Delta x^2}\right) \qquad (11)$$

The criterion (11) defines the maximum value to the time bucket which can be used to simulate the thermal transient condition once upon thermo-physic properties of the material are chosen.

Figure 3 shows the geometry of the finite element discretization in order to highlight the relevant measures.

The system of equations (6), numerically integrated by using the finite differences formulae, can be encoded within an Excel® spreadsheet.

Figure 4 shows the temperature distribution T(x, y) which the software application can perform for the thermodynamic system at the end of thermal transient condition; the mathematics model takes into account a set S that counts 10 points; the temperature of each Pi points is T_{fix}= 0 °C. The temperature of the border edges is T_a = 20 °C. The thermo-physical properties of the pseudo material are imposed to unit values ($\Delta x = \Delta y$=1 m; c=1 J/kg°C; ρ=1 kg/m3). The size of the flat plate was set to L = 20 Δx.

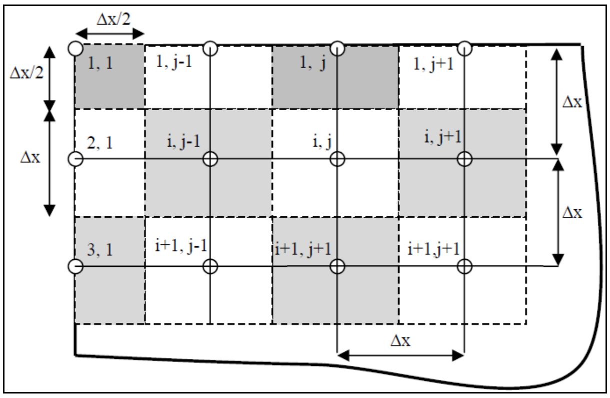

Figure 3: Finite element discretization of the flat plate

The system of Figure 2, is now seen under the light principle of the minimum action (Landau and Lifshitz, 1971). It guarantees that any dynamic or thermodynamic system evolves by minimizing a functional: we can call it energy (or action). The temperature distribution of figure 4 is the result of a thermal transition that leads to the configuration of minimum internal energy. The temperature distribution on the flat plate (the shape of the isotherm curves) shows how the heat is flows.

The latter thermodynamic system exchanges energy and organizes its temperature distribution as a function of its shape, of the temperature imposed along its border edges, of the one imposed in the points $P_i \in S$ and, in particular, of points allocation. The thermodynamic system must bring energy from its border edges to the points P_i which may dissipate outside this heat flow; this last process happens in a manner which adheres to the second principle of thermodynamics; the above mentioned flat plate has to solve a problem that is similar to the TSP. The analogy is therefore established among points to be visited and the sources of internal heat of the system and between the cost of the travelling salesman path and the internal energy level.

Let us consider now the isotherm curve T_{iso}=14 °C (Fig. 3); this is the first closed curve which encircles the set of points Pi; after projecting the points Pi on the isotherm curve T_{iso} the sequence in which they appear on the isotherm (Fig. 5) gives the solution of the problem. We can think about the isotherm T_{iso} = 14 °C as a spacefilling curve which is drawn by the thermodynamic system.

The solution to the TSP may, therefore, be obtained by calculating the steady state thermal condition of a flat plate whose sizes are that of the logistic domain of the TS problem, having a border edges at a constant temperature and heat sources placed on the points to be visited. Once the calculation of the thermal transient condition is performed, which has no computational difficulties, the problem turns in the research of T_{iso} curve and in the projection of points P_i on such curve. The existence of this isotherm is guaranteed by the nature of heat conduction (equation 6), which is in fact an equation of Laplace.

The system of thermal loads and the position of the points Pi in the simulated domain alter the temperature distribution at the thermal balance; so the choice of Tiso curve must be tuned in order to solve the problem.

This behavior seems a drawback of the novel methodology, which has, anyway, the possibility of increasing the number of points to be visited without having to modify the thermodynamic model, but only having to increase the above mentioned projection process according to a directly proportional law.

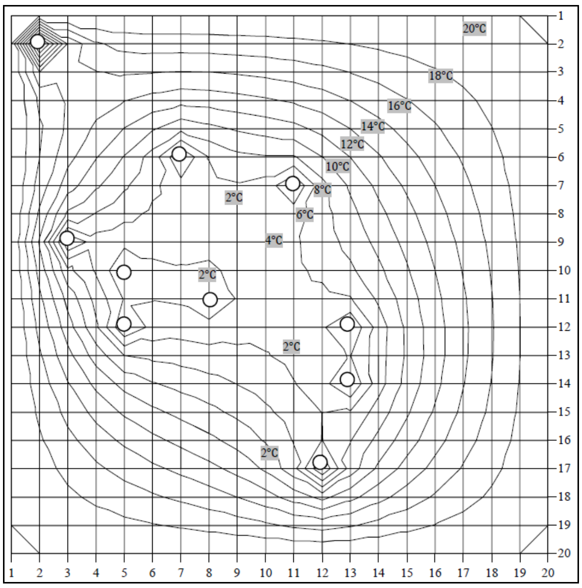

Figure 4: Temperature distribution on the flat plate. P1(2;2), P2(6;7), P3(7;11), P4(12;13), P5(14;13), P6(17;12), P7(11;8), P8(12;5), P9(10;5), P10(9;3) (tsim=120 s; Δt=0,2 s)

Figure 6 shows the end of the thermal transient condition which follows on from a different system of boundary conditions: internal heat sources are continuously placed in the flat plate ($\dot{q}$(x,y) = cost) and the points Pi are kept at fixed temperature Tfix. Also in this scenario, the N points Pi have to be regarded as a target of heat generation; the system reaches again a thermal equilibrium because of the equivalence between heat generated and dispersed. The equations of heat conduction once were integrated numerically by using the finite differences method.

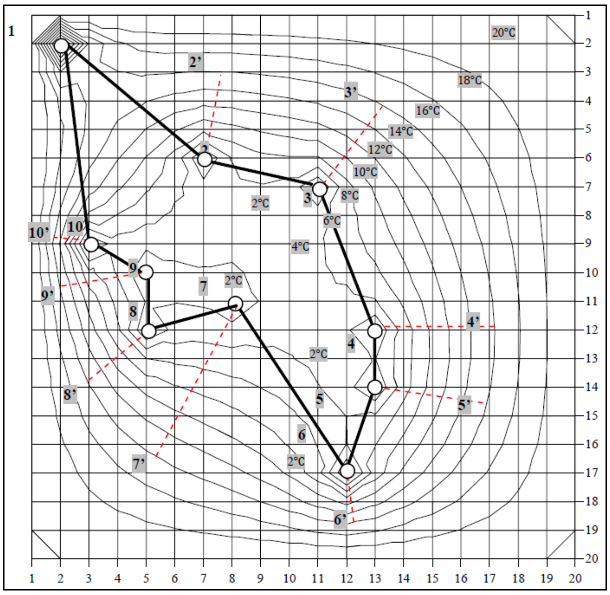

Figure 5: Pi projection process along isotherm curve Tiso=14°C

This time the T_{iso} isotherm curve encircles points Pi only if also the edge of the plate is taken into account; this also shows an approximation whose solution passes through an expansion of the domain in which the conduction thermal transient is calculated; under the above mentioned boundary conditions, the value of isotherm curve is T_{iso}=16 °C (Figure 6). The process of projection of the points P_i along the isotherm curve gives the same final result of the previous one.

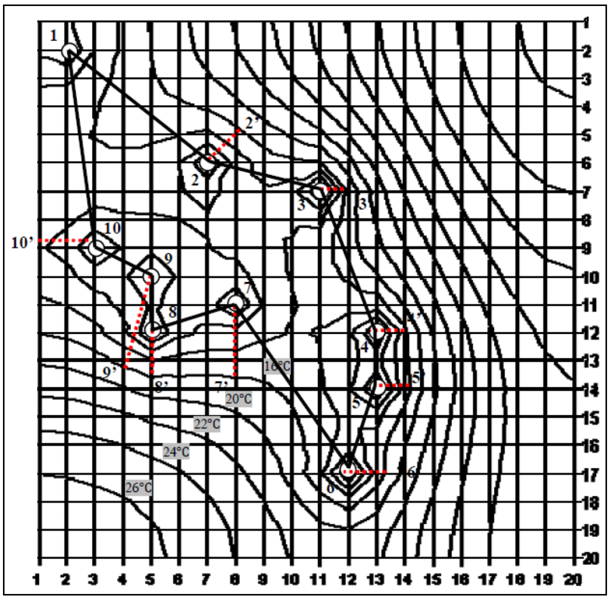

Figure 6: Flat plate temperature distribution with continuous heat source and Pi points projection along T_{iso}=16°C. P_1(2;2), P_2(6;7), P_3(7;11), P_4(12;13), P_5(14;13), P_6(17;12), P_7(11;8), P_8(12;5), P_9(10;5), P_{10}(9;3) (t_{sim} = 120 s; Δt = 0,2 s)

3. RESULTS

The thermodynamic model, previously showed, has been coded in an Excel® spreadsheet; the simulating model can be easily switched from one set of boundary conditions, such as fixed temperature on the border edges and at the points P_i, to another. The software environment allows the circular calculation that enables time driven simulating processes. Once calculated the temperature distribution at thermal equilibrium (a period of simulation t_{sim}=120 s was more than enough), the identification of the T_{iso} curve, on which to project the points of TSP, has been simply obtained by the following relationship: T_{iso} = 3/2 T_m; where T_m is the average temperature of the thermal field.

Although the latter relationship is rough and a more sophisticated check can be encoded in order to find the first closed isotherm around the P_i points to visit, the tests were in most cases fulfilled at the first iteration. The projection of the points P_i on the first closed isotherm consists in calculating for each P_i which is the nearest point belonging to the isotherm curve. To this aim it was codified a routine, by using standard Excel® function; it allows to split the thermal domain of figure 5 in a region warmer than T_{iso} and, consequently, the remaining one; once this partition is performed the elements (cells) of the border are serially numbered in order to establish the rule by which a point is before or after another. Table 2 shows the summary of tests performed for growing number of points to visit. The tests, as many as a hundred for each value of number N, were performed by choosing randomly N-1 points; the first point, that has coordinates P(2,2), has been imposed as a point of departure and arrival of the routes. The results have been compared with those obtained by a genetic algorithm and the nearest neighbor one. The quality of solutions found is midway between that of the solutions found by the two latter; the data processing time appears suitable for business process managing. The implemented algorithm seems open to many improvements particularly as regards the projection of the points P_i on the T_{iso} curve; its computational simplicity and the principle on which it is based ensures positive developments in the next research.

It has to be noted that the coding environment (Excel®) enjoys the favorable property WYSIWYG that allows easier modification when the boundary conditions change; there is evidence also that the same environment on one hand can be integrated with the interfaces of information systems, through barcode and RFID technology; on the other hand, it appears increasingly shared with powerful software applications such as Matlab®; all of the above features are considered of great value in order to support operations management (i.e. order picking; vehicle routing).

Table 2: Results summary (Δx=Δy=1m; L=20 m); the novel algorithm is called thermal space filling curve (Thermal SFC). (*Genetic algorithm is performed by Matlab®)

	Algorithm	N=10	N=20	N=30
TSP point density (N/L^2)		2,5%	5,0%	7,5%
Average value of specific tour lenght, Lm $=L_{tour}/N$	Nearest neighbor	6,03	3,01	3,29
	Thermal SFC	5,70	2,85	3,02
	Genetic	5,50	2,75	2,78
Standard deviation of specific length tour (Lm)	Nearest neighbor	0,70	0,35	0,36
	Thermal SFC	0,68	0,34	0,23
	Genetic	0,50	0,25	0,14
Data processing time (s)	Nearest neighbor	≈ 1	≈ 1	≈ 1
	Thermal SFC	≈ 20	≈ 25	≈ 30
	Genetic	≈ 15	≈ 30	≈ 360
Software size (kB)	Nearest neighbor	100	100	100
	Thermal SFC	500	500	500
	Genetic*	1,11	1,11	1,11

4. CONCLUSIONS

The TSP is one of the lines of research most frequently beaten and, despite the passage of time, it is yet very fascinating. The numerous methods of resolution of TSP are usually dedicated to solving the problem in a large scale and with the highest number of points to visit; the complication of these methodologies has the consequence of their difficult implementation in business processes.

A novel algorithm for solving the TSP of symmetric kind, therefore, it has been proposed; the algorithm is based on the analogy with conduction heat transfer and, in particular, it is the result of the application of the principle of minimum action to the thermal transient of a flat plate; this plate coincides with the logistic domain in which the TSP is defined; within this domain Pi points to visit have to be considered as sources of heat. When the thermal transient condition is elapsed, the resulting temperature distribution contains some closed isotherm curves which can be taken in to account as spacefilling curves; projecting P_i point along one of these isotherm enables to determine the solution sequence of the problem.

Tests carried out on routes with increasing number of point to visit [10, 20, 30] show a quality of solution which is a midway between that of the solutions found by genetic algorithm and by nearest neighbor one. Data processing time appears useful to business process managing (order/batch picking, vehicle routing, machinery sequencing).

The novel model has the advantage of being encoded in a widely distributed electronic environment (Excel®), with ergonomic features useful for the easy correction or amendment; it doesn't suffer, if not in a linear manner, the combinatorial complexity of the problem when the number of points to be visited increases.

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