QUEUING THEORY SIMULATION MODEL FOR CALCULATING NET PRESENT VALUE CORRECTIVE FACTOR IN INVESTMENT PROJECT APPRAISAL

Zoran Petrovic[a], Sladjana Benkovic[b] Ugljesa Bugaric [c], Dusan Petrovic[d] , Gordana Markovic Petrovic[e]

[a] Mokra Gora School of Management
[b] Faculty of Organizational Sciences, University in Belgrade
[c] Mechanical Faculty, University in Belgrade
[d] Mechanical Faculty, University in Belgrade
[e] Dom Zdravlja. Zemun

[a]zoran@tecon.rs,[b] benko@fon.bg.ac.rs, [c]ubugaric@mas.bg.ac.rs , [d] dpetrovic@mas.bg.ac.rs,[e] gm5rovic@gmail.com

ABSTRACT

In contemporary project management, project life cycle is defined by project development phases. Most important phase for project lifecycle is Opportunity phase in which project profitability is evaluated. On the end of this phase is determined if project will be developed in full life cycle, or rejected as non-profitable. Criteria that is used, for project evaluation, is Net Present Value criteria (NPV).

In order to propose methodology for getting more accurate results for NPV, system which is subject of investment is modeled as queuing theory model with balking and reneging. Input parameters of the system are collected from case study. Based on mentioned combined model, probability of service is calculated. In order to make conclusions more versatile, simulation model is build and validated against results from queuing theory model and case study results.

Probability of service, calculated from validated simulation model, is used as corrective factor for calculation of NPV, based on realistic assumption of serviced units, which are participating in income.

Keywords: Queuing theory, Net Present Value, Investment evaluation, Simulation

1. INTRODUCTION

Project life cycle in the industry is divided in several development phases (Newell and Grashina 2004). First and most important phase is feasibility phase in which project economic gain is evaluated. Based on results of this phase, decision is made – either to develop all project cycle phases, or to mark project as non-profitable, cancel it and find alternative one. If project passes initial phase, then it is possible to continue with all other general phases – Intermediate and Final phase (Figure 1).

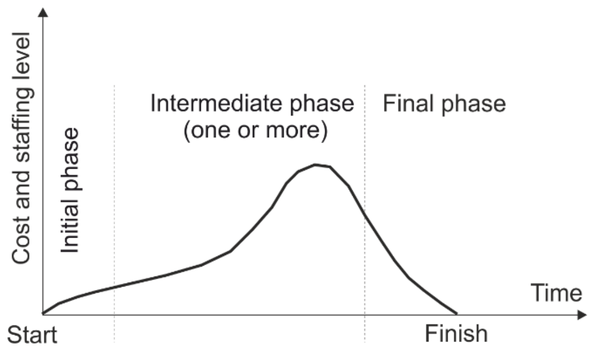

Figure 1: Project life cycle phases (Newell and Grashina 2004)

According to Đuričin and Lončar (Đuričin and Lončar 2012), there is even earlier stage in which project is evaluated – Opportunity phase. In this phase preliminary evaluation of the project is conducted and if project passes this evaluation, Feasibility phase is started. Once again, after finishing of feasibility study evaluation is made, contract is signed and project is officially started (Figure 2).

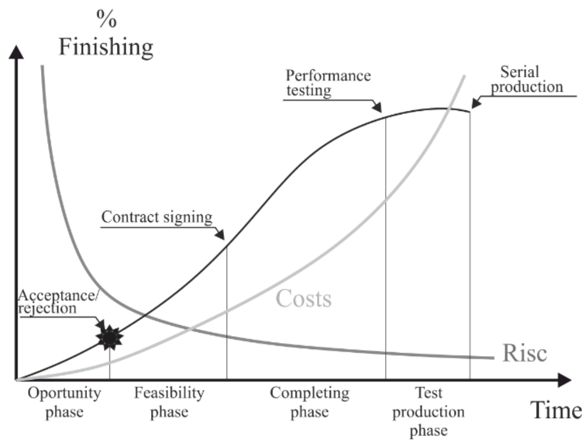

Figure 2: Movement of main parameters through project phases (Đuričin and Lončar 2012)

Regarding project evaluation, lot of different criteria can be used. Some of them are static (Return of Investment, Investment productivity, Employment ratio, etc.), some of them dynamic (Net Present Value, Internal Return Ratio, etc..). There are also lot of different criteria which takes uncertainty into consideration (Break even point analysis, Probability analysis, Game theory analyses, Monte Carlo simulation etc.). One of the most used criteria is Net Present Value (NPV) criteria, which will be also used in this paper.

1.1. Net Present Value criteria

Net Present Value is relatively simple criteria, which takes into consideration Net Income during investment period. In this paper is considered that there is only initial investment in the beginning of the project (I) and that investment period is equal to project life cycle duration (Newnan, D.G., Eschenbach, T.G., Lavelle, J.P., 2004).

$$NPV = \frac{NI_1}{(1+i)^1} + \frac{NI_2}{(1+i)^2} + \cdots + \frac{NI_n}{(1+i)^n} \quad (1)$$

$$NPV = \sum_{k=0}^{n} \frac{NI_k}{(1+i)^k} - I \quad (2)$$

Where:

NPV – criteria of Net Present Value

NI_k – Net Income (difference between Income and Cost) in evaluated period k

i – Discount factor

According to Net Present Criteria, investment is approved if NPV > 0.

NPV is chosen since it is very simple criteria which gives accurate preliminary investment evaluation results, satisfactory for opportunity phase analysis.

1.2. Disadvantages of NPV criteria

Problem with Net Present Value (NPV) criteria is anticipation of Net income in evaluated period. This is usually done by observation of Net Income of similar investments, which were already finished in the past. Such methodology is very risky, since it is very hard to find exactly the same equipment, in the same working surrounding, with same dependent and independent costs.

According to found information, there are two potentially risky scenarios.

First one is that Net Income is underestimated, so result of the NPV can be underestimated, also. This would lead to rejecting of potentially profitable project.

Second one in that Net income is overestimated, which would lead to accepting potentially non profitable project.

Idea of this paper is to suggest certain safety factor, which would be used as corrective factor for NPV Criteria, in order to get better estimation of Net Income.

2. METHODOLOGY

2.1. Queuing theory model

First step in the methodology is to represent potential investment in technological equipment as queuing theory model. Equipment is usually observed as part of internal logistic process (Pfohl 2010).

As it can be seen from Figure 3., potential investment is modelled as queuing theory model, which consist of units coming into the system, waiting line for units in front of the servers, and n-servers. After servicing, units are leaving the system. System is generally considered to be with infinite units arrival, without possibility for units to come back for servicing. Units are considered to be intelligent, so they can decide not to enter the system at all (balking), or they can decide to leave waiting line (reneging). This model is chosen, since it can be used in most cases when investment in production lines is considered. Units which are coming can be in form of: material, spare parts, sub-assemblies, assemblies, which are coming on production line. Waiting line can be considered as buffer before entering on production line and servers can be considered as production machines or lines. For various of reasons units can be rejected before entering system (for example quality control before entering into the system, etc.), which was modelled as balking. Also some units can leave waiting line if waiting in the line is taking too much time – for example in the zinc coating process, casting, etc. System state transitions are represented on Figure 4.

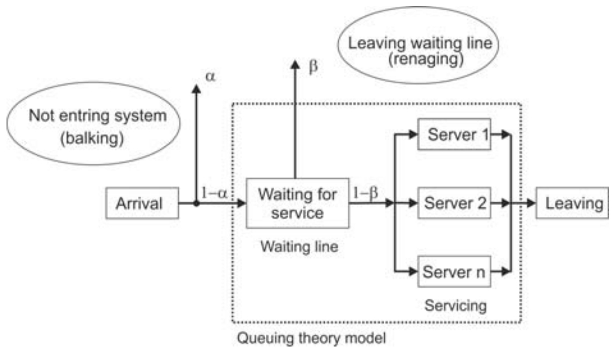

Figure 3: Queuing theory model with balking and reneging

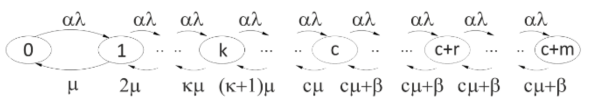

Figure 4: System state transitions

Probabilities of each state of the system are:

$$-\alpha \cdot \lambda \cdot p_0 + \mu \cdot p_1 = 0 \quad (3)$$

$$\alpha \cdot \lambda \cdot p_{k-1} - (\alpha \cdot \lambda + \mu) \cdot p_k + $$
$$+ (k+1)\mu \cdot p_{k+1} = 0 \quad (4)$$

for k= 1,2,…, (c - 1)

$$\alpha \cdot \lambda \cdot p_{c+r-1} - (\alpha \cdot \lambda + c\mu + \beta) \cdot p_{c+r} + (c\mu + \beta) \cdot$$
$$\cdot p_{c+r+1} = 0 \quad (5)$$

for r = 0,1,2,…, (m -1)

$$\alpha \cdot \lambda \cdot p_{c+m-1} - (c\mu + \beta) \cdot p_{c+m} = 0 \quad (6)$$

Solving system of equations by expressing state probability p_o, gives:

$$p_1 = \frac{\alpha \cdot \lambda}{\mu} \cdot p_0 \quad (7)$$

.

.

$$p_k = \frac{1}{k!} \cdot \left(\frac{\alpha \cdot \lambda}{\mu}\right)^k \cdot p_0, \; k = 1,2,..c \quad (8)$$

$$p_{c+r} = \left(\frac{1}{c \cdot \mu + \beta}\right)^r \cdot \frac{(\alpha \cdot \lambda)^{c+r}}{c! \cdot \mu^c} \cdot p_0 \quad (9)$$

$r = 1,2,,\ldots,m$

Sum of all state probabilities has to be equal one:

$$\sum_{k=0}^{c} p_k + \sum_{r=1}^{m} p_{c+r} = 1 \quad (10)$$

Probability p_o is calculated:

$$p_0 = \frac{1}{\sum_{k=0}^{c} \frac{1}{k!} \cdot \left(\frac{\alpha \cdot \lambda}{\mu}\right)^k + \sum_{r=1}^{m} \left(\frac{1}{c \cdot \mu + \beta}\right)^r \cdot \frac{(\alpha \cdot \lambda)^{c+r}}{c! \cdot \mu^c}} \quad (11)$$

Or after transformations:

$$p_0 = \frac{1}{\sum_{k=0}^{c} \frac{\left(\frac{\lambda \cdot \alpha}{\mu}\right)^k}{k!} + \frac{\left(\frac{\lambda \cdot \alpha}{\mu}\right)^c}{c!} \cdot \frac{\lambda \cdot \alpha}{c\mu + \beta} \cdot \frac{1 - \left(\frac{\lambda \cdot \alpha}{c\mu + \beta}\right)^m}{1 - \frac{\lambda \cdot \alpha}{c\mu + \beta}}} \quad (12)$$

All other probabilities can be expressed by using expressed probability po.

Probability of serviced with balking and reneging is:

$$P_{srv} = \sum_{i=0}^{c+m+1} p_i = 1 - p_{c+m} =$$
$$= 1 - \left(\frac{\alpha \cdot \lambda}{c\mu + \beta}\right)^m \cdot p_c \quad (13)$$

Or

$$P_{srv} = 1 - \left(\frac{\alpha \cdot \lambda}{c\mu + \beta}\right) \cdot \frac{1}{c!} \cdot \left(\frac{\alpha \cdot \lambda}{\mu}\right) \cdot p_0 \quad (14)$$

2.2. Case study

In order to validate theoretical queuing theory model, self service car was system was observed. System is consisted of 2 washing bays (servers) and 3 places in waiting line. Layout of observed system is presented on Figure 5.

Number of observed events was chosen based on research of Barlett et al. (Barlett, Kotrlik, Higgins, and Chadwick 2001). Mentioned authors determined minimum sample size, for given population size for categorical and continuous data. Based on their research, for analysis of one year (365 days – population size), for margin of error of 0.05, p= 0.50, t=1.96, sample should be 180. According to this fact, system was observed for 180 randomly chosen days, from opening to it's closing - 12 working hours.

During that time, following data were written in study protocol: exact time of first unit coming in the wide system aria, time between arrival of the next unit – until last one.

Number of units which entered wide system area, but didn't enter system itself, number of units left waiting line and finally time from starting of service, until end of the service and leaving system.

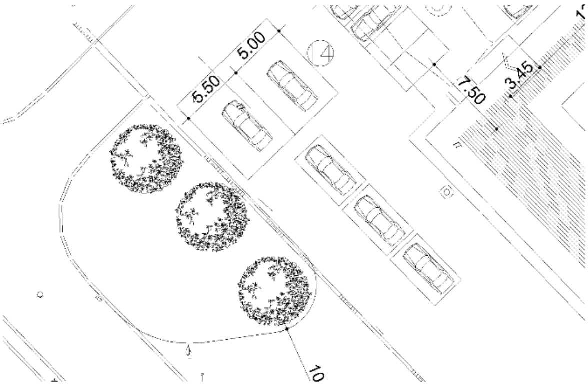

Figure 5: System layout

Based on mentioned data from protocol, following statistical analyses were done. First of all, mean time between units arrival was calculated for each day. Mean time of units servicing was calculated for each day. Both variables were tested by using Kolmogorov-Smirnov test for goodness of fit to exponential theoretical distribution, in order to confirm preliminary assumption, of Markov process of birth and death.

Results from statistical analysis are given in separate chapter - 3. Results.

If both mean time between arrival of the units in the system and mean time of servicing are exponentially distributed, then these data can be used as input values for queuing theory model and probability of servicing can be calculated, using theoretical model.

2.3. Simulation

Theoretical model of queuing theory can be used if assumption of Markov process is fulfilled, so probability of servicing can be analytically calculated. For some special cases, with some assumptions, calculation of probability of servicing is also possible, but for most of the cases, only way to calculate probability of servicing is by using simulation.

Simulation model for calculating probability of servicing was made in Mathlab Simulink, according to general methodology proposed by Mitroff et al. (Mitroff, Betz, Pondy and Sagasti 1974) and more detail methodology proposed by Lopatenok and Merkuryev (Lopatenok, Merkuryev 2000) .

In the model, units are generated according signal from random number generator. After being generated, units are coming to first junction, in which they can go in one of two direction based on the entrance signal. Entrance signal is probability of balking, which is entered as input data in the model. First direction is continue to the waiting line and second one is to the system exit through balking. If unit is leaving the system based on balking, it is noted as balked unit. Algorithm of simulation model is represented on Figure 6.

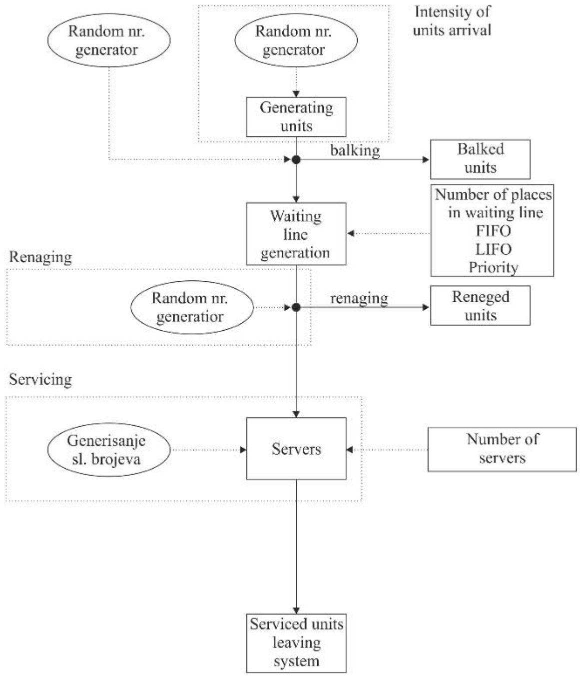

Figure 6: Algorithm of simulation model

Unit which continues goes to the waiting line. Discipline of waiting line can be FIFO, LIFO and Priority. In this paper only FIFO discipline was observed. From the waiting line unit is going into the second junction in which it can go again to the one of two potential directions. Signal which is determining in which direction will unit go, is probability of reneging. If unit goes to the reneging direction it goes out of the system and it is noted as reneging unit. If unit is not going in reneging direction, it goes to the servers. After finishing servicing, unit is leaving system and it is noted as serviced unit. Number of simulation events will be 180, according to research of Barlett et al.

2.4. Probability of service as corrective factor

Observing the system, general behavior of units was noted. Not all units that come in wide system area are entering the system. Also, not all units that enter into the system go to servicing – some of them leave waiting line and go out of the system, without service.

According to proposed methodology, probability of service can be used as corrective factor for calculation of Net Present Value criteria:

$$NPV^{re} = P_{srv}^{sim} \cdot NPV \tag{15}$$

Where:

NPV^{re} – Realistic NPV
P_{srv}^{sim} - Simulated probability of service

3. RESULTS

Based on proposed methodology, according to Kolmogorov – Smirnov test, for distribution fit, with value α =0,05, all samples have exponential distribution, with different mean value. On the Figure 7 – mean time between arrivals distribution is shown and on the Figure 8 – mean time of servicing.

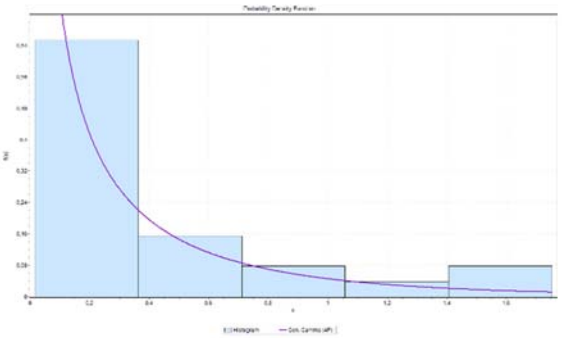

Figure 7: Mean time between arrivals

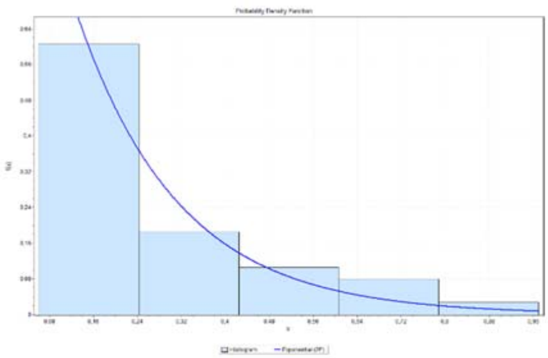

Figure 8: Mean time of servicing

Based on that fact, it is possible to conclude that assumption of Markov process, both in arrival and service is confirmed. From this conclusion, comes second conclusion that it is possible to use theoretical expression for probability of service, for observed case study.

From all gathered samples, mean time between arrival of the units along with mean time of service, were calculated.

Mean time between arrival of the units is $\bar{t}_d = 5{,}15$ min., which gives intensity of arrival of $\lambda = 11{,}65$u/h. Mean time of servicing is $\bar{t}_o = 5{,}05$ min., which gives intensity of service $\mu = 11{,}88$ u/h.

Also, it was noted from results in the protocol, that average balking probability is 10% and intensity of reneging was $0{,}1\mu$.

According to proposed model from queuing theory with g intensity of arrival of $\lambda = 11{,}65$ u/h and intensity of service $\mu = 11{,}88$ u/, gives that probability of service is Psrv = 0,999.

Based on this value, 180 simulating events were started. Average result for probability of servicing was: $P_{srv}^{sim} = 0{,}9892$.

In order to validate simulation model, methodology proposed by Bugarić and Petrović was used (Bugarić and Petrović 2011).

Table 1: Validation of simulation model

Value	**Psrv**
Theoretical	0,999
Simulation	0,989
Average deviation $\sigma = \sqrt{\frac{\sum_1^{br.sim}\left(Vel_{teor} - Vel_{exp}\right)^2}{br.sim}}$	0,0108
Estimated error $\frac{\sigma}{\sqrt{br.sim}}$	0,00108
Relative error $\left\lvert Vel_{teor} - Vel_{exp}\right\rvert$	0.01

4. DISCUSION

Proposed queuing model is used since it can be analytically calculated. Similar model was proposed by Whitt (Whitt 1999) in his study about informing customers about anticipated delays. Boots and Tijms (Boots and Tijms 1999.) were exploring general M/M/M/c queue with impatient customers, with similar model. Model which was proposed in this paper was chosen for two reasons.

First one is that lot of production lines can be very well described by it. Second one is that it gives good bases for validation of simulation model, since it can be solved analytically.

Based on results from combined theoretical queuing model and case study, probability of service was calculated. This value was compared to calculated value from simulation model and according to Table 1. , difference between those two values is not statistically important, meaning that simulation model is validated.

5. CONCLUSION

Methodology that was proposed in this paper helps to calculate Net Present Value of investment project more accurately, which gives certain safety factor in decision making. Simulation model takes into consideration only units that are being served, excluding ones that balked or reneged from system. This model can be used for any investment in production line that can be represented with proposed queuing model.

Benefit from validated simulation model is that it is not sensitive to distribution of time between arrival of the units, or servicing time. It can be used without need for starting assumptions of Markov processes of birth and death. Simulation model can also be used for analyzing worst case scenario of equipment capacity in different theoretical distribution of units arrival and servicing and bulk units arrival, or it can be used for optimization of system configuration based on existing or predicted parameters.

REFERENCES

Barlett, J.E., Kotrlik, J.W., Higgins, M., Chadwick C., 2001. Organizational research: Determining appropriate sample size in surway research. *Informational Technology, Learning and Performance Journal,* 19(1), 43-50.

Boots, N.K., Tijms, H., 1999. An M/M/M/c queue with impatient customers. *Sociedad de Estadistica e Invesigacion Operativa,* 7(2), 213-220.

Bugarić, U., Petrović, D., 2011. *Modeliranje sistema opsluživanja.* Beograd: Mašinski fakultet, Univerzitet u Beogradu.

Đuričin, N.D., Lončar, D.M., 2012. *Menadžment pomoću projekata.* Beograd: Centar za istraživačku delatnost Ekonomskog fakultata u Beogradu

Lopatenok, V., Merkuryev, V., 2000. Simulation and Analysis of Production Facility Operation. *IFAC Symposium on Manufacturing, Modeling, Management and Control*, pp 467-471. July 12-14, University of Patras (Patras, Greece)

Mitroff, I.I., Betz, F., Pondy, L.R., Sagasti, F., 1974. On managing science in the systems age: two schemas for the study of science as whole systems phenomenon. *Interfaces ,* 4(3), 46-58.

Newell, M.W., Grashina, M.N., 2004. *The Project Management Question and Answer Book.* New York: AMACOM.

Newnan, D.G., Eschenbach, T.G., Lavelle, J.P., 2004. *Engineering Economic Analysis.* 9th ed. London: Oxford Press.

Pfohl, H., 2010. *Logistiksysteme Betriebswirtschaftliche Grundlagen.* 8th ed. Heidelberg: Springer.

Whitt, W., 1999. Improving Service by Informing Customers About Anticipated Delays. *Management Science,* 45(2), 192-206.

AUTHORS BIOGRAPHY

Zoran Petrović finished Mechanical Faculty, University in Belgrade in 2003. In past ten years developed his career in different professional and scientific fields. He is working as executive manager in company Tecon Sistem from 2006. He finished his doctoral thesis on Mechanical Faculty in Belgrade in July 2013 and at the moment he is studying EMBA studies at Mokra Gora School of Management.

Slađana Benković has been an associate professor at the Faculty of Organizational Sciences, University in Belgrade for the last 15 years. During 2007/2009 she spent time at the George Washington University, Washington D.C. as a visiting professor. She is Deputy President of the Management Board of the “Endowment of Milivoje Jovanović and Luka Ćelović”, as well as a member of the Management Board of the “Endowment of Đoko Vlajković”. Her teaching and research fields are financial management with a research focus on project finance, modalities of financing development projects of companies, technical evaluation of investment profitability and determination of corporate capital structure.

Uglješa Bugarić is professor at the Mechanical Faculty, University in Belgrade and chairman of Industrial engineering department. His research is mainly oriented to logistics, operations research problems and optimization.

Dušan Petrović has been an associate professor at the Mechanical Faculty, University in Belgrade, since 1999. His research is mainly oriented to logistics and warehouse management problems.

Gordana Marković Petrović finished Medical Faculty, University in Belgrade and Master in Emergency Surgery and Management in Healthcare. At the moment she is working as executive manager in Dom Zdravlja, Zemun and finishing Master studies in traditional medicine at Medical Faculty, University in Belgrade.