ABSTRACT
Modeling of Flexible Manufacturing Systems has been one of the main research topics dealt with by researchers in the last decades. The modeling paradigm chosen can be in many cases a key decision that can improve or give an added value to the example modeling task. Here, the modeling tasks are represented by one of the mostly used one in the academia, the Petri Net paradigm, and in particular the Stochastic Petri Net models. These models constructed will be used to optimize the performance measures that could be interesting for the production systems. The production indicators used here are related with the productivity of the systems and its efficiency in the production. We have added here and extra element to the optimization problem that is related with the energy consumption during the productive process. This will add an extra value to the actual scenario, introducing energy efficiency terms and information into the models and into the optimization process. These productivity and energy consumption measures could be included into an optimization process by changing a certain number of parameters into the model. A two phase optimization process has been applied to the real example we have considered and an improvement of this two phase methodology has been applied, comparing these results with the original two phase method to check whether which approach is more appropriate. Previous results obtained for this systems are improved by this new piece of research.

Keywords: Stochastic Petri nets, flexible manufacturing, simulation, performance measures, energy consumption.

1. INTRODUCTION
Flexible Manufacturing Systems and their representation in an adequate model that expresses their behavior the more accurately is a typical topic treated by many researchers. Here, a comparison between two optimization approaches using a novel representation of the energy consumption process associated to the model is presented. The model representation is done through stochastic Petri nets.

Petri nets have shown their capacities to represent the behaviors that Flexible Manufacturing Systems pose, and specially concurrency and resources representation that are typical features of Manufacturing Systems. Stochastic Petri nets have been used largely to represent systems where an stochastic behavior is associated to tasks. This modeling method has some lacks when dealing with complex models where the state space is clearly untreatable and even simulation can be a great time consuming task. Here we will consider only simulation approaches due to this complexity previously mentioned.

Here, a particular optimization problem will be introduced. This particularity lies on the introduction of terms related to the energy consumption associated to the machining processes and the inclusion of these data into the optimization problem and in particular into the goal function. This will make that the energy consumption information will be considered as a new term of the optimization function.

In order to have the best possible results of this optimization process with a contained computational effort a couple of optimization approaches have been considered. The first one is a two phase optimization method where the second phase uses the solution obtained from the first one, while the second approach takes advantage of the information extracted from the solutions visited during the first phase to reduce the complexity of the problem we are considering here.

The rest of the paper is as follows, in section 2 the FMS that will be used along this paper will be explained and all the elements that will be of interest to be represented in our model will be enumerated. Later on, in sections 3 Petri net model will be depicted. In section 4, the optimization will be depicted and the approaches are extensively commented. Finally, the results we are
interested in are represented associated to the two approaches in section 5 where a comparison of the results is shown. Finally some conclusions are presented in section 6.

2. DESCRIPTION OF THE FMS
The Manufacturing system initially considered is able to perform window frames with the following different features:

**Feature 1**
The first feature to be considered when modeling the system is the type of window where the frame will be included:
- Accessible window,
- tilt and turn window,
- Slide window
- Frames without any other element.

**Feature 2**
This feature is related with the presence of a crosspiece that goes horizontally from one extreme to the other of the window frame.
- With crosspiece
- Without crosspiece

**Feature 3**
The number of leafs that compose the window is the next differentiation element.
- One leaf
- Two leafs

It was considered a third leaf in the initial modeling constraints but finally it was considered that the third leaf could be added as a future improvement of the manufacturing system.

**Feature 4**
The last feature is related with the size of the window that will change the treatment or steps that must be followed in case of considering one size or the other.
- Big size
- Little size

Considering all the features depicted here, there are finally 32 different types of products that the manufacturing cell will be able to produce. Apart from these types of windows, a set of accessories can be added to the different products. These accessories are:
- Box and Guide to include into this device the blind that can be integrated into the window.
- Drip edge to get all the water that can slide through the window

Initially we will describe the processes signing them in bold letters and describing who the operators that perform every process are:

- Operator 1 performs the selection of the materials needed to complete satisfactorily the aluminum profiles depending on the frame to be produced
- Operator 1 also supervises the cutting of the PVC profiles in the corresponding machine.
- Operator 1 performs the operation of introduction of the reinforcement
- Operator 1 supervises operations perform inside the Numerical Control Machine. These operations are 6 different operations that must be performed. We do not enter into more details about these operations because is not the objective of this thesis.
- Finally operator 1 checks the correctness of the reinforcement screwing operation.
- Operator 2 performs the following operations
  - Selection of Material for the reinforcement cutting
  - Supervises the reinforcements cutting in the corresponding machine
  - Distributes the reinforcements to the corresponding profile
  - Performs an extra operation that is the leaf cutting that corresponds to the inverse leaf for the windows that are composed of two leafs
- Operator 3 performs the following tasks:
  - Once all the operations are completed in the Numerical Control Machine, he distributes the completed pieces in the corresponding carriage.
  - For the pieces where the soldering is not needed this operator will retest the strip/post
- In case the previous parts are frames they should be soldered and cleaned passing to operator 4
- Operator 5 will distribute the pieces after coming from the previous task.
- There is one decision important in the production process at this point that is the presence of a crossbar in the window. Depending on this the pieces will take a way or another.
- Operator 6 is in charge of inserting the crossbar into the window frame.
- After this operation all the frames (independently of having or not crossbar) continue to the next operations jointly
- Now the system will need to know if the window is a two leafs window then operator 6 will fix the inverse leaf
- Once finished this operation all the frames will pass to the ironwork placing. This operation will be accomplished by operator 7
- Operator 8 will continue being in charge of the following:
In case the window is a frame he will place the locks and the hinges.
In any case he will hang the window in a place where the other operators will take it to perform the following operations.

After all these processes if the window has a box in its features there will be 7 possible configurations related with the box placing. All these operations related with the box are performed by operators 9 and 9 bis. These seven options are:

1. Guide Assembly + Box + Drip Edge + Silicone Insert
2. Guide Assembly + Box + Silicone Insert
3. Guide Assembly + Drip Edge + Silicone Insert
4. Box + Drip Edge + Silicone Insert
5. Guide Assembly + Silicone Insert
6. Drip Edge + Silicone Insert
7. Box + Silicone Insert

Once the box assembly process is completed the next phase will be glaze the window in case is needed. This task is performed to all windows independently of the box presence.

Operator 10 will continue with:
- Glazing the window
- Inserting the reeds into the window

Operator 11 will:
- Disassemble the leaf/frame
- Pack the finished window

There is an extra operator in the system, operator 12, that will perform ancillary operations helping operator 3 with the distribution of wagons.

Operator 13 selects and distributes the glass supplying them previous to the operation of locks and hinges.

Finally, operator 14 performs the task of cut and distribution of reeds previous to the operation related with them.

Once the operations and the initial operators that are performing the tasks we will consider a table with all the operations and a task numbering that will help us when modeling this example is presented.

The following table presents a description of the different tasks that have to be performed and whom is responsible of performing them.

<table>
<thead>
<tr>
<th>Task</th>
<th>Description</th>
<th>Performed By</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task1</td>
<td>the selection of the materials</td>
<td>Operator 1</td>
</tr>
<tr>
<td>Task2</td>
<td>cutting of the PVC profiles</td>
<td>Operator 1</td>
</tr>
<tr>
<td>Task3</td>
<td>Introduction of the reinforcements</td>
<td>Operator 1</td>
</tr>
<tr>
<td>Task4</td>
<td>Numerical Control Machine 6 Operations</td>
<td>NCM</td>
</tr>
<tr>
<td>Task5</td>
<td>Reinforcements material selection</td>
<td>Operator 11</td>
</tr>
<tr>
<td>Task6</td>
<td>Reinforcements Cutting</td>
<td>Operator 11</td>
</tr>
<tr>
<td>Task7</td>
<td>Reinforcement distribution</td>
<td>Operator 11</td>
</tr>
<tr>
<td>Task8</td>
<td>Screwing reinforcements of Machining Center</td>
<td>Operator 1 and NCM</td>
</tr>
<tr>
<td>Task9</td>
<td>Leaf cutting</td>
<td>Operator 2</td>
</tr>
<tr>
<td>Task10</td>
<td>Inverse Leaves distribution</td>
<td>Operator 2</td>
</tr>
<tr>
<td>Task11</td>
<td>Wagon distribution</td>
<td>Operator 3</td>
</tr>
<tr>
<td>Task12</td>
<td>Retest the strip/post</td>
<td>Operator 3</td>
</tr>
<tr>
<td>Task13</td>
<td>Crossbar distribution</td>
<td>Operator 3</td>
</tr>
<tr>
<td>Task14</td>
<td>Soldering and cleaning</td>
<td>Operator 4</td>
</tr>
<tr>
<td>Task15</td>
<td>Frame distribution</td>
<td>Operator 5</td>
</tr>
<tr>
<td>Task16</td>
<td>Crossbar Mounting</td>
<td>Operator 6</td>
</tr>
<tr>
<td>Task17</td>
<td>Locks and hinges fixing</td>
<td>Operator 8</td>
</tr>
<tr>
<td>Task18</td>
<td>Window hanging</td>
<td>Operator 8</td>
</tr>
<tr>
<td>Task19</td>
<td>Inverse leaf mounting</td>
<td>Operator 8</td>
</tr>
<tr>
<td>Task20</td>
<td>Box assembly (with all options)</td>
<td>Operator 9</td>
</tr>
<tr>
<td>Task21</td>
<td>Glazing</td>
<td>Operator 10</td>
</tr>
<tr>
<td>Task22</td>
<td>Insert the reeds</td>
<td>Operator 10</td>
</tr>
<tr>
<td>Task23</td>
<td>Glass selection and distribution</td>
<td>Operator 13</td>
</tr>
<tr>
<td>Task24</td>
<td>Reeds cut and distribution</td>
<td>Operator 14</td>
</tr>
<tr>
<td>Task25</td>
<td>Disassemble leaf/frame</td>
<td>Operator 11</td>
</tr>
<tr>
<td>Task26</td>
<td>Pack finished window</td>
<td>Operator 11</td>
</tr>
</tbody>
</table>

The flow of parts of the systems under study is represented in figure 1.
3. **STOCHASTIC PETRI NET MODEL**

Here we present the Petri net it has been modeled using stochastic PNs.

The complete model is represented by the following figure.

This complete Petri net model shown before will be more clearly presented in the next figures where it will be divided in substructures that will help understanding the modeling issues.

Figure 3 presents the operations where operators 1, 2 and the numerical control machine are involved. Places Oper1 and Oper2 represent the availability of the operators when marked. Transitions T45, T412, T32 and T431 represent the 4 operations that can be performed or supervised by Operator 1, while T53, T511, T521, T441 and T4111 represent the five operations that the second operator can perform. Finally, the machining tool availability is represented by place Machining_TOOL1 and the operation is shown under transition T421.

Figure 4, represents the operators 3 and 4 and due to their simplicity, because they are only performing an operation we have considered that a simple operator can cover each one of the tasks associated. There is no competition for the operator tasks.
The next figure (Figure 5) represents the tasks where operators from 5 to 9 are involved. This Petri net model represents most of the decisions that must be taken (depending on the type of final product that the FMS is generating). After operator 5 performs its task (transition T611) then the raw parts will take one way or another depending on the type of final product (window or frame). If it is a window will continue through transition window and then a second decision should be taken depending if what has to be built is a leaf of this window or a frame of it (transitions Leaf or Frame). All these operations will be supervised by operator 6. Then operators 7 and 8 will perform their tasks associated to them (transitions T15, T151 and T17). Finally, operator 9 will perform its operation represented by transition T19 but before that a decision should be taken regarding the presence of a BOX in the window structure represented by immediate transitions BOX and NO_BOX.

The last Petri net submodel is represented in Figure 6, where operators from 10 to 14 are modeled. These operators generally are performing simpler operations than the previous ones and their model representation is simpler also.

4. TWO PHASE OPTIMIZATION APPROACHES

The search space corresponding to the optimization problem that it is solved is composed by the following variables:

- Variable \( N_{\text{Oper1}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 2.
- Variable \( N_{\text{Oper2}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 2.
- Variable \( N_{\text{Oper5}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 5.
- Variable \( N_{\text{Oper6}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 6.
- Variable \( N_{\text{Oper8}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 8.
- Variable \( N_{\text{Oper9}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 9.
- Variable \( N_{\text{Oper10}} \) is an integer variable that represents the number of operators that will perform the operations initially assigned to operator 10.
- Variable \( \text{Mach} \_ \text{Delay} \) is a real variable that represents the time that in average takes to the Numerical Control Machine to perform the different tasks.
- Variable \( \text{PROB} \_ \text{BOX} \) is a real variable that represents the percentage of windows that has a box inside its structure.
- Variable \( \text{PROB} \_ \text{FRAME} \) is a real variable that represents the percentage of windows that will be a fixed frame window without any leaf (or with a unique one)
- Variable \( \text{PROB} \_ \text{WINDOW} \) is a real variable that represents the percentage of products that will have a window structure instead of a frame one.

The search space considered for this example will be the one shown in the following text box

<table>
<thead>
<tr>
<th>Parameter definitions</th>
<th>type</th>
<th>minimum</th>
<th>maximum</th>
<th>initial</th>
<th>delta</th>
<th>temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ( N_{\text{Oper1}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1 ( N_{\text{Oper2}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2 ( N_{\text{Oper5}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3 ( N_{\text{Oper6}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4 ( N_{\text{Oper7}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5 ( N_{\text{Oper9}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6 ( N_{\text{Oper10}} ) INT</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7 ( \text{Mach} _ \text{Delay} ) REAL</td>
<td>4</td>
<td>0.01</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 ( \text{PROB} _ \text{BOX} ) REAL</td>
<td>0.05</td>
<td>0.95</td>
<td>0.5</td>
<td>0.01</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9 ( \text{PROB} _ \text{FRAME} ) REAL</td>
<td>0.05</td>
<td>0.95</td>
<td>0.5</td>
<td>0.01</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10 ( \text{PROB} _ \text{WINDOW} ) REAL</td>
<td>0.05</td>
<td>0.95</td>
<td>0.5</td>
<td>0.01</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. Petri Net model Operators 5 to 9 from example 2.

Figure 6. Petri Net model Operators 11 to 14 from example 2.
The optimization function will include the performance measures we are interested in and are mainly related with the throughput of the system and the profit for a 8 hour shift, combined with the costs associated with the presence of more operators in the different positions of the FMC.

The manner how is represented this utilization in a PN model is by the formula represented below that will be explained later on.

**MEASURE Profit**

\[ P\{\#P37>0\}*720000-(P\{\#P34>0\}+P\{\#P26>0\}+ P\{\#P12>0\})*28800*0.3)- 40*(NOper1+NOper2+NOper5+NOper8+NOper9+NOper_ 10)-20000*Mach_Delay; \]

The first expression of this formula \((P\{\#P37>0\})\) represents the throughput of the whole system given that place P37 is the place positioned just before being performed the last task (done by operator 11). This expression will represent the probability that there is more than zero tokens in place P37, and this is exactly the meaning of the throughput (considering that the maximal number of tokens in place P37 is 1 because there is a P-invariant that contains this place and also places P37 and Oper11). The amount of 720.000 corresponds to the gain that the company is having considering a mean selling price for all the windows produced of 25 Euros per unit produced and considering that there is a shift of 8 hours (28800 seconds).

The next term \((E\{\#P34\}+E\{\#P26\}+ E\{\#P12\})*30*8*0.1)\) corresponds to the energy consumption term that is associated with the use of the different machines that are involved in the process. In this particular case there are three operation machines that are represented in places P34, P26 and P12. Computing the utilization of these machines during a shift of eight hours and considering the mean cost of the energy equal to 0.1€/kwh and considering a cost of energy equal to 0.1€/kwh.

The next part corresponds to the cost associated with the utilization of the different operators that has been estimated in 40 Euros for each worker and for an 8 hour shift.

Finally, the last part corresponds to the cost associated to the inclusion into the system of a quicker Numerical Machine Center that will increase the price according to the mean operational speed (20000 €/second)

### 5. RESULTS AND COMPARISON

The results we are interested to compare between the two approaches previously shown are related with productivity measures. It will be considered the number of pieces produced per time unit for each type of product (32 different types can be produced in the FMS). Another performance measure we will consider will be the utilization of the different operators that are present into the system.

Another important comparison measure will be how efficient is the convergence process for the two models and the accuracy they can reach. Also the computational time that the computer will be calculating the measures will be another measure of how good the simulation process is with respect to the colored and the stochastic models.

Also given the complexity of the model is important to consider the quality of the solution obtained combined with more qualitative measures more related with the computational effort and the number of iterations done during the optimization process.

In order to present all this information, the following tables are presented. There are 5 experiments or optimization methods we have applied.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Time (Minutes)</th>
<th>Simulations</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP1</td>
<td>2760.33</td>
<td>3505</td>
<td>271179.9</td>
</tr>
<tr>
<td>EXP2</td>
<td>1990.33</td>
<td>1802</td>
<td>266887.7</td>
</tr>
<tr>
<td>EXP3</td>
<td>589.7</td>
<td>707</td>
<td>266847.9</td>
</tr>
<tr>
<td>EXP4</td>
<td>7286</td>
<td>2351</td>
<td>245902.2</td>
</tr>
<tr>
<td>EXP5</td>
<td>289.12</td>
<td>384</td>
<td>252209.5</td>
</tr>
</tbody>
</table>

| Experiment | QUALITY Timing Simulations Profit |
|------------|----------------------------------|----------------|--------|
| EXP1       | 100.00% 100.00% 100.00%         | 100.00%       |
| EXP2       | 98.42% 39.43% 51.41%            | 51.41%        |
| EXP3       | 99.42% 21.32% 20.32%            | 20.32%        |
| EXP4       | 90.46% 50.12% 67.08%            | 67.08%        |
| EXP5       | 93.00% 10.46% 5.25%             | 5.25%         |

### 6. CONCLUSIONS AND FUTURE RESEARCH

Here we have presented a set of approaches that have been applied to a real Flexible Manufacturing System where a first approach to the introduction of energy consumption information is introduced into the optimization problem adding an extra value to the optimization process and giving another solution to the companies in order to reduce the expenses associated with this energy consumption.

Some possible future research topics will be related with the introduction of more energy related information into the models so that the optimization process will be more concentrated into this topic.

**REFERENCES**


