MATRIX-BASED OPERATIONS AND EQUIVALENTE CLASSES IN ALTERNATIVE PETRI NETS.

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ABSTRACT
Petri net is a modelling paradigm used for discrete event systems (David and Alla 2005), (Cassandras *et al*. 2008). Their transformations are a powerful tool for the validation and verification of Petri nets as models of discrete event systems. If a simplified Petri net verifies a certain set of properties, the original Petri net will verify them if the applied net transformation preserves these properties. On the other hand, the control of Petri nets may also improve with the application of Petri net transformations. If the unobservable transitions are removed by the application of transformation rules, then the state of the simplified net evolves under the firing of the observable transitions.
In this paper, the transformation of nets will be applied to modify the formalism that represents a disjunctive constraint of a decision problem stated on a Petri net model. Some formalisms may be more suitable for the modelling process than others (alternative Petri nets), while others are more compact and suitable for the development of optimization processes in an efficient way (compound Petri nets, alternatives aggregation Petri nets or disjunctive coloured Petri nets).
A set of transformation rules that preserve the structure of the associated graph or reachable markings are provided in this paper, as well as an example of application in a net transformation between two formalisms. As a consequence of the application of these rules, both the original and resulting formalisms will show an equivalent behaviour. Hence, any of them can be used to state a decision problem but the efficiency of the algorithm to solve the problem may be different when considering the required computer resources and the quality of the obtained solution.

Keywords: equivalence operations, Petri net transformations, decision support system, compound Petri nets, alternative Petri nets.

1. INTRODUCTION
One of the stages that can be considered in the modeling process of a discrete event system is the validation and verification (Peterson 1981), (Jimenez *et al*. 2005). According to (Silva, 1993), it is possible to reduce the cost and the duration of the design process of a SED by checking if certain properties are verified by the model. One of the techniques of qualitative analysis is based in the transformation of the Petri net structure. Some important issues of these techniques are described in (Berthelot 1987), (Silva 1993) and (Haddad and Pradat-Peyre 2006).

These early developments in the theory of the Petri nets have led to other transformation techniques in the static structure and to new formalisms based on PN that are aimed to simplify the modeling of DES whose structure varies with time. For example (Van der Aalst 1997) provides with eight transformation rules, based in the previously mentioned techniques, applied to systems that experience the frequent changes in their structure.

An undefined Petri net can be interpreted as a model of a discrete event system that includes freedom degrees in its structural characteristics (Latorre *et al.*, 2009b). The undefined Petri net is an abstraction that can be particularized in a specific formalism. A classical approach to obtain a model of an undefined discrete event system is a set of alternative Petri nets (Latorre *et al.*, 2007). This type of nets verifies the property of mutually exclusive evolutions, hence in the same Petri net alternative structural configurations of an original discrete event system can be included (Latorre *et al.* 2011).

In this paper the concept of equivalence class is applied to every alternative Petri net. This concept allows substituting any Petri net belonging to a set of alternative Petri nets by another one whose behaviour and properties are the same than the original one. Hence, the resulting set of alternative Petri nets will verify the same properties and show equivalent behaviour. Every equivalence class will be composed by Petri nets with the same behaviour and all of them will be said to be equivalent.

Given an alternative Petri net, the methodology to obtain equivalent Petri nets to the first one will be based in matrix-based operations, applied to the incidence matrix of the net. These matrix-based operations will lead to new Petri nets but the graphs of reachable markings will be isomorphous in the original and the resulting net. This fact ensures that the properties and the behaviour of both Petri nets is the same and, hence, that they can be considered as equivalent ones.

The matrix based operations can be applied with the aim of transforming the set of alternative Petri nets into a compound Petri net (Latorre *et al.* 2010). This process requires the merging of the sets of parameters of all the nets belonging to the set of alternative Petri nets. In particular, it is necessary to merge the structural parameters that are the elements of the incidence matrices. In order to obtain a compound Petri net with the smallest set of undefined structural parameters and hence to obtain a compact model that requires a reduced computation resources to simulate the evolution of the original DES, it is convenient to apply the matrix-based operations to the set of alternative Petri nets.

These operations might lead to alternative Petri nets whose incidence matrices have more similarities in the same positions (common elements). This fact imply that when the element of all the incidence matrices in a given position is the same, there is not an associated undefined parameter and when most of the elements are the same in a certain position, then the set of feasible values for the undefined structural parameter that appears is reduced.

2. TRANSFORMATION OPERATIONS

Once the objectives of the transformations are clear, the matrix-based operations will be presented and some examples will be given.

Definition 1. Operation of swapping two rows of a matrix.

The operation of swapping two rows of a matrix is defined as the following function:

$$\begin{array}{rcl} \text{swapr}: \mathbf{M}_{m\times n}\times\{1,2,\ldots,m\}\times\{1,2,\ldots,m\} & \rightarrow & \mathbf{M}_{m\times n} \\ (\mathbf{A}, i, j) & \rightarrow & \mathbf{B}\in\mathbf{M}_{m\times n} \end{array}$$

$$\text{where, } \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \in \mathbf{M}_{m\times n},$$

$$\mathbf{B} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \in \mathbf{M}_{m\times n}.$$

In other words, **definition 1**, describes the swapping of the ith and jth rows in a matrix $\mathbf{A}$. This operation is denoted by swapr($\mathbf{A}$, i, j)

Remark 1. When applying this operation to the incidence matrix of a Petri net it has to be taken into consideration that the ith row represents the input and output arcs of a place of the PN. Let us call this place p_i. For this reason, swapping two rows, the ith and the jth, implies that the arcs associated to p_i are no longer present in the ith row of the incidence matrix but in the jth. As a natural consequence, if this new incidence matrix is to be included in the characteristic equation of the Petri net it has to be considered that the ith element of $\mathbf{m}$, the marking of the Petri net, does not represent $\mathbf{m}(p_i)$ the marking of the place p_i anymore. The same considerations can be made for p_j, the jth row of the incidence matrix and $\mathbf{m}(p_j)$. Therefore, the swapr($\mathbf{A}$,i,j) operation implies the swapping of the ith and jth elements in the marking vector $\mathbf{m}$ of the Petri net. This statement is also true for the particular case of $\mathbf{m}_0$. It is clear then that it is necessary to apply the same swapping operation that is applied to the incidence matrix, to the marking of the Petri net. In a subsequent section, it will be mentioned the reference name and the alias of any place of a Petri net. With these concepts it will be generalized the previous considerations on the swapping of rows of an incidence matrix and the application of the same operation in the marking of the associated Petri net.

Definition 2 . Operation of swapping two columns of a matrix.
The operation of swapping two columns of a matrix is defined as the following function:

$$\text{swapc: } \mathbf{M}_{m\times n}\times\{1,2,\ldots,n\}\times\{1,2,\ldots,n\} \to \mathbf{M}_{m\times n}$$
$$(\mathbf{A}, i, j) \quad \to \quad \mathbf{B} \in \mathbf{M}_{m\times n}$$

where,

$$\mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1i} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2i} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mi} & \cdots & a_{mj} & \cdots & a_{mn} \end{pmatrix} \in \mathbf{M}_{m\times n},$$

and

$$\mathbf{B} = \begin{pmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1i} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j} & \cdots & a_{2i} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mj} & \cdots & a_{mi} & \cdots & a_{mn} \end{pmatrix} \in \mathbf{M}_{m\times n}$$

In other words, **definition 2**, describes the swapping of the columns i and j in matrix **A**, which is denoted by swapc(**A**, i, j)

Remark 2. The state equation of a Petri net requires representing the characteristic vector that summarizes the information contained in the sequence of transitions fired. The characteristic vector (also called firing count vector) contains elements that are different to zero in the positions that correspond to the transitions fired. If an operation swapc is applied to an incidence matrix and the state equation is represented, the characteristic vector should be modified according to this same swapc operation.

Definition 3. Operation of adding a row of zeros to a matrix.
The operation of adding a row of zeros to a matrix is defined as the following function:

$$\text{addr: } \mathbf{M}_{m\times n} \to \mathbf{M}_{(m+1)\times n}$$
$$\mathbf{A} \to \mathbf{B}, \text{ such that}$$

$$\text{Given } \mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \in \mathbf{M}_{m\times n} \Rightarrow$$

$$\text{addr}(\mathbf{A}) = \mathbf{B} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \\ 0 & \cdots & 0 \end{pmatrix} \in \mathbf{M}_{(m+1)\times n}$$

The operation described in the **definition 3** is denoted by addr(**A**) and adds a row of zeros to the matrix **A**.

Remark 3. The operation addr applied to the incidence matrix of a Petri net implies the addition of a new place with a particular property: every input and output arc has weight zero. In other words, this new place is an isolated node of the Petri net.

The marking of the Petri net that results from the application of this operation should include the marking of the new place, which will occupy the last position of the vector. However, being isolated, the place cannot experience any variation of its initial marking in the evolution of the Petri net. Furthermore, the marking of other places will not be influenced by the added place, hence the marking of the new Petri net, excluding the added place, will be the same to the original one. If the new place is considered in this comparison it is possible to say that the significant marking (the one that varies in at least an evolution of the PN) is the same in both Petri nets, hence the graphs of reachable markings are isomorphous.

Definition 4. Operation of removing a row of zeros of a matrix.
The operation of removing a row of zeros of a matrix is defined as the following function:

$$\text{removr: } S \to \mathbf{M}_{(m-1)\times n}$$
$$\mathbf{A} \to \mathbf{B}, \text{ such that}$$

$S = \{\mathbf{A} \in \mathbf{M}_{m\times n} \mid a_{m*} = (0\ 0\ \ldots\ 0)\ \}$, in other words, S is the set of matrices whose mth (last) row is a row of zeros.

$$\text{Given } \mathbf{A} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \cdots & \cdots & \cdots \\ a_{m-1,1} & \cdots & a_{m-1,n} \\ 0 & \cdots & 0 \end{pmatrix} \in \mathbf{M}_{m\times n} \Rightarrow$$

removr(**A**) = **B**

$$\mathbf{B} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \cdots & \cdots & \cdots \\ a_{m-1,1} & \cdots & a_{m-1,n} \end{pmatrix} \in \mathbf{M}_{(m-1)\times n}$$

The operation described in the **definition 4** is denoted by removr(**A**) and removes the last row of a matrix **A**, which should contain only zeros.

Remark 4. The operation removr applied to the incidence matrix of a Petri net implies the removal of a place with a particular property: every input and output arc has weight zero. In other words, this new place is an isolated node of the Petri net. Moreover, the place should be associated to the last row of the incidence matrix (if this last condition is not verified it is always possible to apply an operation swapr to guarantee this fact).

The marking of the Petri net that results from the application of this operation should not include the marking of the removed place (which occupied the last position of the vector before the operation). However, being isolated, the place could not experience any variation of its initial marking in the evolution of the Petri net. Furthermore, the marking of other places will not be influenced by the removed place, hence the marking of the new Petri net, will be the same to the original one (excluding the added place). If the removed place is included in this comparison it is possible to say, as it was mentioned in the **remark 4**, that the reachable significant markings are the same in both Petri nets, hence the graphs of reachable markings (including the non-significant markings) are isomorphous.

The swapping of rows and columns of the incidence matrices simply locates in a different place of the matrices the information (weights) related to the arcs that link a certain place with the transitions of the PN or a certain transition with the places of the Petri net. Furthermore, if the parameters associated to a place or a transition that changes its position in the incidence matrices do not remain attached to the position in the matrix but move with the place or transition, the behaviour of the net, and its structure, will be the same. For this reason, to apply such a transformation as the swapping of rows and columns of the incidence matrices, it is necessary to ensure that the appropriate amount of parameters are associated to the moving places and transitions. In order to facilitate this operation it is convenient to define a reference name for every place and transition, to which its parameters will be also referred. This reference name will be attached to the information (weights of arcs) of the rows and columns of the incidence matrices (that can change its position). On the other hand, it is also convenient to define an alias for every place and transition. The alias will be attached to the position in the incidence matrices, in the way that the first row of the incidence matrix will always be associated to the alias p_1, the second to the alias p_2 and so on. The same may happen with the transitions: the first column will always be associated to t_1, the second with t_2 and so on.

3. APPLICATION OF THE TRANSFORMATIONS

Example 1.

Let $\mathbf{A} \in \mathbf{M}_{m\times n}$ be the incidence matrix of a Petri net R.

The names of places and transitions of the Petri net can be shown in the following representation of **A**:

$$\mathbf{A} = \begin{array}{cc} \begin{array}{ccccccc} t_1^r & \cdots & t_u^r & \cdots & t_v^r & \cdots & t_n^r \end{array} & \\ \begin{pmatrix} a_{11} & \cdots & a_{1u} & \cdots & a_{1v} & \cdots & a_{1n} \\ \cdots & & \cdots & & \cdots & & \cdots \\ a_{i1} & \cdots & a_{iu} & \cdots & a_{iv} & \cdots & a_{in} \\ \cdots & & \cdots & & \cdots & & \cdots \\ a_{j1} & \cdots & a_{ju} & \cdots & a_{jv} & \cdots & a_{jn} \\ \cdots & & \cdots & & \cdots & & \cdots \\ a_{m1} & \cdots & a_{mu} & \cdots & a_{mv} & \cdots & a_{mn} \end{pmatrix} & \begin{array}{c} p_1^r \\ \cdots \\ p_i^r \\ \cdots \\ p_j^r \\ \cdots \\ p_m^r \end{array} \end{array}$$

Let us now apply two operations to the incidence matrix **A**:

$\mathbf{B}$ = swapr($\mathbf{A}$, i, j); $\mathbf{C}$ = swapc($\mathbf{B}$, u, v)

In other words: $\mathbf{C}$ = swapc(swapr($\mathbf{A}$, i, j), u, v)

The resulting incidence matrix, with the new alias for the places and transitions is presented below:

$$\mathbf{C} = \begin{array}{ccc} \begin{array}{ccccccc} t_1^r & \cdots & t_v^r & \cdots & t_u^r & \cdots & t_n^r \\ t_1 & \cdots & t_u & \cdots & t_v & \cdots & t_n \end{array} & & \\ \begin{pmatrix} a_{11} & \cdots & a_{1v} & \cdots & a_{1u} & \cdots & a_{1n} \\ \cdots & & \cdots & & \cdots & & \cdots \\ a_{j1} & \cdots & a_{jv} & & a_{ju} & & a_{jn} \\ \cdots & & \cdots & & \cdots & & \cdots \\ a_{i1} & \cdots & a_{iv} & & a_{iu} & & a_{in} \\ \cdots & & \cdots & & \cdots & & \cdots \\ a_{m1} & \cdots & a_{mv} & \cdots & a_{mu} & \cdots & a_{mn} \end{pmatrix} & \begin{array}{c} p_1 \\ \cdots \\ p_i \\ \cdots \\ p_j \\ \cdots \\ p_m \end{array} & \begin{array}{c} p_1^r \\ \cdots \\ p_j^r \\ \cdots \\ p_i^r \\ \cdots \\ p_m^r \end{array} \end{array}$$

In this representation of the incidence matrix **C**, it can be seen that the reference names of the swapped rows and columns have changed the position according to these swaps. This change is a consequence of the fact that the reference names are related to structural and marking parameters (among others) such as the elements of the incidence matrices and not to positions in the matrix. If the Petri net is the model of a real system, the reference names are likely to be associated to a physical meaning as well.

However, in certain applications it is very useful to define an alias for every place and transition. This alias is associated to the position that the elements of the incidence matrix occupy. The aliases do not bear the superindex “r”. For example, in the matrix **C** = swapr(**A**, i, j), the alias of the place whose input and output arcs are stated in the jth row is p_j, whereas its reference name is p_i^r.

Example 2.
Let us consider the simple alternative Petri nets presented in the figure 1 and their incidence matrices shown in the figure 2. Some equivalence operations will be applied to transform the simple alternative Petri nets into matching ones able to be merged. The result of this merging is obtaining an equivalent compound Petri net with the smallest size of the set of undefined structural parameters and the smallest size of the feasible combination of values of these parameters. In this example it is not intended to obtain the optimal compound Petri net, just to illustrate the application of some equivalence operations.

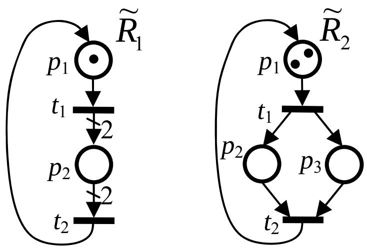

Fig. 1. Simple alternative Petri nets.

$$\mathbf{W}(\widetilde{R}_1) = \begin{array}{cc} t_1 & t_2 \\ \left(\begin{matrix} -1 & 1 \\ 2 & -2 \end{matrix}\right) & \end{array} \begin{array}{c} p_1 \\ p_2 \end{array}$$

$$\mathbf{W}(\widetilde{R}_2) = \begin{array}{c} \begin{array}{cc} t_1 & t_2 \end{array} \\ \left(\begin{matrix} -1 & 1 \\ 1 & -1 \\ 1 & -1 \end{matrix}\right) \end{array} \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array}$$

Fig. 2. Incidence matrices of the simple alternative Petri nets.

The first equivalence operation consists of increasing the size of the incidence matrix to reach the dimensions 4×3. This process require the addition of isolated places and transitios as it can be seen in the figure 3.

The operations that have been applied are the following:

$\mathbf{W}(R_1^m)$=addc(addc(addr($\mathbf{W}(\widetilde{R}_1)$)))

$\mathbf{W}(R_2^m)$=addc(addc(addr($\mathbf{W}(\widetilde{R}_2)$)))

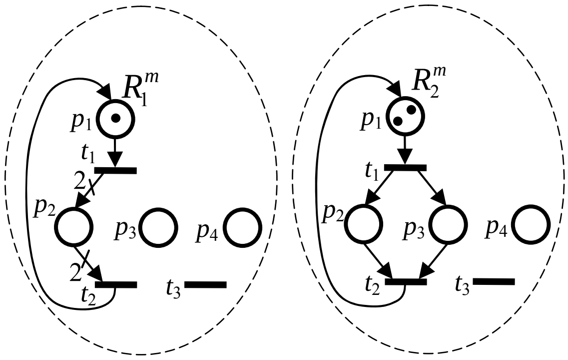

Fig. 3. Addition of isolated places and transitions to increase the size of the incidence matrices.

The new incidence matrices are shown in the figure 4.

$$\mathbf{W}(R_1^m) = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \left(\begin{matrix} -1 & 1 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right) \end{array} \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}$$

$$\mathbf{W}(R_2^m) = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \left(\begin{matrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{matrix}\right) \end{array} \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}$$

Fig. 4. Incidence matrices of the alternative Petri nets after increasing their size.

The second set of equivalence operations will be the swapping of one row and one column in $\mathbf{W}(R_1^m)$. The purpose of this operation may be to make the largest number of elements placed in the same position of both incidence matrices to coincide in order to reduce the number of undefined structural parameters of the resulting Petri net.

swapc

$$\mathbf{W}(R_1^m{}') = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \left(\begin{matrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & -2 \end{matrix}\right) \end{array} \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array}$$

swapr

Fig. 5. Incidence matrix of $R_1^m{}'$ after the swapping operations.

The process may be seen in the figure 5 and correspond to the operations:

$\mathbf{W}(R_1^{m}{}')$=swapr(swapc(addr($\mathbf{W}(\widetilde{R}_1)$,2,3),2,4)

After this last operation it is possible to merge the incidence matrices of both alternative Petri nets to obtain a single compound Petri net.

4. CONCLUSIONS AND FURTHER RESEARCH

In this paper it has been shown how it is possible to apply matrix-based operations to the incidence matrices of a Petri net that preserve the structure of the graph of reachable markings. As a consequence, the properties and behaviour of the original and resulting Petri nets are equivalent. This idea constitutes a powerful tool that allows, in the application described in this document, to transform a set of alternative Petri nets into a compound Petri net with a set of undefined parameters is expected to be reduced, comparing with a case where the matrix-based operations are not applied.

As future research it is expected to extend the application of these ideas to other fields in the modeling of discrete event systems, as well as to develop new matrix-based operations that allow obtaining equivalent Petri nets.

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