THE EXCLUSIVE ENTITIES IN THE FORMALIZATION OF A DECISION PROBLEM BASED ON A DISCRETE EVENT SYSTEM BY MEANS OF PETRI NETS

Juan Ignacio Latorre-Biel[(a)], Emilio Jiménez-Macías[(b)]

[(a)] Public University of Navarre. Deptartment of Mechanical Engineering, Energetics and Materials. Campus of Tudela, Spain
[(b)] University of La Rioja. Industrial Engineering Technical School. Department of Electrical Engineering. Logroño, Spain

[(a)]juanignacio.latorre@unavarra.es, [(b)]emilio.jimenez@unirioja.es

ABSTRACT

The design of discrete event systems (DES) can be seen as a sequence of decisions, which allows obtaining a final product that comply with a set of specifications and operates with efficiency. A decision support system can alleviate the decision making as well as provide with more information and tools to make the best choice to the decision-maker.
The decisions related to the design of a DES may include the choice among a set of alternative structural configurations. These alternatives may be defined by the designer by mere combinations of subsystems that solve subproblems associated to the specifications and behaviour of the DES. As a consequence, it is possible that the alternative configurations share redundant information that lead to improvements in the classical approaches to solve this type of decision problems.
In this paper, the formalization of a decision problem based on a DES, underlining the characteristic feature of exclusivity between alternative configurations is presented as a tool that broadens the classical approach with new ideas and techniques to improve the efficiency in the solving of decision problems.

Keywords: decision support system, discrete event systems, Petri nets, exclusive entity.

1. INTRODUCTION

Many technological systems can be described as discrete event systems (DES) due, in a large number of cases, to the presence of digital computers in their control. Mobile phones, computer networks, manufacturing facilities or logistic systems constitute examples of DES whose presence is common in our technological society (Cassandras *and* Lafortune 2008). The efficiency and correctness in the operation of these systems can save important quantities of money to companies and users. The design of discrete event systems is likely to influence in a decisive way the performance in their operation. For this reason to define adequately the design process of a discrete event system to be manufactured or constructed is a very productive activity with clear consequences in the whole life of the system (Balbo and Silva 1998). On the other hand, to consider the performance of the operation of the system is an adequate approach to afford a design process that aims to achieve a desired behaviour for the system once it is in a running stage. Nevertheless, to forecast the future behaviour of a system in process of being designed and that is not a reality yet presents several handicaps.

On the first hand, it is necessary to approximate the results, since a model of the system and not the real one should be used. On the other hand, it is needed to select the type of model to be used. In certain DES it is possible to develop physical prototypes to test certain properties of the behaviour of the systems. More commonly, formal models are developed to apply algorithmic methodologies to analyze the behaviour of the original system. Sometimes it is possible and productive to combine the construction of several models of different nature in the design process of a system: physical systems that model specific characteristics of the real system can be combined with formal models developed on a computer to forecast the behaviour of the real system.

The formal models developed to forecast the performance of a discrete event system in process of being designed are usually complemented with simulation in order to evaluate the behaviour of the model (Piera *et al.* 2004). In fact, simulation allows exploring the region of the state space of the system under specific conditions. On the other hand, it is common that modelling and simulation present a rate between the information obtained from the system and cost associated to the modelling process, which is more favourable than other techniques that imply the physical implementation of the model by means of prototyping.

Furthermore, the design process of a DES requires stating and solving several decision problems. In particular, it is usual that it is necessary to choose among a set of alternative configurations or structures for the system (Latorre *et al.* 2009). This is the case, for example, when different layouts for the material or products conveying can be chosen to define a final configuration for a chain supply in process of being designed. A classical approach to this type of problems is intensive in computer resources, when the solution is searched by means of formal algorithms. This fact is due to the analysis by simulation of every one of the alternative configurations, knowing that this analysis requires launching not only one but sometimes an undetermined number of simulations.

An analysis of the design process described in the previous paragraph, characterizes the discrete event system to be designed among a set of alternative structures with the property of mutual exclusive evolution between the structures. This idea allows developing methodologies that reduces the computational cost of performing simulations to the different alternative configurations by the removal of the redundant information present in the models of every one of them (Latorre et al. 2010b).

This concept of exclusive entity is abstracted in a more general idea defined by the exclusive entities, associated to an undefined model. On the other hand, this new idea can be particularized in a variety of formalisms to be able to represent the model of the system in a compact way able to develop fast sets of simulations to support the decision making process in the design of discrete event systems.

The general approach given by a set of exclusive entities to the exclusiveness associated to the different alternative structural configurations for a discrete event system to be designed, constitutes a characteristic feature of a model defined as a disjunctive constraint in the formalization of a decision problem based on a DES. A model of this kind can be called undefined Petri net since it contains certain parameters whose values should be chosen among a domain set as result of decisions.

The model of the system is only a part of the formalization process of a decision problem stated on a discrete event system. There are other elements that can be included in the resulting formal problem as the type of solution expected for the problem, the solution space and, depending on the decision problem, the objective function that evaluates the cost or the performance of the DES after the selection of a certain solution from the solution space.

In this paper, an overview on the statement and formalization process of a decision problem based on a discrete event system is given underlying the exclusiveness feature in the different alternative structural configurations for the DES that can be particularized by different formalisms. On the other hand, this exclusiveness can be abstracted into the concept of set of exclusive entities that leads to an interesting property that will be defined in this paper.

2. DEFINITIONS

A discrete event system can be defined in the following way:

Definition 1. Discrete event system.
Dynamic system whose behaviour can be described by discrete state variables and is governed by asynchronous and instantaneous incidences, called events, which are solely responsible for the state changes.

A discrete event system may be defined in a more or less ambiguous way by a set of specifications and some constraints and expectations in its dynamic behaviour. The ambiguity in the definition of a DES can be interpreted as freedom degrees, some of which should be particularized in the design process, while others can be specified in the operation processes. The mentioned freedom degrees may be called undefined characteristics of the discrete event system.

The previous paragraph allows to define a particular type of DES.

Definition 2. Undefined DES.
A discrete event system with at least one undefined characteristic is said to be an undefined DES.

The type of decisions stated in the design process of an undefined DES try to reduce the ambiguity in the description of the discrete event system, by the transformation of undefined characteristics in defined ones. As a consequence, it is possible to state a decision problem in the way described in the following.

Definition 3. Decision problem based on a DES.
Let D be an undefined discrete event system.
A **decision problem based on *D*** is a choice, among several alternatives, in response to a question posed on any set of the undefined characteristics of D or its evolution.

Once a decision problem has been stated, it is necessary to solve it. For this purpose, it is convenient to represent it in a formal language.

A formal language shows important advantages from a natural language to state a decision problem. On the one hand, it provides with precision to the description of the problem, removing ambiguity and allowing the application of an algorithmic solving methodology. On the other hand, the consequence of the successful application of a solving methodology to a decision problem expressed in a formal language is one or several quantitative results, which can easily be compared with numerical references or the results of other methodologies.

A first element that is convenient to include in the formal statement of the decision problem is the discrete event system itself. There are a number of formal languages that can cope with the modelling of a generic discrete event system. However, the decision of the formal language to be considered in this paper, the Petri nets (Petri 1962), is based in the versatility and double representation that may be matrix-based or graphical. On the other hand complex behaviours of collaboration and competence may be modelled in an easy and natural way (David and Alla 2005), (Jiménez *et al.* 2005).

In particular, it is possible to define an autonomous unmarked Petri net in the following way (Cassandras and Lafortune 2008) and (Silva 1993), where an introduction to the Petri net paradigm can be found as well as in (Peterson 1981) or (David and Alla 2005).

Definition 4. Petri net graph
A (generalized) *Petri net graph* (or *Petri net structure*) is a weighted bipartite graph

$$N = \langle P, T, F, \mathrm{w}\rangle$$

where

$P = \{p_1, p_2, \dots, p_n\}$ is the finite, non-empty, set of places (one type of node in the graph).
$T = \{t_1, t_2, \dots, t_m\}$ is the finite, non-empty, set of transitions (the other type of node in the graph)
$F \subseteq (P \times T) \cup (T \times P)$ is the set of directed arcs (from places to transitions and from transitions to places) in the graph, called flow relation.
$\mathrm{w} : F \rightarrow \mathrm{N}^*$ is the weight function on the arcs.

In this paper, a new approach in the definition of a Petri net will be defined.

Definition 5. Unmarked Petri net.
A (generalized) *unmarked Petri net* (or *Petri net structure*) is a triple

$$N = \langle n_p, S_\gamma, S_{val\gamma}\rangle$$

where

$n_p \in \mathrm{N}^*$ is the number of places.
$S_\gamma = \{ \gamma_1, \gamma_2, \dots, \gamma_n \}$ is a set of structural parameters.
$S_{val\gamma} = \{ cv_1, cv_2, \dots, cv_m \}$ is the set of feasible combinations of values for the parameters of S_γ.

It is verified that $n = k \cdot n_p$, where $k \in \mathrm{N}^*$ and $\forall cv_i \in S_{val\gamma}$, $cv_i = (v_1, v_2, \dots, v_n)$.

This new definition of unmarked Petri net allows constructing the incidence matrices of the formalism and underlines the approach of this paper, focussed on the formalization process of a decision problem based on a discrete event system.

Reducing the concept of Petri net to a collection of parameters and their feasible values, the formalization process from a discrete event system can be considered as the translation of a subset of characteristics of the DES into a set of parameters of the Petri net. The characteristics of the DES translated and included in the Petri net will depend on the degree of detail of the model.

Subsequently, an undefined characteristic of the discrete event system will be modelled by means of one or several undefined parameters in the Petri net.

The process of obtaining a formal model from an original system, the modelling process, can be interpreted by means of the translation of the undefined characteristics of the DES into a set of undefined parameters. As a consequence, an undefined parameter can be defined as indicated below.

Definition 6. Undefined parameter
Any numerical variable of a Petri net model or its evolution that has not a known value but it has to be assigned as a consequence of a decision from a set of at least two different feasible values. The value assigned to the undefined parameter must be unique.

A parameter of a Petri net may belong to the set of structural parameters; nevertheless, it is possible to define other types of parameters according to the role they play in the model. For example, there is a category of marking parameters that includes the initial marking of all the places of the Petri net. As a consequence, an autonomous marked Petri net can be defined in the following way.

Definition 7. Marked Petri net.
A (generalized) *marked Petri net* (or *Petri net system*) is a triple

$$N = \langle n_p, S_\gamma, S_{val\gamma} \rangle$$

where

$n_p \in \mathrm{N}^*$ is the number of places.
$S_\gamma = \{ \gamma_1, \gamma_2, \dots, \gamma_n \}$ is a set of structural parameters.
$S_{val\gamma} = \{ cv_1, cv_2, \dots, cv_m \}$ is the set of feasible combinations of values for the parameters of S_γ.

It is verified that $n = (k+1) \cdot n_p$, where $k \in \mathrm{N}^*$ and $\forall\ cv_i \in S_{val\gamma}$, $cv_i = (v_1, v_2, \dots, v_n)$.

It is possible to notice from the comparison of the **definition 5** and the **definition 7** that the addition of the n_p marking parameters have modified the definition of the unmarked Petri net by increasing the size of the set S_γ and hence the number of values in the feasible combinations of values belonging to S_γ.

Furthermore, it is easy to deduce that this parametric definition of a Petri net allows easily to be extended to Petri nets with extended features as interpreted Petri nets, including timed Petri nets, and other nets that include priorities, colours, etc.

As it has already been explained previously, the design process of a discrete event system is usually associated to several alternative structural configurations for the DES. A classical approach for the modelling of such a system is associated to so many different Petri nets as alternative structural configurations for the DES can be found. These Petri nets can be called alternative Petri nets and belonging to the same model for the original DES should comply with a property of exclusiveness (Latorre *et al.* 2011). Of course it is not possible that several of them can be chosen as solution for the DES design process. The only option for the model to be coherent with the reality of the decision problem is to comply with a property of exclusiveness. This property can be imposed by means of the concept of mutually exclusive evolution defined below.

Definition 8. Mutually exclusive evolution
Given two Petri nets R and R'. They are said to have mutually exclusive evolutions if it is verified:
i) If $\mathbf{m}(R) \neq \mathbf{m_0}(R) \Rightarrow \mathbf{m}(R') = \mathbf{m_0}(R')$
ii) If $\mathbf{m}(R') \neq \mathbf{m_0}(R') \Rightarrow \mathbf{m}(R) = \mathbf{m_0}(R)$

As a consequence, a set of alternative Petri nets can be described as:

Definition 9. Set of alternative Petri nets.
Given a set of Petri nets $S_R = \{ R_1, \dots, R_n \}$, S_R is said to be a set of alternative Petri nets if $n>1$ and $\forall\ i, j$ such that $1 \leq i, j \leq n$, R_i and R_j verify:

i) R and R' have mutually exclusive evolution.

ii) $\mathbf{W}(R) \neq \mathbf{W}(R')$.

R_i is called the *i*-th alternative Petri net of S_R.

This classical approach of modelling an undefined DES with alternative structural configurations by means of a set of alternative Petri nets is not the only option. Even it is not necessarily the most efficient option (Latorre *et al.* 2009) for posing and solving a formal statement of the decision problem based on the DES.

In this search for new formalisms, it is interesting to abstract the representation of the undefined DES performed with the set of alternative Petri nets. On the first hand, it is possible to obtain a general abstraction for the mutually exclusive evolution of the alternative Petri nets by means of the concept of the exclusive entities. The alternative Petri nets can be considered exclusive entities since only one of them can be chosen at a time. In fact, a set of exclusive entities can be defined in the following way:

Definition 10. Set of exclusive entities.
Given a discrete event system, a set of exclusive entities associated to it is a set $S_x = \{ X_1, \ldots, X_n \}$, which verifies that

i) The elements of S_x are exclusive, that is to say, only one of them can be chosen as a consequence of a decision.

ii) $\forall\, i, j \in \mathrm{N}^*$, $1 \leq i, j \leq n$ it is verified that $X_i \neq X_j$.

iii) $\exists$ f: $S_x \rightarrow S_R$, where

$S_R = \{ R_1, \ldots, R_n \}$ is a set of alternative Petri nets, feasible models of D.

f is a bijection $\Rightarrow \forall\, X_i \in S_x\ \exists!\ \mathrm{f}(X_i) = R_i \in S_R$ such that R_i is a feasible model for D and $\forall\, R_i \in S_R\ \exists!\ \mathrm{f}^{-1}(R_i) = X_i \in S_x$.

Definition 11. Undefined Petri net.
An *undefined Petri net* is a 4-tuple

$$N = \langle n_p, S_\gamma, S_{val\gamma}, S_x \rangle$$

where

$n_p \in \mathrm{N}^*$ is the number of places.
$S_\gamma = \{ \gamma_1, \gamma_2, \ldots , \gamma_n \}$ is a set of structural parameters.
$S_{val\gamma} = \{ cv_1, cv_2, \ldots , cv_m \}$ is the set of feasible combinations of values for the parameters of S_γ.
$S_x = \{ X_1, X_2, \ldots , X_q \}$, where $q > 1$, is a set of exclusive entities.

It is verified that $n = (k+1) \cdot n_p$, where $k \in \mathrm{N}^*$ and $\forall\, cv_i \in S_{val\gamma}$, $cv_i = (v_1, v_2, \ldots , v_n)$.

In fact, the set of exclusive entities S_x does not provide with more structural or marking parameters to the model. It simply organizes or classifies the parameters into exclusive subsets. The specific representation of this undefined Petri net can be made according to different formalisms that should include a set of exclusive entities.

3. PROPERTIES AND APPLICATIONS

Several properties can be stated in relation with the idea of an undefined Petri net:

Proposition 1. The feasible combinations of values for the undefined structural parameters of a compound Petri net is a set of exclusive entities.

Proposition 2. A set of choice variables is a set of exclusive entities.

Proposition 3. A natural choice colour is a set of exclusive entities.

Proposition 4. A set of alternative Petri nets is a set of exclusive entities.

All these valid representations of a set of exclusive entities lead to different formalisms able to model a discrete event system with alternative structural configurations. For more details on the elements that appear in the statements of the propositions see (Latorre *et al.* 2010a) and (Latorre *et al.* 2010c).

An additional property should be guaranteed for any representation of a set of exclusive entities.

Theorem. Given an undefined Petri net R^U associated to a set of exclusive entities S_x, any representation of the set of exclusive entities S_z verifies that

$$\mathrm{card}\ (S_x) = \mathrm{card}\ (S_z)$$

This last property implies that no matter which representation is chosen for the set of exclusive entities of an undefined Petri net, the cardinality of its representation is the same than its abstraction S_x. In other words, the number of alternative structural configurations of the original discrete event system is constant.

The applications of the concept of set of exclusive entities can be found in the decision field that is associated to the discrete event systems.

Each exclusive entity can be related to a feasible configuration of the original discrete event system. The set of all these configurations determine the complete set of possible choices to define univocally the controllable parameters of the associated Petri net model.

One interesting application of the concept of set of exclusive entities consists of restricting the association to the structural configurations of the original DES to the exclusive entities. In this case it is possible to develop a methodology to choose among different structures to design, modify or control certain systems modelled by Petri nets. Every exclusive entity may be associated to several undefined or controllable parameters that lead to diverse behaviours, however, the exclusive entities are reserved, in this approach, to the structural parameters. In this methodology, it is possible to state the following theorem:

The search for an efficient representation of an undefined Petri net, can lead to formalisms that profit from the search methodology, from the similarities between the different structural configurations of the DES, from a single solution space, etc, enhancing the performance of the optimization algorithms aimed to take the best decisions.

Some formalisms that can be mentioned are the sets of alternative Petri nets, the compound Petri nets, the alternatives aggregation Petri nets and the disjunctive coloured Petri nets. Their suitability for representing an undefined Petri net can be deduced from the propositions 1 to 4.

4. CONCLUSIONS AND FUTURE RESEARCH

In this paper, a decision problem based on a discrete event system is analysed. Moreover, some important topics in the formalization process of this problem are considered. In particular, it has been underlined a relevant type of decision problem stated in the design process of a discrete event system. The new approach of considering a Petri net from a parametric point of view leads to an abstraction of a model of a discrete event system with alternative structural parameters. Some properties allow relating the set of exclusive entities with different representations that comply with the invariance of their number of elements. This property is related with the fact that the number of alternative structural configurations for the DES being designed is constant.

The topic presented and summarized in this paper is an important part in the theory that affords the solution of the decision problems based on DES with different alternative structural configurations by means of the removal of redundant information and obtaining compact formalisms that behave efficiently in the algorithms to solve the associated problems.

Open research lines so far are the search for new formalisms to represent undefined Petri nets, as well as to develop criteria and algorithms to choose the best formalism to solve a given decision problem.

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