SIMULATING COMPLEX SYSTEMS: THE CHALLENGE OF ERRATIC OUTPUT BEHAVIOR FOR DECISION SUPPORTIVE FORECASTING

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ABSTRACT
In many complex systems a small change of the independent variables X (input) may completely change the value of the dependent variable Y (output). We call this behavior erratic and try first to find a formal definition for it. Erratic behavior can also be reproduced in corresponding simulation systems. By calibration it is even possible fine-tune the simulation to the real world data, achieving high descriptive validity. However, since the real system is highly sensitive to minor changes of the initial conditions, the simulation model must reach an equally high level of fidelity if ti comes to prediction. Even a qualitatively appropriate forecast could be of low value if the erratic behavior leads to high quantitative deviations. Unfortunately, the fidelity of simulation models is limited by many factors including available system data, money, and time. Thus, the initial conditions of the real system can only be approximated in the model input. Modeling necessarily introduces a certain amount of uncertainty with respect to the real world situation. Consequently, a tolerable level of deviation has to be defined which might be easily exceeded in the case of erratic behavior. Based on two examples we generalize this problem and try to systemize its investigation on the basis of some preliminary formal definitions. The ultimate goal of this research endeavor is the ability to assess the power of simulation-based predictions with respect to the future behavior of systems that have shown erratic behavior in the past.

Keywords: System behavior prediction, erratic and discontinuous system behavior, decision support,

1. INTRODUCTION

In complex systems the interrelation between different measurands is often erratic. Introducing a small additional amount of X (input) may completely change the value of Y (output). This behavior is sometimes even unpredictable as has been shown by chaos research. As more and more investigations on complex systems are performed using simulations it has become clear, that such erratic behavior can also be produced in corresponding simulation systems. It is, however, not obvious that real world and simulated erratic behavior are causally interconnected, since there are always many possible simulation systems (or parameterizations) that can produce such data. It is extremely seldom possible to prove that a model is isomorphic to its real world correspondent. Therefore, empirical validation seems to be of uttermost importance, especially in the context of practical decision making, decision support and risk analysis (Hofmann and Krieger, 2008).

The general goal of simulation-based decision support is to use the simulated behavior as a forecast for the real behavior. Erratic functional relations in real world systems, however, put a serious challenge to this approach. Since the real system is highly sensitive to minor changes of the initial conditions, the simulation model must reach an equally high level of fidelity. Even a qualitatively appropriate forecast could be of low value if the erratic behavior leads to high quantitative deviations. Unfortunately, the fidelity of simulation models is limited by many factors including available system data, money, and time. Thus, the initial conditions of the real system can only be approximated in the model input. Modeling necessarily introduces a certain amount of uncertainty with respect to the real world situation. Consequently, a tolerable level of deviation has to be defined. We will show that this is a question of choosing an adequate measure.

The paper will not address the issue, how sufficient observations of a real system producing erratic output are collected, although this might be the most challenging task at all. We will focus mainly on the interpretation of simulation runs and, secondary, on the conditions of a justified transfer of simulation results into reality.

The paper illustrates the challenge with two examples, and provides approaches for the formalization of “erratic behavior” of models.

2. ILLUSTRATIVE EXAMPLES

In order to illustrate the rather theoretical verbal description of the problem in the introduction the paper starts with two examples highlighting the issue. The first example is taking from military combat simulation, the second stems from evacuation simulation.

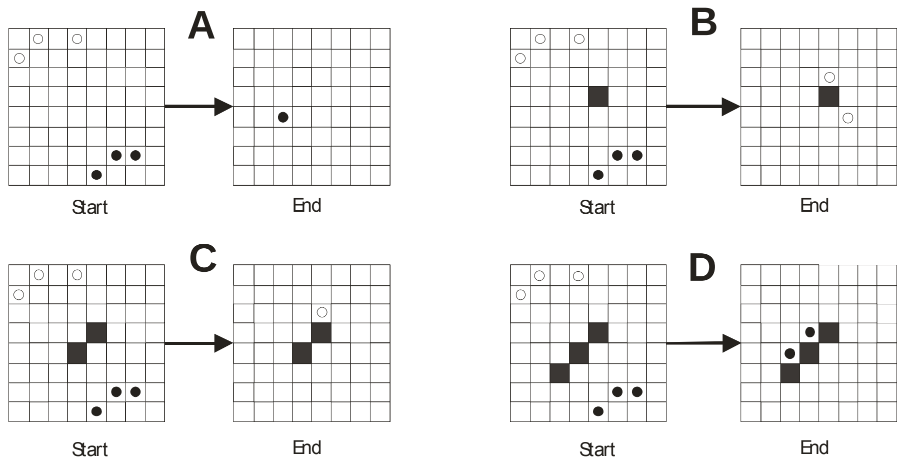

Figure 1: Sensitivity of a simplified high resolution combat simulation model towards minor changes of terrain

2.1. Military Combat Simulation

The phenomenon of unpredictably erratic output in simulation systems behavior - in the sense described above - is well known. It can, at least, be traced back to the dissertation of (Schaub, 1991), who could prove that the dependency of duel-like combat outcome from terrain representation (input variation) in high resolution combat simulation models is extremely discontinuous. His original goal was to refine an aggregated combat model (high level of abstraction) by generating context-specific effectiveness measures for it via a high resolution simulation model (low level of abstraction). He could clearly demonstrate that these measures (called "Lanchester coefficients"; they are used in almost all aggregated combat models in differential equations (Lanchester-Type models of warfare)) were highly affected by the slightest change of the terrain representation of the high resolution model (introducing a single tree representation, for example, see Figure 1). Notice that the initial positions of the troop representations (circles of different color) are identical in all four variants, and that from A to D only a single change of terrain occurs. The end states, however, differ significantly.

Schaub could trace back this effect in the model to the high sensitivity of reconnaissance and attrition processes on lines of sight, which was absolutely no contradiction to reality (which might be seen as a *qualitative concordance* of model and reality). However, the effect was much too erratic to allow sensible abstraction. Schaub concluded that Lanchester coefficients can hardly be based on high resolution models. The context-specific effectiveness measures calculated with the high resolution model output show unpredictably erratic behavior which could not be validated quantitatively by comparisons with reality.

It has to be mentioned that Figure 1 is a simplification, since it depicts high resolution combat models as deterministic models. Actually, they are stochastic models and the sensitivity effect is more appropriately visualized in Figure 2. The general problem, however, is not affected by this difference.

At least part of the challenge is simply that the resolution of the simulation model (the size of a cell) restricts depiction of reality. All terrain features must be condensed into single cell states. Thus, a group of real trees must be translated into an arrangement of cells. Due to the limits of resolution this is an underspecified transformation (see Figure 3).

Note that this is only an example, but every map (terrain representation) is limited in resolution with respect to reality. Hence, higher resolution is in most

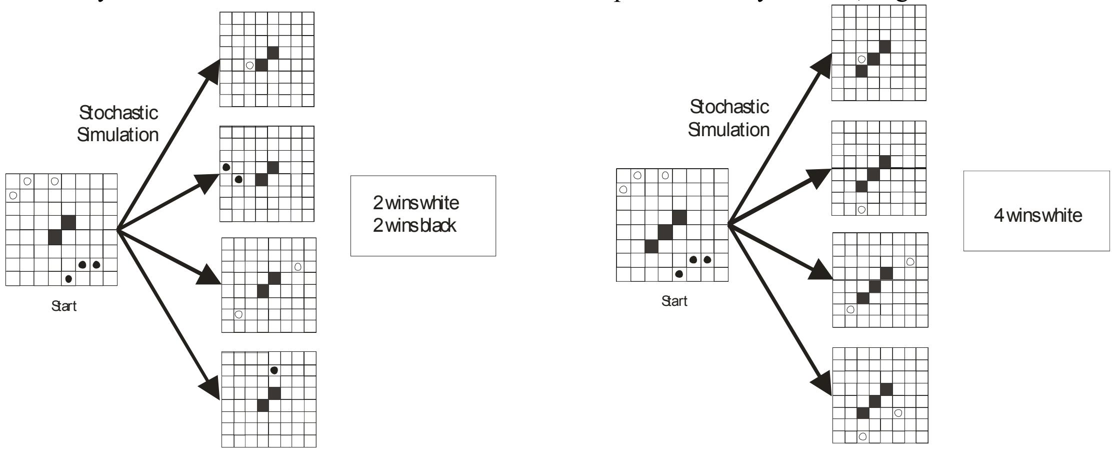

Figure 2: Stochastic combat simulation and its sensitivity to single terrain features

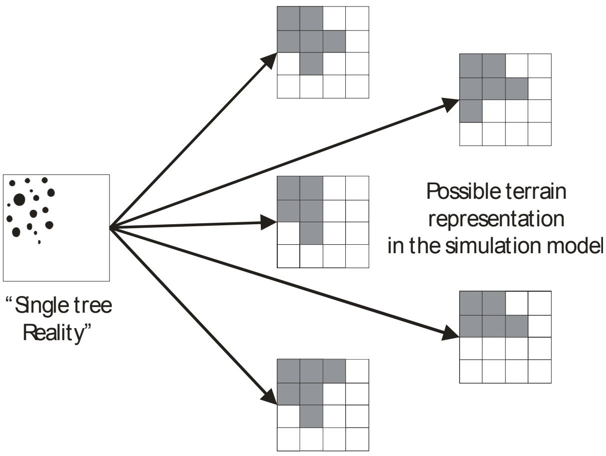

Figure 3: Uncertainty introduced into the combat simulation model by limited terrain resolution

cases only a gradual improvement, sometimes even degradation, since validation gets more and more difficult (think of the challenges of a "real time ultra high terrain representation").

Thus, we can summarize the challenge as follows: Although the erratic behavior produced by the simulation model qualitatively matches the corresponding erratic real world behavior, limits in model resolution introduce more uncertainty into the model as tolerable for such an erratic process, highly sensitive to minor changes of input.

Consequently, there is a fundamental limitation to the application of combat simulation models for decision supportive prognosis in real military conflicts.

High resolution combat simulation models are extremely complicated models when it comes to details, based on a subject not everybody is familiar with. Hence, we will use a second example to illustrate some subtleties of the problem. This second example, an evacuation simulation model, is also included to show the ubiquity of the problem.

2.2. Evacuation Simulation

Figure 4 shows two hypothetic screenshots of a simulation called ESCAPE, used for didactic purpose at the University of the Federal Armed Forces. The goal of the model is to investigate the effects of different room geometry and obstacles on the evacuation time. The lines in Figure 4 represent walls, the breaches in the walls doors, the dark squares people (agents) trying to escape through the doors of the building, the circles obstacles, and the triangle a source of danger (fire, explosion etc.). The (stochastic) model generates flight behavior of agents according to some simple movement rules, like for example:

1. If you are within a certain range of the source of danger, move in the opposite direction.
2. If you can see an outward leading door, and if there is no obstacle and no source of danger between you and the door, move towards it.
3. If you can see an internal door, and if there is no obstacle and no source of danger between you and the door, move towards it.
4. If you can see neither an internal nor external door nor a source of danger, move in line with your nearest neighbor.
5. If you can see neither an internal nor external door nor a source of danger nor any neighbor, move randomly.

The most important output parameter of the model is the total evacuation time, defined as the time from the beginning of the simulation (when the source of danger is recognized) until the departure of the last agent (see Figure 5).

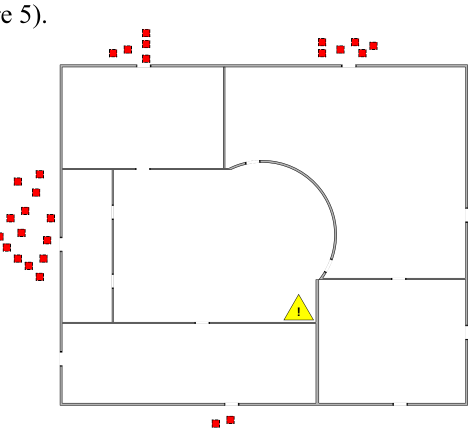

Figure 5: Final state of the ESCAPE simulation

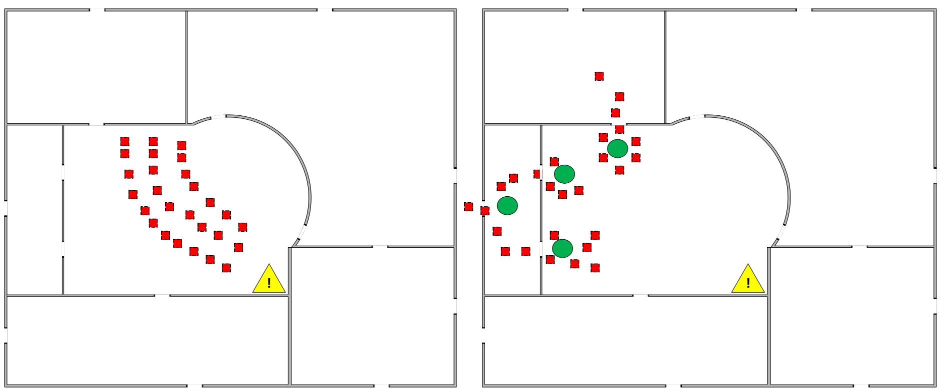

Figure 4: Example of an initial situation (left) and a simulation snapshot (right) of a slightly changed door arrangement with additional obstacles (green) in ESCAPE

The model can be modified and implemented in numerous ways. There is no single best implementation of the model and a lot of assumptions and simplifications must be made about human behavior, about textural attributes of the floors, walls, ceilings, about accidents to be considered and so on. The resolution of the models is therefore limited even with respect to crucial input parameters. The variety of reasonable assumptions and simplifications manifests itself as structural variation of the evacuation model. For example, in all models the representation of dimensions of agents and obstacles as well as their spacing is critical, and, at least to certain extent, arbitrary. For instance, the distance between columns, walls and doors and the vertical size of "persons" are closely related to the number of people who can move simultaneously. Only slight changes of these parameters in a model or slightly different modeling of geometry can cause tremendous (discontinuous) changes in total evacuation time. Consequently, there is a certain amount of irreducible uncertainty about the concordance of the real initial situation, and model input.

Evacuation time (output) is highly sensitive, both in reality and in the model, to minor changes of the location of obstacles and doors (input). The model can be calibrated in order to achieve a good qualitative concordance of model and reality, for example, with respect towards the positive effect of pillars in front of doors. However, for decision supportive prediction (designing a new building, for example) such a qualitative concordance might not suffice. Prediction of erratic real system and model behavior must match quantitatively, too.

3. GENERALIZATIONS

Based on the examples we can now try to first generalize and then formalize the concordance problem of erratic real world and corresponding model behavior.

1. Some real systems show "erratic" behavior, designating large and irregular output changes from minor input changes (Subsequently, we try to formalize this verbal description).
2. Such erratic behavior can be imitated with simulation models.
3. Using techniques of calibration concordance can be established between both erratic behaviors (for descriptive purposes, at least).
4. Due to limits of resolution the simulation cannot match the real situation perfectly.
5. Hence, deviations between both erratic behaviors are unavoidable (in predictive applications).
6. In contrast to non-erratic systems, such deviations might be substantial.
7. The concordance between reality and model is therefore often only qualitatively (A term which has to be formally defined, too).
8. For many practical decisions quantitative concordance is essential.
9. Due to the limits of model resolution erratic system behavior might lead to insufficient quantitative concordance of simulation models.

Such a verbal description might be helpful to get aware of the problem; however, it is unsatisfying with respect to further inspection and treatment of the challenge. At least three conceptual descriptions have to be stated more precisely:

First of all, "*erratic behavior*" has to be defined formally. This is obviously the most important task. Second, we need a general, formal description of the *amount of uncertainty introduced into prognosis by limited model resolution*. Third, we have to render more precisely what *qualitative concordance* actually means. All three definitions must finally be *aligned*.

In this paper we concentrate on the first task of defining the erratic/non-erratic model or system behavior in general. The task of finding corresponding measures for the second and third challenges as well as the alignment of all three definitions is postponed to later work.

4. ERRATIC FUNCTIONAL RELATIONS: IN SEARCH FOR A CLASSIFICATION SCHEME

In the first abstract of this paper we used "discontinuous" instead of "erratic"; during the elaboration of the full paper we realized, however, that the common meaning of "discontinuous" might be two narrow in capture the essence of the problem. Discontinuous is a technical term used intensively in (mathematical) calculus. It denotes a saltus in a continuous function. However, such a saltus can be large or infinitesimally small (see Figure 6, graphics A and B). Obviously, the local saltus in graphic B is much less significant with respect to the global range of Y than the saltus in graphics A. Moreover, as shown in the examples, the interpretation of simulation outputs depends on the precision of input data (besides of the general model fidelity), implying that the input X is only determined with a certain range ΔX. Within this local variation of X a significant change of Y might occur (ΔY), both in a continuous and in a discrete system (Graphics C and D of Figure 6). This change of Y has to be seen in connection with the global range of Y ($\blacktriangle Y$) over a predefined global range of X ($\blacktriangle X$).

It is self-evident, that for each application of a simulation model, a specific tolerance threshold (max ΔY) exists for the output variation ΔY, especially if caused by uncertainty of X (ΔX), and that, in general, this threshold may also depend from the global range of Y ($\blacktriangle Y$). With other words, there is always a model-purpose specific tolerable level of output uncertainty.

Formal definition of erratic behavior should reflect these considerations.

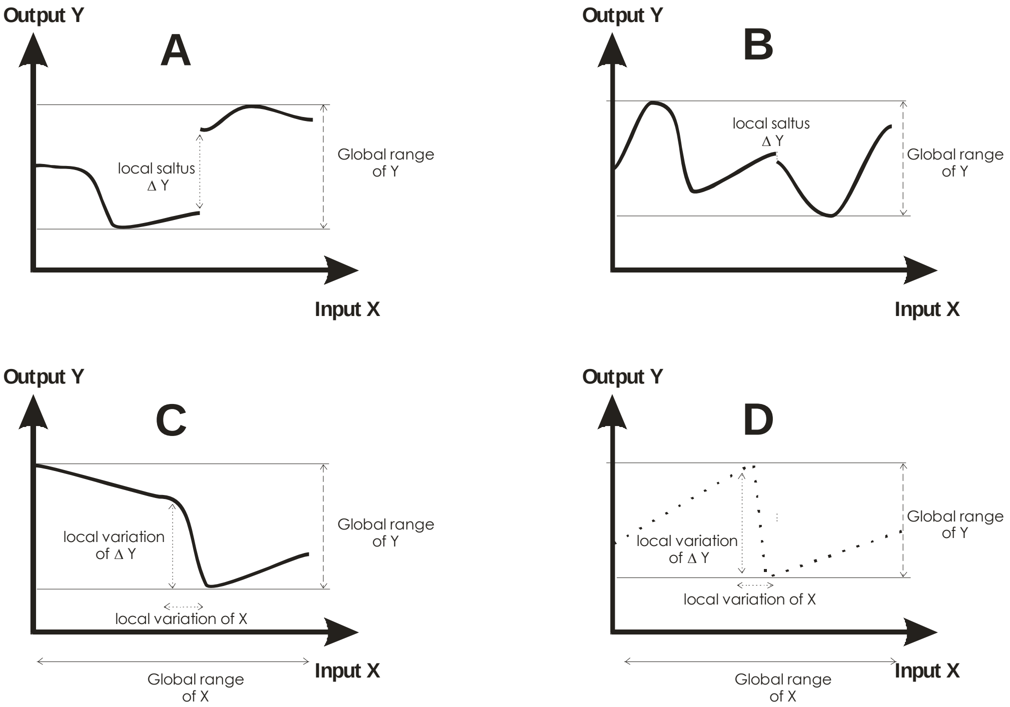

Figure 5: Erratic behaviour of output functions

5. FORMAL DEFINITIONS OF ERRATIC SYSTEM BEHAVIOR

This section is work in progress. We show the most straightforward approaches for defining erratic/non-erratic model or system behavior with mathematical formalisms. Let $f(t)$ be the system state at time t (let us assume that t is the only input parameter **x**) of system *S*. For the sake of simplicity we assume that $f(t)$ is real-valued.

First approach: We call **system S non-erratic (1)** if there exist a number $h > 0$ and a constant $K(h) > 0$ such that for all $t \in [t_0, \infty)$ the condition $|f(t+h) - f(t)| \leq K(h)$ is fulfilled. This definition assures that variations of the input **x** (here: t) smaller than h causes output variations $|f(t+h) - f(t)|$ which do not exceed a certain value $K(h)$. This limitation may help for the prognosis of future system behavior. It should be noted that that a non-erratic function may not be continuous or Lipschitz continuous functions (in the mathematical sense) at all.

This definition is extremely simple; however, at least for some applications it might be sufficient. If the data hitherto (t) gathered from a real system is non-erratic(1) it is very likely, that the deviation of a prognosis for $t+h$ will be smaller than $K(h)$.

The consideration of ▲Y (the maximal range of the output) can be easily included into the first approach by defining K(h) := (▲Y / ▲X) h.

The disadvantage of this approach is that it only considers the local behavior of the system. The common understanding of erratic behavior, however, also includes global effects. These can be taken into account with our second approach.

Second approach: Here we use the idea of the *Total Variation* of a function f and change it for our needs. For our practical purposes we simplify this concept: Let $h > 0$ be fixed and

$$V(f,t) := \sum_{i=1}^{n} \; |f(t + i\,h) - f(t + (i-1)\,h)|, \quad t \in [t_0, \infty).$$

Then we say that **system S is non-erratic (2)** if there exist a constant $K(h) > 0$ such that $V(f,t) < K(h)$. In this definition we consider the cumulative effect of the next n time steps starting with an arbitrary t.

The disadvantage of this definition is that substantial local deviation might not be taken into sufficient account.

Third approach: In mathematics a function is called Lipschitz continuous, if for any t_1 and t_2 in the domain of f the inequality holds:

$$\frac{|f(t_2) - f(t_1)|}{|t_2 - t_1|} \leq L, \qquad L > 0.$$

Intuitively, a Lipschitz continuous function is limited in how fast it can change. In this approach we call **system S is non-erratic (3)** if it is Lipschitz continuous. The difference $|t_2 - t_1|$ can be interpreted as time difference of two model/simulation time points or as the resolution of the formal model. In the example of the military combat simulation $|t_2 - t_1|$ might be the size of the cells. If one considers the same model and merges 4 cells to a new cell, then Lipschitz continuity of the output function would mean, that the $|f(x) - f(y)|$ is less than $2L$.

In general -- following these lines -- one has to consider a metric d_X on the domain of f and a metric d_Y on the set of all outputs. Then Lipschitz continuity is defined as (**system S is non-erratic(4)** if)

$$\frac{d_Y(f(t_2), f(t_1))}{d_X(t_2, t_1)} \leq L, \qquad L > 0$$

for all t_1 and t_2 in the domain of f.

The major problem in practice is the specification of the strongly context depending metrics.

However, it seems to us that this definition captures most of the notion of non-erratic behavior.

The way ahead from these definitions is clear. First, one might interpret h as the irreducible *amount of uncertainty introduced by limited model resolution* (which might also be interpreted as limits of precision in practical measurement). Thus, if we can actually determine h exactly, and also find domain specific reasonable levels for $K(h)$, we might be able to judge system behavior with respect to the possibilities of making trustworthy predictions for a future $t+h$. Second, we have to construct a measure for comparing system and simulation model output behavior in order to define qualitative concordance in the light of formally defined erratic behavior. We strongly hope, that all necessary mathematical alignment can be made, and that, finally, we might be able to assess the power of simulation-based predictions with respect to the future behavior of systems that have shown erratic behavior in the past.

6. SUMMARY AND CONCLUSION

In this paper we try to illustrate, generalize, and then formalize the challenge of erratic output behavior for decision supportive simulation-based forecasting. The examples are taken from military and evacuation simulation.

We demonstrate that some real systems show "erratic" behavior, designating large and irregular output changes from minor input changes and that such erratic behavior can be imitated with simulation models. Using techniques of calibration concordance can be established between both erratic behaviors. Due to limits of resolution, however, the simulation cannot match the real situation perfectly. Hence, deviations between both erratic behaviors are unavoidable (in predictive applications). In contrast to non-erratic systems, such deviations might be substantial. The concordance between reality and model is therefore often only qualitatively. For many practical decisions, however, quantitative concordance is essential. Due to the limits of model resolution erratic system behavior might therefore lead to insufficient quantitative concordance of simulation models for predictive purposes.

Since such a verbal description is insufficient for operationalization and therefore insufficient for practical use, we have presented four possible mathematical definitions for our central concept of "erratic behavior".

Although a lot of work has still to be done to align these definitions to a formal description of the amount of uncertainty introduced into prognosis by limited model resolution, and to a formal definition qualitative concordance, we are optimistic that such alignments are possible.

If successful, we might, finally, be able to assess the power of simulation-based predictions with respect to the future behavior of systems that have shown erratic behavior in the past.

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