ON THE SOLUTION OF OPTIMIZATION PROBLEMS WITH DISJUNCTIVE CONSTRAINTS BASED ON PETRI NETS

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ABSTRACT
Manufacturing facilities, chain supplies and other systems of technological interest can be usually described as discrete event systems. The Petri nets are a modeling paradigm able to cope with complex behaviour of DES. The design of this kind of systems and their efficient operation usually lead to the statement of optimization problems with disjunctive constraints. Those constraints are given by the Petri net models with variables that represent the freedom degrees of the designer or the process engineer that defines the working parameters of a production line. Disjunctive constraints are difficult to handle in optimization problems. In this paper an analysis of the disjunctive constraints is performed and an overview of four different representations for this type of constraint is developed: a set of alternatives Petri nets, a compound PN, an alternatives aggregation PN (AAPN) and a coloured Petri net (CPN). The advantages and drawbacks of every one of these representations as well as some examples of transformation algorithms are given.

Keywords: alternatives aggregation Petri net, compound Petri net, alternative Petri net, optimization.

1. INTRODUCTION
In industrial and chain supply systems it is usual to state optimization problems with disjunctive constraints when some operation or design decisions must be taken (Latorre, Jiménez and Pérez 2007). The solution procedure for those optimization problems can be afforded from different points of view. One of the possibilities, a classical approach, can be considered. In this case it is common to take into account a number of simpler problems in which the original problem can be decomposed, to solve all of them independently and to choose the best solution among them (Zhou and Venkatesh 1999). Nevertheless, a comprehensive and exhaustive analysis of the different possibilities to afford the solution of an optimization problem with disjunctive constraints has not been developed so far.

In this paper, a general view of the optimization problems with disjunctive constraints based on Petri nets is provided, with a detailed description of the disjunctive constraints and different representations of them with the purpose of analysing how a particular representation might be more suitable to solve a certain optimization problem with an enhanced performance.

On the other hand, some of the mentioned representations are associated to the classical approach of decomposition of the optimization problem into a set of simpler ones. As a consequence, a comprehensive view of the different representations of a disjunctive constraint is offered in this paper.

2. OPTIMIZATION PROBLEM WITH DISJUNCTIVE CONSTRAINTS BASED ON PETRI NETS
A design decision in an industrial or logistic problem can be associated to an optimization problem based on a disjunctive constraint. The disjunctive constraint may appear in the case a Petri net can model different alternative systems and any of them can be considered as feasible systems, which comply with the specifications given by the problem (Latorre et al. 2009c).

A disjunctive constraint shows the particular property that it is composed of a family of restrictions. Moreover, for every one of the alternative models of the system that verify the specifications of the problem, one and only one of the restrictions must be complied. For this reason an important property of every representation of the disjunctive constraint, as it will be shown in the following section, is the exclusiveness property. According to that principle, every subset of a disjunctive constraint is exclusive in the sense that it must be complied only in a single alternative or in a set of them.

3. DIFFERENT REPRESENTATIONS OF A DISJUNCTIVE CONSTRAINT
A disjunctive constraint related to an optimization problem based on a Petri net which is seen under the classical approach of dividing the original problem into

a set of simpler problems, is related to what can be called set of alternative Petri nets.

In this case a set of alternative models for the system can be considered. They have structural differences. In other words, their incidence matrices have at least a parameter which is different among them (Latorre, Jiménez and Pérez 2009a).

These alternative Petri nets are exclusive in the sense that only one of them can comply with the specifications of the problem, while the rest must be discarded after a solution procedure. Being exclusive, they verify the definition of disjunctive constraint. On the other hand, they constitute the more intuitive approach to the solution process of an optimization problem with disjunctive constraints based on Petri net, since it is very natural to interpret the problem by means of different and discrete alternative Petri net models. Nevertheless, this approach may not be the more efficient one to perform a solution procedure for the problem (Latorre, Jiménez and Pérez 2010a). This sole fact justifies the search for other representations more suitable for the solution process of other optimization problems.

In the figure 1, a set of two simple alternative Petri nets can be seen. They are simple since there is not any undefined structural parameter in them. In other words, every one of the parameters of the incidence matrices is associated to a unique value. On the other hand they are alternative for a certain system if both of them comply with the specifications and only one of them can be chosen as model for the considered system. It is necessary to choose among the two alternative PN if the system should be determined univocally.

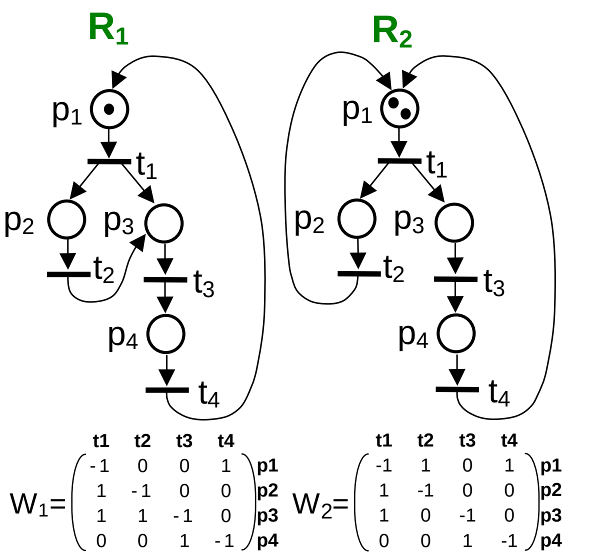


Figure 1: Simple alternative Petri net

Another representation for the disjunctive constraint implies the merging of a set or a subset of simple alternative Petri nets into a compound alternative Petri net. This merging process allows to reduce the volume of information needed to store the incidence matrices of the alternative Petri nets when there are enough similarities between them. As a result of the merging process, a particular incidence matrix for every compound alternative Petri net is obtained. In these incidence matrices some parameters are necessary to represent the different values that some elements of the matrices can have. The set of feasible values for every parameter of the incidence matrix of a compound alternative Petri net, called undefined structural parameters, is composed of elements that are exclusive. In other words, when one of these combinations of values for the undefined structural parameters of a compound Petri net is chosen, the rest are discarded. Hence, the exclusiveness of the original simple alternative Petri nets is transformed into this characteristic of the compound alternative Petri net.

In the figure 2, a compound Petri net is shown. This compound Petri net is equivalent to the set of simple alternative Petri nets of figure 1. It contains three undefined parameters. In other words, there are three variables in the Petri net: α_1, α_2 and α_3. The two first variables belong to the incidence matrix of the PN, hence they are called undefined structural parameters. The third variable is the initial marking of the place called **p1**, therefore, it is called an undefined marking parameter.

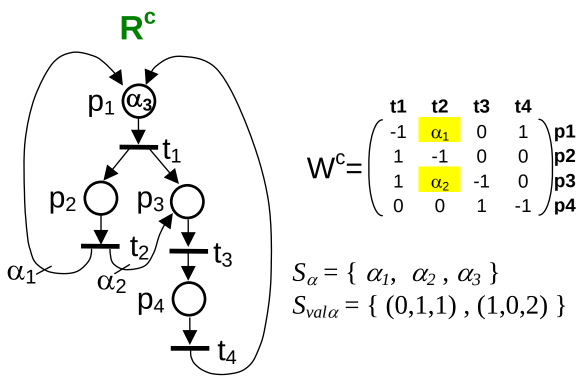


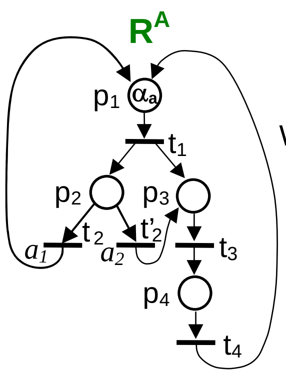
Figure 2: Compound Petri net

As it can be seen from a comparison between the set of simple alternative Petri nets in figure 1 and the compound Petri net in figure 2, both the matrix-based and the graph-based representations of the latter look simpler and seem to require less computational resources to be stored and processed by automatic calculation. Nevertheless, there is a piece of additional information that require the description of the compound Petri net that is not required by the representation of the set of simple alternative Petri nets: the set of undefined parameters, S_α, and the set of feasible combinations of undefined parameters, $S_{val\alpha}$. If the set of alternative Petri nets are not simple, that is to say if they contain undefined structural parameters, the mentioned additional information is also needed to complete the description of the set of Petri nets.

Another interesting comment on the nets of figures 1 and 2 is related to the exclusiveness property associated to the freedom degrees present in the original system. This exclusiveness is the source that creates the disjunctive constraint that make an optimization problem associated to this undefined system difficult to solve. The presence of the disjunctive constraint, due to the exclusiveness property, can be solved by means of decisions. This property is represented in the set of simple alternative Petri nets, as their name states, by the presence of different alternative models for the system. One of them should be chosen and when this is done, the system is univocally specified. On the other hand the exclusiveness property is defined in the compound Petri net by a set of feasible combinations of values for the undefined structural parameters, given by the set $S_{val\alpha}$.

Moreover, it is also possible to describe a third additional way to perform the representation of a set of simple alternative Petri nets. This new representation can be obtained from a different process of mixing the simple alternative Petri nets called aggregation. The resulting Petri net, which can also be a simple or a compound Petri net, is called alternatives aggregation Petri net. This Petri net can be obtained from a set of simple alternative Petri nets as well as from a compound Petri net.

The alternatives aggregation Petri nets can be an efficient way to represent a disjunctive constraint based on Petri nets. The property of exclusiveness that characterizes the original simple alternative Petri nets is also included in this AAPN. In this case, some choice variables are defined in order to implement the different exclusive alternative Petri nets. The choice variables verify that only one of them can and must be active as a consequence of a decision.



$$W^A = \begin{array}{c} \begin{array}{ccccc} t1 & t2 & t'2 & t3 & t4 \end{array} \\ \begin{pmatrix} -1 & 1 & 0 & 0 & 1 \\ 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \\ \begin{array}{cc} a_1 & a_2 \end{array} \end{array} \begin{array}{c} p1 \\ p2 \\ p3 \\ p4 \end{array}$$

$S_\alpha = \{ \alpha_a \}$; $S_{val\alpha} = \{ 1,2 \}$
$S_A = \{ a_1, a_2 \}$

Figure 3: An alternatives aggregation Petri net

In the figure 3, an example of alternatives aggregation Petri net (AAPN) can be seen. It is equivalent to the different representations based on Petri nets shown in the figures 1 and 2. In the figure 3 there is an undefined marking parameter and a set of choice variables. Those variables are Boolean ones and verify the exclusiveness property in the sense that one of them and only one of them can be true at a time. In fact this “activation” of a single choice variable may happen after a decision on that subject has been taken. The choice variables are associated to some transitions as Booleans or Boolean functions which allow the firing of the associated transition when it is enabled and the choice function of choice variables is true.

Those choice variables can be associated to a set of choice colours linked to an equivalent Petri net. This transformation allows to develop a representation of an AAPN by means of a coloured Petri net. This last representation can be useful for the reuse of simulation and optimization software for CPN and the application of the theoretical results of the CPN to the AAPN.

4. SOLUTION ALGORITHM OF THE OPTIMIZATION PROBLEM BASED ON DIFFERENT REPRESENTATIONS FOR THE DISJUNCTIVE CONSTRAINT

Every one of the representations of a disjunctive constraint based on a Petri net allow the development of a process to obtain a solution for an optimization problem with certain particularities that can provide with an enhanced or a reduced performance according to the specific problem that is aimed to be solved.

A set of alternative Petri nets allow a solution procedure based on a set of optimization problems that can be solved independently. In fact, this approach, traditionally known as “divide and conquer”, overcomes the disjunctive constraint by solving a single problem for every restriction of the family that configures the disjunctive constraint. Once every problem is solved, a choice can be done among the solutions. The chosen solution complies a restriction which is associated but none of the rest that belong to the disjunctive constraint.

This strategy of “divide and conquer” is a classical approach that can be afforded by means of independent and simple processes. The drawbacks that can be considered for this technique are the need to perform a set of different optimizations, some of which will not provide with a good solution, hence they may constitute a waste of time. On the other hand, it is necessary to develop a subsequent stage of comparison of the results of the independent optimizations to obtain the best solution for the global problem. On the other hand, a clear advantage can be found for this approach. Usually, the definition of the set of simple alternative Petri nets arises directly from the statement of the optimization problem and it is a very intuitive and natural way of representing the possible alternative models for the system to be analysed.

This idea is easy to understand when the design of a manufacturing facility is aimed. In this case, it is possible to choose different machines to be acquired and set up. Sometimes there are different suppliers for the same stage of a certain production line. In this case the feasible combinations of different models of machines lead to a set of alternative models for the manufacturing facility that is being designed. Every one of those models is an alternative where one and only one of them must be chosen.

From the design of a manufacturing facility it is usually expected to obtain a system able to optimize certain objectives, for instance, to maximize the operational benefit and the machine utilization rate and to minimize the costs and the work in process. As a consequence, an optimization problem can be stated in order to develop the design process of the system. The feasible solutions of the optimization problems should comply with a set of constraints. In particular, these constraints define the solution space. As it can be easily deduced, every alternative Petri net constraints the values a solution can take. Moreover, any solution should only verify the constraints imposed by a single alternative Petri net, not by the rest of them. As a consequence, the set of alternative Petri nets constitutes a set of alternative constraints to the optimization problem, in other words, a disjunctive constraint.

A compound Petri net consists of a representation of a disjunctive constraint that is associated to a single incidence matrix, where in the previous case with the set of simple alternative Petri nets, a set of incidence matrices could be found. In principle, the reduction of the number of incidence matrices to be considered in the statement of the optimization problem may imply a reduction in the volume of information to be stored in the memory of the computer that would perform the solution process. This statement is especially true when the simple alternative Petri nets that are merged into a compound Petri net have similarities that reduce the size of the sets of undefined structural parameters and of feasible combination of values for those parameters.

In fact, given an optimization problem based on a Petri net, with disjunctive constraints, it is difficult to decide which representation to take for the Petri net model of the system. There are, nevertheless, certain characteristics that are more suitable for one of the representations previously mentioned. In particular, the larger the similarities between the alternative Petri nets, the more efficient a compound Petri net might be with respect to an equivalent set of alternative Petri nets.

From a given problem it may be easier to obtain one or other representation for the Petri net model of the system (a set of alternative Petri nets or a compound Petri net). The less evident representation may be obtained from the other one. In fact, it is possible to obtain any of both descriptions from the other one.

The process to obtain a set of alternative Petri nets from a compound Petri nets consists on associating an alternative Petri net to every one of the feasible combinations of values for the undefined structural parameters. This decomposition of the compound Petri net will eliminate the undefined structural parameters but it will increase the number of incidence matrices needed to represent the undefined Petri net. The exclusiveness among the feasible combination of values for the undefined structural parameters is transformed into the exclusiveness between the different alternative Petri nets.

On the other hand, the process to obtain a compound Petri net from a set of alternative Petri nets is based on a process of merging the alternative Petri nets into a single compound Petri net. This process will reduce the n incidence matrices of the alternative Petri nets into a single one with a set of undefined structural parameters.

It is interesting to notice that the set of alternative Petri nets that can be obtained from a compound Petri net is not unique, and the compound Petri net that can be built up from a set of alternative PN is neither unique. There is, in fact, a large number of Petri nets or sets of them that can be deduced from the other. Moreover, there are some interesting operations that can be performed on any representation of a disjunctive constraint of this type, also called undefined Petri net with undefined structural parameters. Those operations preserve the equivalence between the representations.

Nevertheless, it is possible to proof that given a set of alternative Petri nets, it can always be obtained a set of canonical Petri nets, which is the equivalent set of PN, where any of them presents an incidence matrix of minimal size. As a consequence it is possible to state that given a compound Petri net, the equivalent set of canonical Petri nets is unique.

Other alternative representation of an undefined Petri net with undefined structural parameters, an alternatives aggregation Petri net, will allow to develop a representation of a disjunctive constraint based on a Petri net which is associated to a single incidence matrix. It has to be noticed that any equivalent compound Petri net also presents a single incidence matrix, but the AAPN may not contain undefined structural parameters (if it is a simple AAPN) or it may contain them (if it is a compound AAPN), whereas the compound Petri net includes always variables in the incidence matrices by definition. The handling of a single incidence matrix without undefined parameters in it (that require the storage of an additional set of feasible combinations of values for them), may imply a better performance for the optimization algorithm than an equivalent compound Petri net if the size of the resulting incidence matrices are similar.

In previous paragraphs it has been explained that it is possible to develop algorithms to transform sets of alternative Petri nets into compound Petri nets and vice versa. It is also possible to perform any transformation between the AAPN and other equivalent representations of an undefined Petri net as the compound PN and sets of alternative PN. For example, the development of an equivalent AAPN from a set of alternative Petri net, can be performed from the concepts of shared subnets, link transitions and reduction and simplification rules. In more detail, a set of shared subnets is searched in the set of alternative PN. Then, an aggregation process of the non-shared subnets, related by the link transitions is done. The following step is the association of the choice variables related to the original alternative Petri nets to the link transitions provided by the corresponding alternative Petri net. Finally, reduction and simplification rules are applied in order to decrease the complexity of the functions of choice variables associated to the resulting AAPN.

The last equivalent representation of a disjunctive constraint, a coloured Petri net, will provide with a model, very similar to the original AAPN. As a consequence, the performance of the associated optimization processes is expected to be the same.

In this case, the transformation between an AAPN and an equivalent CPN is immediate. It is only necessary to relate the set of choice variables with a set of colours.

5. COMPARISON BETWEEN DIFFERENT SOLUTION PROCESSES OF AN OPTIMIZATION PROBLEM WITH A DISJUNCTIVE CONSTRAINT BASED ON A PETRI NET

The purpose of the different representations for a disjunctive constraint based on Petri nets that have been researched so far is to develop an exhaustive and comprehensive analysis of the possible ways to handle this constraint in the solution process of an optimization problem. As it has been explained in the previous section, there are significant differences in the algorithm that implements a solution process of an optimization problem regarding to the representation of the disjunctive constraint that has been considered (Latorre, Jiménez and Pérez 2010b).

The approach based on a set of simple alternative Petri nets and the one based on a compact mix of these simple alternative PN, obtained by merging (obtaining a compound PN) or aggregating the PN (leading to an alternatives aggregation Petri net), can be compared considering the following facts:

1. A set of simple alternative Petri nets usually provide with incidence matrices associated to every simple Petri net. These incidence matrices are smaller than the ones associated to a more compact equivalent representation, like a compound PN, an AAPN or a CPN.

2. When the similarities between the different simple alternative Petri nets are significant, the size of the incidence matrix of the compact representation of the disjunctive constraint is reduced dramatically. For this reason the time needed to operate with the compact incidence matrix in the solution process of the optimization problem may be less than the addition of the optimization processes of the simple alternative Petri nets which compose the complete set.

The approaches based on the compound Petri net and on the alternatives aggregation Petri nets can also be compared by the following considerations:

1. 1. As a general rule, an AAPN profits from similarities between subnets of the simple alternative Petri nets.

2. Also as a general rule, a compound Petri net profits from similarities between more distributed features of the simple Petri nets.

As a consequence, the same disjunctive constraint can lead to optimization algorithms with significantly different performance, regarding to the specific problem that is aimed to be solved.

6. CONCLUSIONS

An optimization problem based on the Petri net model of a discrete event system may contain disjunctive constraints. The time required to obtain a solution for this problem is usually an important requirement for practical applications in such a degree that the solution might not be useful if the delay time to obtain it is important.

Any reduction in the time needed to solve a problem of this kind can have important consequences in the applicability of a certain technique.

A classical approach to solve optimization problems with disjunctive constraints consists of dividing the problem into simpler ones to be solved independently and compared to choose one of the solutions as the best one.

In this paper this classical approach has been integrated in a systematic analysis of the different representations of the disjunctive constraint.

As a conclusion, it has been seen that the classical approach is associated to a set of simple alternative Petri nets.

On the other hand, a number of additional representations of the disjunctive constraints can be deduced from the systematic analysis mentioned before.

Every one of the representations found for the disjunctive constraint lead to optimization algorithms that have been presented in this paper may show very different performances. As a consequence, the study of the best representation of a disjunctive constraint can be an important issue to obtain an efficient algorithm to solve optimization problems with disjunctive constraints based on a Petri net.

This paper describes for the first time the systematic analysis that have allow to discover the different representations of the disjunctive constraint and to analyse the possible applications in the solution algorithms of the optimization problems.

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