COLOURED PETRI NETS AS A FORMALISM TO REPRESENT ALTERNATIVE MODELS FOR A DISCRETE EVENT SYSTEM.

Juan-Ignacio Latorre[(a)], Emilio Jiménez[(b)], Mercedes Pérez[(c)]

[(a).] Public University of Navarre. Dept. of Mechanical Engineering, Energetics and Materials. Campus of Tudela, Spain
[(b)] University of La Rioja, Department of Electrical Engineering. Logroño, Spain
[(c)] University of La Rioja, Department of Mechanical Engineering. Logroño, Spain

[(a)] juaningnacio.latorre@unavarra.es, [(b)] emilio.jimenez@unirioja.es [(c)] mercedes.perez@unirioja.es

ABSTRACT
Coloured Petri nets (CPN) constitute a formalism that belongs to the paradigm of the Petri nets, used to model discrete event systems (DES). This formalism has been extensively used to represent complex systems and shows its full potential when arise a large number of subnets with the same static structure thanks to the folding process. In this paper a completely new application of the Coloured Petri nets is presented. It implies a conceptual variation in the traditional scope of use of the CPN. The coloured Petri nets will be used to represent sets of alternative Petri nets. In other words, they will represent a set of exclusive models for a single DES by means if a unique CPN. The main advantage in this application of the coloured Petri nets is that they can be used to develop efficient algorithms to solve optimization problems based on Petri net models.

Keywords: coloured Petri nets, alternatives aggregation Petri net, compound Petri net, alternative Petri net

1. INTRODUCTION
The coloured Petri nets are a broadly used formalism to model discrete event systems. They present particular interest when the real system to be modelled includes several subsystems which have the same static structure. In this case a folding procedure can be afforded in order to obtain a single subnet from the original set of subsystems with the same static structure. The subnet that results from the folding process need to include additional information which is presented as the colour attributes of every token. In this way it is possible to distinguish when a certain token present in the folded subsystem belongs or not to a certain subsystem of the original unfolded Petri net (Silva 1993, David and Alla 2005).

In this paper, it will be presented a completely new application of the coloured Petri nets, which profits from the property mentioned in the previous paragraph. This new application, which implies an important conceptual variation from the previous systems where the coloured Petri nets have been used so far, consists of the modelling of different alternative discrete event systems by means of a single coloured Petri net.

A set of discrete event systems are said to be alternative when they have the property of the exclusiveness among them. This property means that when one of the systems is chosen for being analysed, the others cannot be included in the analysis since only one of them can exist at a time in a real environment (Latorre et al. 2009c). As it has been seen, alternative DES are related to decisions. It is necessary to choose one alternative system among a set of them to define an initially undefined discrete event system. In a sense, it is possible to state that every Petri net in a set of alternative Petri nets is a model suitable for a different problem (Latorre, Jiménez and Pérez 2009a). This characteristic is very useful to reuse the model for the posing of diverse problems on a certain real system and to perform an efficient application of an optimization algorithm. As it has been seen, the conceptual leap from previous applications of CPN is important.

If the formalism of the Petri nets is used to model the original alternative discrete event systems, the result of the modelling process is a set of alternative Petri nets. This set of alternative Petri nets can be used to perform simulations and determine the quality of every one of the Petri nets regarding the achievement of a certain objective (or group of objectives). In fact, an optimization problem can be stated in order to apply a solving technique which may provide with a good solution for the problem.

The solving algorithm may have different characteristics according to the representation considered for the set of alternative Petri nets (Latorre, Jiménez and Pérez 2010a). When the information needed to define in an unambiguous way the set of Petri nets is efficiently compacted, by means for example of a folding procedure, the resulting description will not include redundant information which may lead to an efficient storage of the definition in a computer and, as a consequence, an efficient solving procedure. Coloured

Petri nets allow to obtain a single model that contains the description of every Petri net which belongs to a certain set of alternative Petri nets.

2. THE ALTERNATIVES AGGREGATION PETRI NETS

Coloured Petri nets developed to represent a set of alternative Petri nets are conceptually based in the alternatives aggregation Petri nets (AAPN).

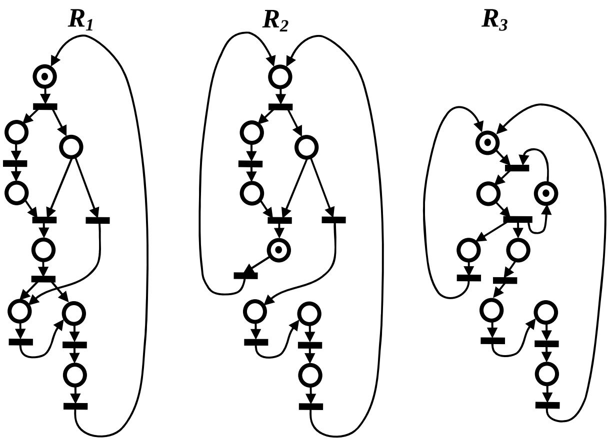

Figure 1: Simple alternative Petri nets

An AAPN is a Petri net obtained from the aggregation of Petri nets from a set of alternative Petri nets. The aggregation process of alternative PN to obtain an AAPN can be regarded as a folding procedure, in the sense that subnets with the same static structure that belong to different alternative PN are reduced to a single subnet in the AAPN. The subnets which have the same static structure are called shared subnets and they can be R-shared if the static structure and initial marking are the same in all of them or Q-shared when the static structure but not the initial marking is the same in all the cases (Latorre et al. 2009c).

In the figure 1 it can be seen a set of simple alternative Petri nets. They are alternative PN because in the example that is being presented, they are exclusive models to the same discrete event system, which behave in different ways. Nevertheless, they comply with the specifications of the DES, which allow certain freedom degrees to be specified by means of the appropriate decisions. These freedom degrees are the cause of the existence of three alternative Petri nets, any of which might be chosen as model for the system.

In order to build up an alternatives aggregation Petri net from a set of alternative Petri nets, an aggregation procedure of the different alternative PN should be performed. In fact, a specific alternative PN to be aggregated is previously divided into subnets and link transitions that connect the different subnets. The non-shared subnets are added to the AAPN and all the link transitions. All the link transitions of a certain alternative Petri net are associated to the choice variable that is related to this alternative Petri net.

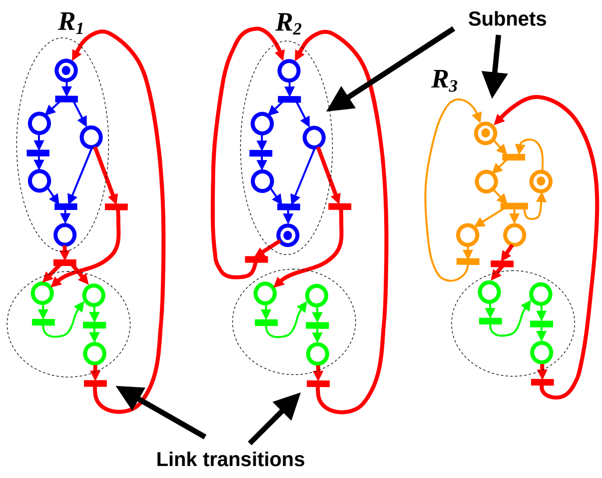

Figure 2: Decomposition in shared and non-shared subnets and link transitions.

In the figure 2 it is shown an efficient decomposition of the original set of alternative Petri nets into shared and non-shared subnets and link transitions. In fact there is a subnet that is shared by only R_1 and R_2, another subnet shared by the three alternative PN and a last subnet that belongs only to R_3. All the transitions that are not included in the subnets, because link different subnets or even a subnet with itself, are called link transitions. This decomposition is efficient because a large amount of shared subnets between the different alternative PN has been found.

The choice variables are Boolean parameters defined in a number which is equal to the cardinality of the set of alternative Petri nets and each one of them is associated to a different alternative Petri net, such that a bijection is established between the set of choice variables and the set of alternative Petri nets. The choice variables represent the information lost in the aggregation process in the same way that the attributes of the tokens (colours) represent the information that is lost in the folding procedure when a CPN is built up from a Petri net.

The resulting alternatives aggregation Petri net is a Petri net that includes in its description all the alternative models for a real discrete even system that have been aggregated. The same AAPN can be reused for the statement of different decision problems on the same real system and to perform an optimization procedure with enhanced performance.

Figure 3 shows an alternatives aggregation Petri net, which is equivalent to the original set of simple alternative Petri nets represented in the figure 1. In this net there is a set of choice variables: $S_A = \{ a_1, a_2, a_3\}$.

The static structure of the original set of alternative Petri nets were described by means of a set of incidence matrices, while the resulting AAPN requires a single incidence matrix for an unambiguous description. In general the incidence matrix of the AAPN has larger size than any of the incidence matrices of the alternative Petri nets. Nevertheless, it is also usual that the shared subnets among the alternative Petri nets lead to an incidence matrix for the AAPN where some redundant information present in the matrices associated to the alternative Petri nets has been removed. As a consequence, the storage and operation with the incidence matrix of the AAPN may be more efficient than the handling of so many incidence matrices as alternative Petri nets are in a certain optimization problem. In other words, the AAPN help to develop efficient algorithms with enhanced performance.

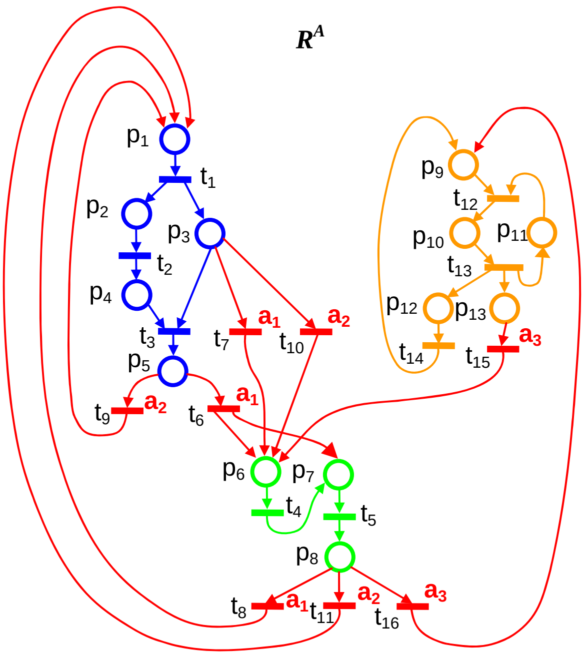

Figure 3: Alternatives aggregation Petri net

3. ALGORITHM TO TRANSFORM AN AAPN INTO A CPN

The procedure to obtain a CPN from an AAPN is almost immediate. The static structure of the AAPN and the one of its equivalent CPN are the same. Both of these Petri nets can be obtained by a process whose nature is similar: an aggregation of alternative Petri nets in the case of AAPN and the folding of different subnets in the case of a CPN. Nevertheless, it is important to realize that aggregation and folding are not the same process, since the former starts from a set of alternative models for a certain DES and the latter starts from a set of subnets that represent different subsystems of a certain discrete event system. In other words, the CPN is built up from a set of subsystems that may exist in a real environment, since the alternative Petri nets are models of systems from which only one might exist, while the rest are discarded models for a certain application.

The choice variables comply with a property of exclusiveness, because only one of them may be active at a time, meaning that a decision has been made to choose a certain alternative PN from the original set as a model for the real discrete event system. These variables also represent the information that has been lost in the process of aggregation, since a token which is present in a shared subnet of the AAPN belongs, in fact, to a certain alternative Petri net. The choice variable that is active when the token exists will point to the alternative Petri net to which it belongs.

In the translation procedure from the AAPN to the CPN, the choice variables should be transformed into the attributes of the tokens. In particular, each choice variable will lead to a choice colour. The activation of a choice variable in the original AAPN will be equivalent to the initialization of the equivalent PN with the initial marking associated to the choice variable and with all the tokens associated to the corresponding choice colour.

As it can be deduced, bijections can be defined between the set of alternative Petri nets, the set of choice variables and the set of choice colours. The decision of choosing a certain alternative Petri net as model for a real discrete event system can be done after the selection of a choice colour in an appropriate CPN model.

The exclusiveness property present in the original set of alternative Petri nets, where only one of them can be chosen as model for the original DES, and also present in the set of choice variables, where only one of them can be activated at a time, should also be present in the set of choice colours. The set of choice colours shows also the property of exclusiveness since the CPN that results from a certain AAPN has a marking with a monochrome choice colour. That is to say only one choice colour can be active at a time.

The name of choice colour implies that there is a possibility of having non-choice colours. The non-choice colours may not comply with the monochrome property (exclusiveness). In fact, the original AAPN might also be a coloured Petri net. Nevertheless, the translation process from the AAPN to the equivalent CPN adds the choice colours that should be clearly different from the other colours.

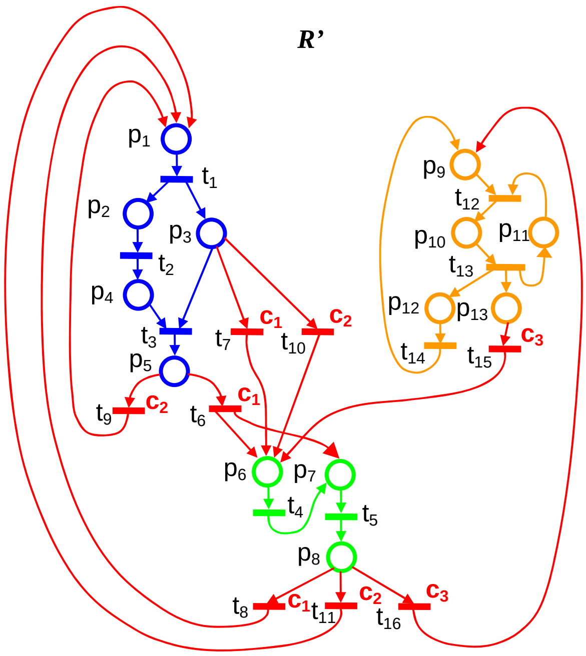

Figure 4: Coloured Petri net

The monochrome choice colour implies that the functions associated to the transitions to allow their firing, cannot change the choice colour of a token. During the evolution of a CPN, only one choice colour may exist for the tokens.

The last step in the transformation of an AAPN to a CPN is to translate the functions of choice variables associated to the transitions of the AAPN into functions of choice colours associated to the transitions of the equivalent CPN. This transformation is immediate by the substitution of the choice variables by the choice colours.

As it has been seen, three steps are included in the algorithm to transform an AAPN to a CPN: The static structure is left the same, the choice variables are translated into monochrome choice colours and the functions of choice variables are transformed into functions of choice colours.

In the figure 4, a coloured Petri net has been obtained from its equivalent alternatives aggregation Petri net. It can be noticed that the structure of both Petri nets are the same. In other words, their incidence matrices have the same components. In order to obtain the set of choice colours { c_1, c_2, c_3}, a correspondence with the set of choice variables of the AAPN has been set.

As a result of the algorithm described in this section, a coloured Petri net can be built up to describe the alternative Petri net models of a certain discrete event system, by means of monochrome choice colours and functions of choice colours associated to some transitions. The decision to choose one or another behaviour for the real discrete event system (given by the associated alternative Petri net) is performed by the activation of a certain choice colour and the deactivation of all the rest of them.

Even considering that it is possible to build up a CPN directly from a set of alternative Petri nets, the singularity of this application where a single CPN can represent a set of alternative Petri net that leads to the existence of an underlying AAPN related to the CPN has moved the authors to present in this paper the intermediate step of constructing an equivalent AAPN. The authors believe that this detailed process will help to understand the complete process and to take into account the particularities of this process of building up a CPN from a set of alternative Petri net compared to the process of constructing of a conventional CPN in a classical application where there are not any set of alternative models for the original DES.

4. REVERSE TRANSFORMATION

Given a certain CPN with monochrome choice colours, it is possible to obtain an equivalent AAPN with a direct transformation that will have the following characteristics:

1. The static structure of both nets are the same.

2. A set of choice variables should be defined in order to establish a bijection between this new set and the set of choice colours.

3. The property of exclusiveness between the choice colours, represented by making them monochrome, should be transformed into the property that only one choice variable can be active at a time, i.e. after a decision has been taken.

4. The functions of choice colours associated to some transitions should be translated into functions of choice variables, just y substitution of every choice colour by its corresponding choice variable, according to the bijection defined in 2.

The result of this reverse transformation is an alternatives aggregation Petri net that is equivalent to the original CPN. A further step can be developed in order to transform the AAPN into an equivalent set of alternative Petri nets.

5. OTHER TRANSFORMATIONS

In this paper, several representations of a set of alternative Petri nets have been presented: an alternatives aggregation Petri net and an equivalent coloured Petri net. It is possible to transform any of them into any other of them. Nevertheless, there are other representations for the original set of alternative Petri nets as the compound Petri nets that also allows to perform transformations with the representations presented in this paper.

6. APPLICATIONS AND PROPERTIES

As it has been explained in previous sections, there are several applications and properties of the new concept presented in this paper: the use of a coloured Petri net to represent a set of alternative models for a certain discrete event system. Some of them are briefly described below:

1. Reduce the computing effort required to solve a certain optimization problem. The development of a CPN equivalent to a certain set of alternative Petri nets where there are similarities between different alternative models (shared subnets) may increase the efficiency of an optimization algorithm. This property arises as a consequence of the removal of redundant information in the description of the Petri net, mainly the incidence matrix (Latorre, Jiménez and Pérez 2010a).

2. Reuse of models. Given a certain undefined discrete event system, a set of alternative Petri nets can be developed as exclusive models. As a consequence an equivalent AAPN-CPN can be obtained from the set of alternative Petri nets. For a given DES, and hence for a given AAPN-CPN model, several different optimization algorithms can be stated, for example by means of the definition of different objectives to be achieved by the DES. In other words, by defining different objective functions it is possible to state different optimization problems and reuse the same AAPN-CPN model to represent and analyse the evolution of the same discrete event system.

3. Reuse of software. Given an optimization problem based on an AAPN as model for the original DES, it is possible to obtain the equivalent CPN and reuse the software developed for general applications of CPN in order to implement the optimization algorithm to solve it.

4. Recycle the models developed for certain alternative Petri nets. In the case of there are similarities between the alternative Petri nets, something usual in the design process of discrete event systems, it is possible to reuse the models of some alternative Petri nets and avoid the development of shared subnets that have already been created, since an equivalent AAPN-CPN may remove the redundant information. The non-shared subnets can be taken from the alternative Petri nets to the equivalent AAPN-CPN and inserted as blocks in the resulting incidence matrix or as subnets for the graphical representation.

7. CONCLUSIONS

The coloured Petri nets have been extensively used in the last years for the modelling of technological discrete event systems of very different sectors.

In this paper a conceptually new application is presented: the obtaining of a single CPN for the representation of a set of alternative Petri nets which have been developed as exclusive models for a certain discrete event system.

This new application arises from the definition of an underlying alternatives aggregation Petri net and has interesting applications and properties that are described in this paper.

The use of CPN as formalism to represent sets of alternative models allows to make the decision-taking and optimization processes based on discrete event systems easier to develop and more efficient to apply in a computer.

REFERENCES

R. David, H. Alla, "Discrete, Continuous and Hybrid Petri Nets," Springer, Berlin. 2005

J. Heizer, B. Render, "Operations Management," Prentice Hall. 2008.

E. Jiménez, "Techniques of advanced automation in industrial processes" "Técnicas de automatización avanzada en procesos industriales," PhD Thesis. Ed. Univ. de La Rioja, Logroño. 2002

E. Jiménez, , M. Pérez, , J.I. Latorre, "On deterministic modelling and simulation of manufacturing systems with Petri nets". Proceedings of European Modelling Simulation Symposium. Marseille, pp. 129-136. Marseille. 2005

E. Jiménez, , M. Pérez, I. Latorre. Industrial applications of Petri nets: system modelling and simulation. Proceedings of European Modelling Simulation Symposium, pp. 159-164. Barcelona. 2006

A. Kusiak, "Computational Intelligence in Design and Manufacturing". Wiley-Interscience. 2000

J.I. Latorre, E. Jiménez, M. Pérez, "Comparison of optimization techniques applied to a flexible manufacturing system," Proceedings of the 18th European Modelling and Simulation Symposium (EMSS 06). Barcelona, pp. 141-146. October 2006.

J.I. Latorre, E. Jiménez, M. Pérez, "Macro-Reachability Tree Exploration for D.E.S. Design Optimization," Proceedings of the 6th EUROSIM Congress on Modelling and Simulation (Eurosim 2007). Ljubljana, Slovenia, September 2007.

J.I. Latorre, E. Jiménez, M. Pérez, "Decision taking on the production strategy of a manufacturing facility.

An integrated methodology," Proceedings of the 21st European Modelling and Simulation Symposium (EMSS 09). Puerto de la Cruz, Spain, vol. 2, pp. 1-7, September 2009.

J.I. Latorre, E. Jiménez, M. Pérez, E. Martínez, "The design of a manufacturing facility. An efficient approach based on alternatives aggregation Petri," Proceedings of the 21st European Modelling and Simulation Symposium (EMSS 09). Puerto de la Cruz, Spain, vol. 2, pp. 33-39, September 2009.

J.I. Latorre, E. Jiménez, M. Pérez, J. Blanco, "The problem of designing discrete event systems. A new methodological approach," Proceedings of the 21st European Modelling and Simulation Symposium (EMSS 09). Puerto de la Cruz, Spain, vol. 2, pp. 40-46, September 2009.

J.I. Latorre, E. Jiménez, M. Pérez, "A genetic algorithm for decision problems stated on discrete event systems." Proceedings of the UKSim 12th International Conference on Computer Modelling and Simulation. Cambridge, United Kingdom, March 2010.

J.I. Latorre, E. Jiménez, M. Pérez, "The alternatives aggregation Petri nets as a formalism to design discrete event systems." International Journal of Simulation and Process Modeling, Special Issue. 2010

D.Y. Lee, F. DiCesare. "FMS scheduling using Petri nets and heuristic search". Robotics and Automation. Proceedings, Volume 2, 1057 – 1062. 1992.

G. Music, "Petri net based scheduling approach combining dispatching rules and local search, " Proceedings of the 21st European Modelling and Simulation Symposium (EMSS 09). Puerto de la Cruz, Spain, vol. 2, pp. 27-32, September 2009.

M.À. Piera, M. Narciso, A. Guasch, D. Riera, "Optimization of logistic and manufacturing system through simulation: A colored Petri net-based methodology," Simulation, vol. 80, number 3, pp 121-129, May 2004

M. Silva. "Introducing Petri nets, In Practice of Petri Nets in Manufacturing" 1-62. Ed. Chapman&Hall. 1993

M. Zhou. K. Venkatesh. "Modelling, Simulation and Control of Flexible Manufacturing Systems. A Petri Net Approach", WS World Scientific, 1999

AUTHORS BIOGRAPHY

Juan Ignacio Latorre is electrical engineer. He has developed part of his professional career in the manufacturing private sector. Nowadays he is associated professor in the Public University of Navarre. He is developing his PhD thesis in optimization of manufacturing processes based on Petri net models. His research interests include automation, optimization and Petri nets.

Emilio Jiménez Macías studied Industrial Engineering (with Computer Science, Electronics and Automation specialty) by the University of Zaragoza, and in 2001 presented his PhD thesis about Industrial Automation, in the University of La Rioja, where he works presently in the Electrical Engineering Department (Coordinator of the System Engineering and Automation Group). His research areas include factory automation, modeling and simulation (Petri nets), and industrial processes.

Mercedes Pérez de la Parte received the BS degree in Telecommunications Engineering from the University of Seville in 1997. She has held research and teaching positions, since 1997 until present, at the University of Seville (Spain), at the University of Los Andes (Venezuela), and the University of La Rioja, where she is an Assistant Professor at the Mechanical Engineering Department.