A STOCHASTIC APPROACH FOR THE DESIGN OF END-OF-LINE STORAGE AREA

Sara Dallari[a], Elisa Gebennini[b], Andrea Grassi[c], Magnus Johansson[d]

[a, b, c] Dipartimento di Scienze e Metodi dell'Ingegneria, Università degli Studi di Modena e Reggio Emilia
Via Amendola 2 – Pad. Morselli, 42122 Reggio Emilia, Italy

[d] Elettric 80 S.p.A.
Via G. Marconi 23, 42030 Viano (RE), Italy

[a]sara.dallari@unimore.it, [b]elisa.gebennini@unimore.it, [c]andrea.grassi@unimore.it, [d]johansson.m@elettric80.it

ABSTRACT
Nowadays, many companies tend to minimize inventories throughout the entire logistic process by adopting a cross-docking policy, i.e. preferring direct shipping rather than to store products.

In these systems the objective is to maintain synchronization between production and shipping processes, but storage areas are still necessary in practice to work as buffers able to compensate short period mismatches.

The paper proposes a stochastic approach to design such storage areas. In particular, the case in which a good trade-off between the number of storage zones (called *bins* in the following) and their size is addressed. The simulation is adopted to determine storage requirements from a stochastic point of view, and an analytic formulae is derived to evaluate an approximate coverage probability.

Keywords: cross-docking, storage requirements, stochastic aspects, block-stacking , lane depths

1. INTRODUCTION

The situation addressed in this paper is representative of many industrial companies, especially companies producing final goods with a shelf life quickly absorbed by market (e.g. beverage, foods). In this case, cross-docking policies are generally preferred (see Apte and Viswanathan 2000; Baker 2007).

In this paper it is assumed that Stock-Keeping Units (SKUs) are placed in specific zones of the storage area called *bins*. Such bins are limited zones where a number of pallets of the same SKU type can be stored. We assume that each bin has a single access point. Usually, bins are positioned on the ground or by gravity shelves.

Bins are not dedicated to a particular SKU type, but once just one pallet of a certain type of SKU is placed into a bin, that bin must hold that specific SKU type only. Once a bin gets empty again, it could be assigned to another SKU type as needed.

Configurations with an high number of low-capacity bins allow to hold different SKU types contemporaneously but the space available for storing products is reduced as a consequence of an high number of aisles needed to access the bins. On the other hand configurations with few high-capacity bins makes it possible to have an increased total storage space but it can hold only few SKU types. A trade off should be found to determine a good configuration.

This paper deals with the design and management of such storage areas. Thus, *tactical decisions*, i.e. medium term decisions regarding the dimensioning of the system and the layout determination (see the classification by Rouwenhorst et al. 2000) are addressed.

Even if various frameworks about warehouse design are available in the literature (refer as exemples to Baker and Canessa 2009; Ashayeri and Gelders 1985), the problem addressed in this paper is quite innovative and it is related to the pallet block-stacking problem. Block-stacking refers to unit loads stacked on top of each other and stored on the warehouse floor in lanes.

Gu et al. (2010) provided a comprehensive review of existing research results about warehouse design and performance evaluation, in particular for what concerns the pallet block-stacking problem.

This specific problem was treated by several authors and in particular the decision about the selection of lane depths was addressed.

Moder and Thornton (1965) developed mathematical models to evaluate ways of stacking pallets. Berry (1968) proposed an analytic model to evaluate a tradeoff between the material handling costs and the space utilization. Marsh (1979) provided a simulation model to estimate the space utilization of different lane depths. Marsh (1983) compared the layout design found using these two aforementioned approaches. A dynamic algorithm to maximize the space utilization by selecting lane depths was developed by Goetschalckx and Radliff (1991). Larson et al. (1997) proposed a heuristic approach to this problem, having the purpose to maximize the utilization of the storage space minimizing material handling costs.

Thus, the specific problem is of interest but it was mainly approached in a deterministic way and, as Gu et al. (2010) pointed out, it was addressed usually with restrictive assumptions. Thus, further improvement is needed taking into account the uncertainty of storage and retrieval requirements.

The aim of this paper is the development of an approach and a methodology to define a good configuration of bins in the storage area. The goodness of a specific configuration is evaluated in terms of capacity of meeting storage requirements, under a stochastic point of view. Space constraints, SKUs' storage requirements and the contemporary presence of different SKUs in the storage area are taken into account.

2. THE METHODOLOGY

The methodological approach proposed in this paper aims to compare different bin configurations by taking into account stochastic aspects (e.g. production and shipping patterns).

In particular the methodology provides:

- a framework to model input data so that a stochastic representation of the space requirements in the storage area is given. This phase is carried out by using a simulation approach;
- the individuation of a performance function able to evaluate the satisfaction of the storage requirements.

The methodology, applied to a significant case study in Section 3, is organized in the following steps:

- Step 1: data collection;
- Step 2: analysis of production and shipping;
- Step 3: simulation model for each SKU type;
- Step 4: simulation campaign;
- Step 5: simulation output analysis;
- Step 6: bin configuration evaluation.

Each of the above steps is described in more detail in the following.

Step 1: Data collection. Production and shipping data referring to a specific period of time can be collected from the Enterprise Resource Planning (ERP) of the system under study.

The focus is on those SKU types that are frequently produced but not in a continuous manner, i.e. when the risk of temporary mismatches between production and shipping is significant. In this case, a proper storage area decoupling the two processes is needed.

Step 2: Analysis of production and shipping. Production and shipping are stochastic processes. We assume that the processes under study behave according to the conceptual model reported in Figure 1.

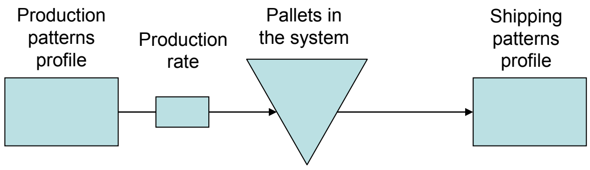

Figure 1: Conceptual model.

On-field production and shipping data can be grouped according to the SKU type so that both order sizes and interarrival times can be derived.

Then, these sequences of interarrival times and sizes of both production and shipping orders are fitted to achieve characteristic mass probability distributions. The probability distributions are the key input data of Step 3 where a simulation model of the system is developed according to the above conceptual model.

Therefore, in order to model realistic operative conditions, the conservation of the mean flow across the system must be guaranteed by the following equation:

$$\textit{Mean production flow} = \textit{Mean shipping flow} \qquad (1)$$

Moreover, any unrealistic values that can derive from the above distributions (e.g. too much large order sizes) are neglected.

Step 3: Simulation model for each SKU type. A simulation model is developed for each SKU type using the discrete-events simulator Fleχsim™, in which production and shipping processes are represented by the mass probability distributions obtained in previous steps (Figure 2). The production orders can be satisfied on different production lines, even at the same time. Pallets processed by production lines enter endless queues waiting to be shipped. Thus, the queues between production and shipping provide the storage requirements over time for each SKU type.

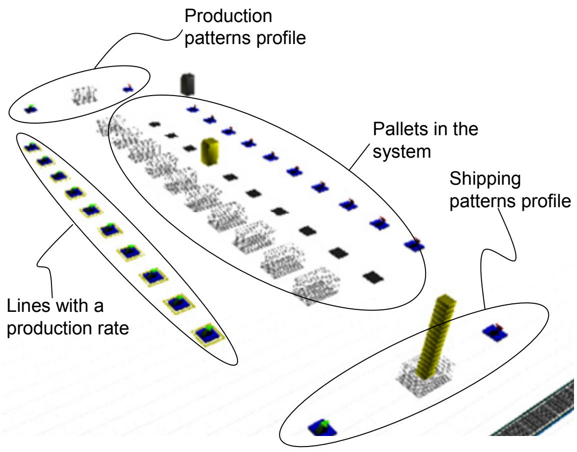

Figure 2: Simulation model.

Step 4: Simulation campaign. The simulation campaign consist in a long simulation run for each SKU type. Periodically, during the run, storage requirements are logged to obtain a representative sample for each SKU type.

Step 5: Simulation output analysis. The simulation output is the trend of storage requirements for each SKU type. Note that the values of interest are those related to the steady state and do not include the warm-up period. As shown in Figure 3 the warm-up period can be easily identified.

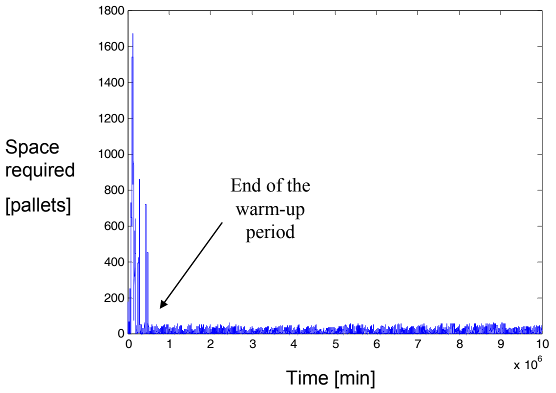

Figure 3: Example of the identification of the warm-up period.

Similarly as input data, output data are fitted as well, so as to achieve the probabilistic representation of the required storage space for each SKU type of interest. Thus, probability distributions of such space requirements are obtained and denoted as $f^i(x)$, $i=1,\ldots,N$, where N is the number of SKU types. An example is shown in Figure 4.

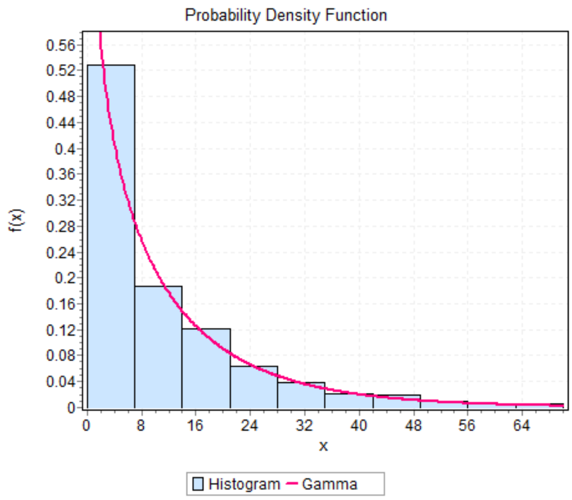

Figure 4: Example of mass probability distribution of storage requirements.

Note that the contemporary presence of different SKU types in the storage area must be taken into account. As an example, if some SKU types are very rarely present at the same time in the storage are (e.g. they are produced in different periods of time), the actual required storage space may be less than the sum of the single space requirements.

Step 6: Bin configuration evaluation. Given the probability distributions related to all the SKU types under study, this step provides a method to evaluate the goodness (or badness) of a certain bin configuration.

Specifically, let k be the number of bins in the storage area and y be the capacity of each bin, the objective is to compute the service level of the possible storage requirements that may occur at the storage area according to a certain probability. Note that the probability of any storage requirement depends on the SKUs' probability distributions.

By varying the parameters k and y, a set of bin configurations, denoted as the BC set, is generated. Among the bin configurations belonging to BC, the feasible ones are that satisfying the following constraint: the space occupied by a certain bin configuration (k,y) (considering both the zones for storing pallets and the aisles) must be at most equal to the space available in the storage area. Thus, a set of feasible bin configurations BC' is obtained.

Each bin configuration (k,y) belonging to BC' can be evaluated considering the mass probability distributions of storage requirements for all the SKU types and the contemporary presence of different SKU types in the storage area.

For the sake of clarity, we focus on the approximate probability of not satisfying the storage requirements (i.e. of exceeding the required storage space) related to a certain bin configuration (k,y), i.e. a badness function $BF(k,y)$ is formulated. The lower the obtained BF value, the better the bin configuration is.

The following notation is adopted:

$i=1,\ldots,N$	SKU type;
k	number of bins;
y	bin capacity;
$f^i(x)$	probability distribution of SKU i, for $i=1,\ldots,N$;
$z_i=0,1,\ldots,k$	variable identifying the number of bins assigned to SKU i, for $i=1,\ldots,N$.

The proposed method consists in selecting all the possible combinations $\{z_i\}_{i=1,\ldots,N}$ so that $\sum_i z_i = k$. As an example, a possible combination consists in assigning all the k bins to the first SKU and none of them to the other SKUs, i.e. $z_1 = k$ and $z_i = 0$ for $i = 2,\ldots,N$.

Then, the badness function can be formulated for a certain bin configuration (k,y) as follows:

$$BF(k,y) = \sum_{\forall \{z_i\}_{i=1,\ldots,N}} \left(\prod_{i=1}^{N} \int_{y \cdot z_i}^{\infty} f^i(x)dx \right). \qquad (2)$$

Among all the bin configurations belonging to BC', the one with the lowest BF value is selected.

3. CASE STUDY

In the case study, we apply the methodology for designing bin configurations proposed in the previous section to the actual storage area of a company operating in the beverage field.

The company under study produces different SKU types (e.g. 1 l still water bottle, 1.5 l still water bottle, 1 l sparkling water bottle, etc.). They can be divided into three SKU classes.

The SKU class I includes all the SKU types produced in high and constant volumes. Such SKUs present a continuous flow from the production process to the shipment area so that they are not significant for the specific problem addressed in this paper, i.e. it is possible to manage both the processes so that the synchronization between them is guarantee.

The SKU class II involves SKU types with a less regular demand. In this case the difficulty in maintaining synchronization between the production process and the shipping process leads to the necessity of an intermediate storage area.

The SKU class III includes SKU types occasionally produced, i.e. only when a specific order is coming.

The methodology proposed in this paper is suitable for the SKU class II. Specifically, seven SKU types are identified in the proposed case study and denoted as *SKU A*, *SKU B*, …., *SKU G*.

The application of all the methodology steps introduced in Section 2 is described in the following.

Step 1: Data collection. Data on a 6 months period are collected from the Enterprise Resource Planning (ERP) of the system. Data are arranged so that production and shipment orders of each SKU type are available over time.

Step 2: Analysis of production and shipping. Data from the previous step are fitted by using a statistical software tool so that the mass probability distributions of both size and interarrival times of production and shipping orders are obtained for each SKU type. In this case, Gamma distributions (location equal to zero) are selected to represent these processes.

The parameters of the Gamma distributions representing the production process are reported in Table 1 as regards the interarrival times and in Table 2 as regards the order sizes. The corresponding Gamma distribution parameters for the shipping process are given in Table 3 and 4.

Table 1: Gamma distribution parameters of the production process – Interarrival times [min]

SKU Type	Shape	Scale	Mean
SKU A	0.743	3001	2229
SKU B	0.104	22878	2372
SKU C	0.140	19061	2667
SKU D	0.380	9376	3567
SKU E	0.290	15845	4593
SKU F	0.155	26365	4089
SKU G	0.228	28561	6521

Table 2: Gamma distribution parameters of the production process – Order sizes [pallets]

SKU Type	Shape	Scale	Mean
SKU A	1.360	87	119
SKU B	1.421	78	111
SKU C	1.404	96	135
SKU D	1.690	77	130
SKU E	1.546	68	105
SKU F	2.161	41	88
SKU G	1.672	69	116

Table 3: Gamma distribution parameters of shipping process – Interarrival times [min]

SKU Type	Shape	Scale	Mean
SKU A	0.133	2050	273
SKU B	0.040	13012	520
SKU C	0.052	13497	698
SKU D	0.133	6447	859
SKU E	0.116	12317	1424
SKU F	0.139	6950	969
SKU G	0.067	19378	1305

Table 4: Gamma distribution parameters of shipping process – Order sizes [pallets]

SKU Type	Shape	Scale	Mean
SKU A	0.642	23	15
SKU B	1.021	24	24
SKU C	3.947	9	35
SKU D	2.117	15	31
SKU E	1.647	20	33
SKU F	1.365	15239	21
SKU G	1.085	21	23

The obtained probability distributions are used to choose, in a probabilistic way, appropriate and realistic input data for the simulation model explained in the next step.

Thus, it is important to assure that the balancing equation (1) is satisfied. Moreover, to avoid unrealistic values for the order size and the interarrival times, proper limits are adopted for the probability distributions describing both the production process (see Table 5) and the shipping process (see Table 6).

Table 5: Distribution limits for the production process

SKU type	Max interarrivaltime [min]	Max size [pallets]
SKU A	11963	470
SKU B	36997	430
SKU C	35619	525
SKU D	27504	465
SKU E	41186	391
SKU F	51697	282
SKU G	66753	418

Table 6: Distribution limits for the shipping process

SKU type	Max interarrivaltime [min]	Max size [pallets]
SKU A	3745	84
SKU B	12286	111
SKU C	14976	89
SKU D	11775	101
SKU E	21012	118
SKU F	12964	82
SKU G	25028	103

Step 3: Simulation model for each SKU type. The simulation model consists in parallel production lines processing pallets of a certain SKU type at a predefined production rate and according to the order size and interarrival times obtained from the previous step. Similarly, shipment orders are generated according to the related probability distributions.

Pallets from the production lines enter a queue waiting for the shipment. When a new shipping order arrives and the number of waiting pallets matches the order size, the queue is reduced of this amount.

Step 4: Simulation campaign. The simulation run, for each SKU type, corresponds to a period of 10.000.000 minutes. During the run, space requirements (i.e. the length of pallets queues) are logged every 100 minutes to obtain a representative sample for each SKU type.

Step 5: Simulation output analysis. Once the warm-up period have been identified and cut-off, data about space requirements of each SKU type are fitted in order to find a representative mass probability distribution.

Figure 5 shows the probability distributions for all the SKU types of interest. Note that Gamma distributions are chosen and the corresponding parameters are reported in Table7.

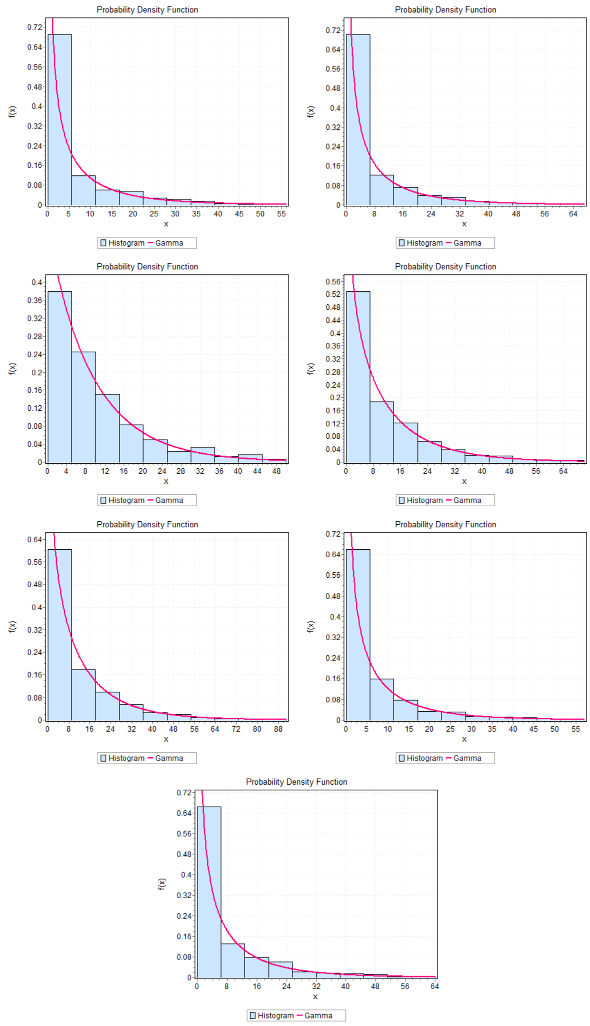

Figure 5: Mass probability distributions of storage requirements for the SKU types of interest

Table 7: Gamma distribution parameters – Storage requirements

SKU types	Shape	Scale	Mean
SKU A	0.392	15	6
SKU B	0.379	18	7
SKU C	0.964	10	10
SKU D	0.728	15	11
SKU E	0.701	17	12
SKU F	0.446	14	6
SKU G	0.461	16	7

Step 6: Bin configuration evaluation. Given the probability distribution $f^i(x)$ for i=A,…,G by Step 5, it is possible to apply equation (2) for any couple (k,y) so that the badness value is compute for a specific number of bins k having capacity y each.

Since functions $f^i(x)$ are continuous functions while the number of pallets to store into the bins is an integer, a further observation is needed: the x-axis can be divided into intervals centered on any integer value so that if $x<0.5$ we consider not to have any pallet to store, if $x < 1.5$ we consider to have at most 1 pallet to store, and so on.

Then, various possible solutions (k, y) belonging to the set of feasible bin configurations BC' can be examined.

Specifically, Table 8 shows the *BF* values for $k = 7$ bins of different capacities, i.e. $y = 9, 10, \ldots, 21$

Table 8: *BF* solutions for *k*=7 and different capacities *y*

Solution #	*y*	*BF*
1	9	0.705
2	10	0.400
3	11	0.229
4	12	0.132
5	13	0.077
6	14	0.045
7	15	0.026
8	16	0.015
9	17	0.009
10	18	0.005
11	19	0.003
12	20	0.002
13	21	0.001

We notice from Table 8 that if a *BF* value less than 0.01 is considered to be acceptable, Solution #9 ($y = 17$) is the first feasible solution. Note that from Solution #9 to Solution #13 the *BF* values are low and closed to each other so that it may be convenient to avoid occupying too much space by choosing the Solution #9, i.e. the one with the less capacity *y* associated.

Similarly, Table 9 reports the *BF* value for a different *k*, i.e. 8 bins in the storage area.

Table 9: *BF* solutions for *k*=8 and different capacities *y*

Solution #	*y*	*BF*
1	9	0.557
2	10	0.294
3	11	0.156
4	12	0.084
5	13	0.045
6	14	0.025
7	15	0.014
8	16	0.007
9	17	0.004
10	18	0.002
11	19	0.001
12	20	0.001
13	21	0.000

In this case, having one more bin, assuming the same *BF* limit of acceptance 0.01, Solution #8 ($y = 16$) is the more convenient.

Therefore, solutions (*k*=7,*y*=17) and (*k*=8,*y*=16) are both able to assure an acceptable *BF* value.

The same procedure can be applied to other bin configurations.

4. CONCLUSIONS

The paper proposes a methodological approach to design storage areas in cross-docking systems. Stochastic aspects of the production and shipping processes are taken into account. The approach consists in identifying storage requirements from a statistical point of view, and then in formalizing a function to compare different bin configurations.

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