DEMAND FORECASTING AND LOT SIZING HEURISTICS TO GENERATE COST-EFFECTIVE PRODUCTION PLANS: A SIMULATION STUDY ON A COMPANY IN THE WOOD FLOORS SECTOR

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ABSTRACT
Master Production Scheduling (MPS) is a very important activity in manufacturing planning and control because the quality of the MPS can significantly influence the total cost. Unfortunately, many companies do not know their future demands and have to rely on demand forecasts to make production planning decisions. Thus in many cases companies must first select a good forecasting method, and then use forecasted demand as input to the planning phase. The paper presents a simulation study, conducted on an industrial case, of these two phases: selection of the most appropriate forecasting method, and production planning through forecasted demands. The aim is double: to quantify improvements, in terms of total costs decrease, with respect to the actual company policy; to evaluate the impact on total costs of the demand forecasting method inaccuracy.

Keywords: Master Production Scheduling, lot sizing, demand forecasting, inaccuracy.

1. INTRODUCTION
The paper presents an industrial case of a company in the wood floors sector of lot sizing under demand uncertainty.

Numerous researches have studied lot sizing and setup scheduling problems, with reviews by Drexl and Kimms (1997) and Karimi, Fatemi Ghomi and Wilson (2003). Many of them assume deterministic demand, and measure performances of different production planning algorithms and procedures in terms of minimization of total costs and computational time. However in many real cases future demand is unknown, and master production scheduling is based on demand forecasts rather than on actual demand.

In this study we simulate master production scheduling activities, performed by a lot sizing heuristic, using data originated by a forecasting procedure. Outputs from the simulation study are used both to evaluate improvements with respect to the actual company policy, and to evaluate the impact of demand forecasting inaccuracy on total costs.

The simulation study considers the most 5 representative items in terms of sales. The product is a two layers parquet. The top layer is constituted from noble wood, with thickness between 3.5 and 5 mm. The inferior layer, technologically more complex, has a support function and allows keeping the pavement without deformation, contrasting the natural tendency to the movement; it consists in a multilayer cross-sectional fiber that guarantees a final product not deformable.

To investigate the impact of demand forecasting method on total costs through computer simulation, we must obtain the demand forecasts. Two alternative approaches have been used to produce the forecasts in previous studies. One approach is to generate the forecasting error according to some probability distribution and add it to the actual demand, as for example in Lee and Adam (1986). The other is to use a forecasting model to make forecasts based on previous demand (see Zhao, Goodale and Lee 1995; Xie, Lee and Zhao, 2004). In this paper we follow this second approach.

For each item demand data on monthly base, for years 2003-2007, are available.

Different demand forecasting methods (see next paragraph) are applied to each item. The first two years (2003-2004) are used for initialization. Then, each method is applied to obtain forecasts (always on a monthly base) for years 2005-2006 and to calculate per period forecasts errors. The method obtaining the minimum Mean Absolute Deviation for each item is selected to perform forecasts for year 2007.

Note that the forecast models are tested with an entirely different set of data (year 2005-2006) from which the model is formed (2003-2004)

Monthly forecasted demands for year 2007 are then used as input for the production planning algorithm. The production planning phase is performed each month, with a rolling time horizon of 4 months. This length is imposed by the lead time needed to obtain the inferior layer needed to manufacture the final product. If the inferior layer is available, requirements for a periods can be covered by the production planned for the same period; in effect, the application of the noble wood layer can be performed in a relative small amount of time.

The simulation study consisted in generating production plans for each months of year 2007. Outputs (inventory levels, setup costs, holding costs) have been compared to what has been done by the company during the same period.

In order to evaluate the impact of demand forecasting methods inaccuracy on total costs, a Mixed Integer Programming model has been implemented to solve to optimality the lot sizing problem under deterministic demand (that is, the actual demand of year 2007). By comparing the optimal solution (obtained with actual demand), and the one carried out by using forecasted demand, it is possible to quantify the influence of using a certain method to obtain forecasts.

1.1. Notation

t = months index
A = setup cost [€]
v = item value [€]
r_y = annual carrying rate [€/€/year]
$r = r_y/12$ = monthly carrying rate [€/€/month]
d_t = actual demand for period t
$F_{t,j}$ = forecasted demand at period t for period $t+j$
I_t = inventory level at the end of period t
Q_t = quantity produced for period t

The index indicating items is omitted because they are considered independently from each others.

2. DEMAND FORECASTING

Four demand forecasting methods have been tested:
- Moving Average.
- Simple Exponential Smoothing.
- Trend Corrected Exponential Smoothing.
- Trend and Seasonality Corrected Exponential Smoothing.

Each of the above method has been implemented using different values of its characterizing parameters in order to find values that, for each item, minimize the MAD during 2005-2006. Four values for the number of preceding periods (N=2, N=3, N=4 and N=5) have been tested for moving averages, while as far as exponential smoothing methods are concerned, a search procedure has been performed in order to find optimal values for the smoothing parameters α, β and γ.

Figure 1 summarizes the results obtained applying each forecasting method to each demand item historical data, measuring the MAD in years 2005-2006. It can be observed that in all cases the Trend and Seasonality Corrected Exponential Smoothing provides the smaller error, and it is therefore selected to forecast demand for year 2007.

In particular, the values that provide better result, in terms of minimization of MAD, are respectively: α=0.3, β=0.1 and γ=0.1 for item#1; α=0.05, β=0.1 and γ=0.2 for item#2; α=0.05, β=0.01 and γ=0.5 for item #3; α=0.25, β=0.2 and γ=0.1 for item #4; α=0.001, β=0.2 and γ=0.3 for item #5.

Figures 2 to 6 shows, for each item, the actual demand against the forecasted demand obtained by applying the selected method for year 2007.

MAD				
Method / Article	Moving Average	Simple Exponential Smoothing	Trend Corrected Exponential Smoothing	Trend and Seasonality Corrected Exponential Smoothing
item #1	≈970	952	954	**874**
item #2	≈210	180	165	**160**
item #3	≈1110	798	815	**722**
item #4	≈550	690	665	**545**
item #5	≈530	517	497	**374**

Figure 1. MAD of different demand forecasting methods on 2002-2006

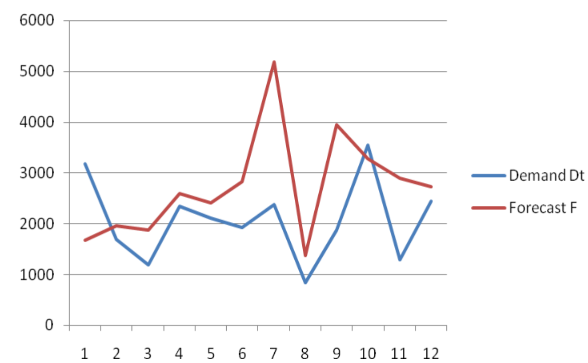

Figure 2. Item#1: comparison between actual and forecasted demand during 2007.

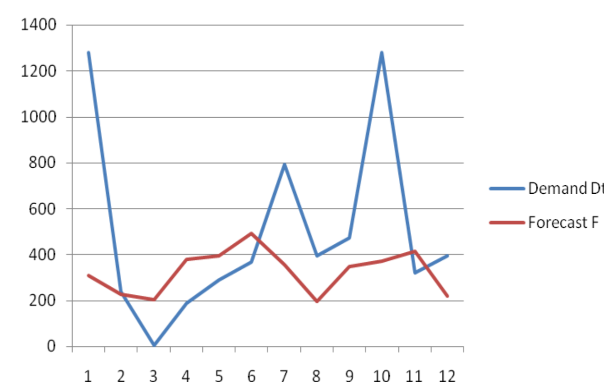

Figure 3. Item#2: comparison between actual and forecasted demand during 2007.

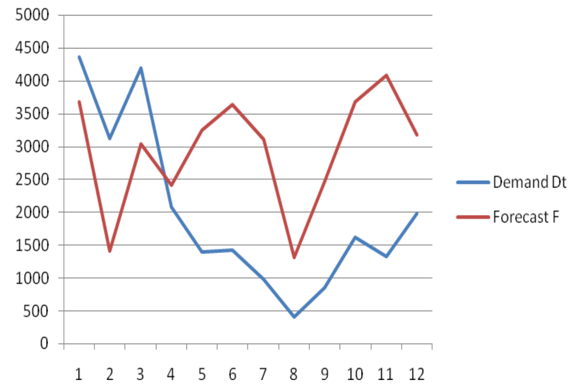

Figure 4. Item#3: comparison between actual and forecasted demand during 2007.

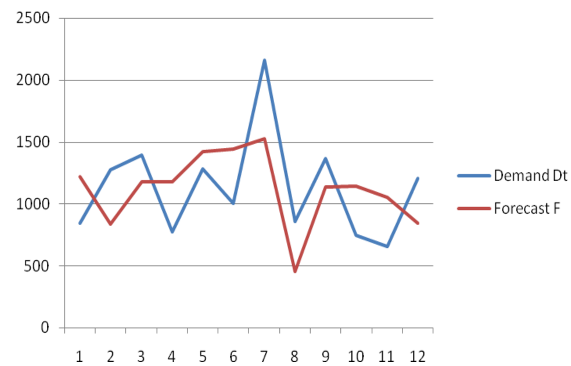

Figure 5. Item#4: comparison between actual and forecasted demand during 2007.

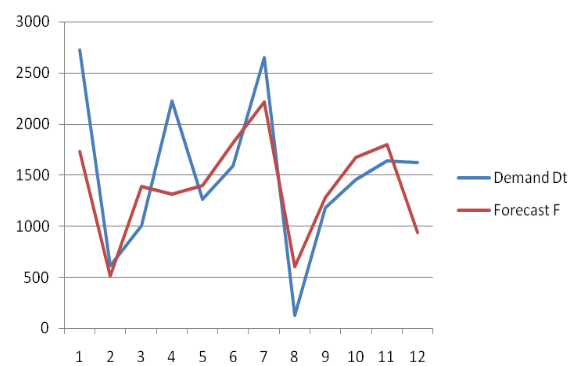

Figure 6. Item#5: comparison between actual and forecasted demand during 2007.

3. PRODUCTION PLANNING

The production planning phase is performed each month, with a rolling time horizon of 4 months. This length is imposed by the lead time needed to obtain the inferior layer needed to manufacture the final product. If the inferior layer is available, requirements for a periods can be covered by the production planned for the same period; in effect the application of the noble wood layer can be performed in a relative small amount of time.

Starting for example by January 2007 (t = 1), the related production plan for this month will indicate production quantities that have to be delivered in January, February, March and April. To do this we need demand requirements for the same periods. Demand requirements for the first month, in this case January, are equal to orders already collected (d_1) and do not need to be forecasted. On the contrary, demand requirements for February, March and April have to be forecasted ($F_{1,1}$, $F_{1,2}$, $F_{1,3}$). We obtain these forecasts using, as mentioned in the previous paragraph, the Trend and Seasonality Corrected Exponential Smoothing:

$$F_{t,j} = (L_t + j \cdot T_t) S_{t+j} \tag{1}$$

where L_t and T_t, are respectively level and trend calculated at time t, and S_{t+j} is the seasonal factor estimated for period $t + j$.

The four data, namely d_1, $F_{1,1}$, $F_{1,2}$, $F_{1,3}$, are used by the planning algorithm to generate (trying to minimize the sum of setup and holding costs, see Paragraph 3.1) productions requirements for periods t = 1, 2, 3 ,4. However, only the first period is implemented, while period 2, 3 and 4 allow estimating raw material requirements. In particular period 4 will indicate the quantity of a new raw material requirement, while period 2 and 3 will be used to correct and refine previous estimates. Naturally, safety stocks of raw materials have to be provided to allow for uncertainty.

The procedure is then repeated in the next month, February (t = 2), and so on. Table 1 shows orders collected for the current month (d_t) and forecasts for the next three months ($F_{t,1}$, $F_{t,2}$, $F_{t,3}$) in each period t for item#1 during 2007. Each row represents the input to the planning algorithm for the current month. Note that forecasts for succeeding periods differ, of course, from orders that will be actually collected.

Table 1 reports all other inputs required by the algorithm to generate the production plan, that is: the initial inventory level, the setup cost, the item value and annual carrying rate.

3.1. Production planning heuristics

The planning algorithm is an adapted version of the Silver Meal heuristics (Silver and Meal 1973), that is usually adopted for deterministic lot sizing. When requirements are deterministic, the heuristic calculates the exact number of periods that have to be covered by a replenishment. For example, if we are in period 1, and T is the number of period covered by a replenishment, the ordered quantity will be equal to:

$$Q(T) = \sum_{j=1}^{T} d_j \tag{2}$$

Table 1. Demand forecasting during the simulation study for Item #1.

	d_t	$F_{t,1}$	$F_{t,2}$	$F_{t,3}$
January	3167	2509	2424	3402
February	1692	2127	2907	2650
March	1188	2507	2272	2619
April	2346	2258	2635	4777
May	2111	2589	4701	1241
June	1926	4319	1134	3215
July	2369	976	2750	2250
August	839	2665	2205	1942
September	1866	2001	1758	1645
October	3541	2217	2122	1805
November	1282	1804	1496	1597
December	2437	1685	1832	1753
	Initial Inv.	A	v	r_y
	3623	296	32	10.00%

The heuristic tries to chose the number of periods T (and the associated quantity of replenishment) that minimize future total costs in the time unit. If we indicate with $TRC(T)$ the total relevant costs (setup costs + holding costs) associated to a replenishment that covers T periods, then the total relevant costs per unit time ($TRCUT(T)$) associated to the same replenishment are equal to:

$$TRCUT(T) = \frac{TRC(T)}{T} = \frac{C_r + C_c(T)}{T} \tag{3}$$

The heuristic evaluates the $TRCUT(T)$ for increasing values of T (starting from T = 1) and when the following condition is satisfied:

$$TRCUT(T+1) > TRCUT(T) \tag{4}$$

T is chosen as the number of periods. In this way, due to the deterministic nature of demand, inventory lasts always an integer number of periods, and each replenishment occurs exactly when the inventory level is equal to 0.

In the non deterministic case, on the contrary, we have to use forecasts instead of actual demand; so it is not guaranteed that ordered quantities will last an integer number of periods.

Thus, if for example we are in period t (we are planning to fulfil requirements for periods t, $t+1$, $t+2$, $t+3$), first we have to check the inventory level: if $I_{t-1} \geq d_t$ stocks are enough to satisfy requirements for the next month; thus $Q_t = 0$. If $d_t > I_{t-1}$, stocks on hand are not sufficient, and we have to order at least the quantity $Q_t = I_{t-1} - d_t$ to cover the next period requirement. In general, the order quantity $Q_t(T)$ that cover the expected requirements for the next T periods will be equal to:

$$Q_t(T) = \begin{cases} I_{t-1} - d_1 & \text{for } T = 1 \\ I_{t-1} - d_1 + \sum_{j=1}^{T-1} F_{t,j} & \text{for } T > 1 \end{cases} \tag{5}$$

Similarly to the deterministic case, the heuristic will decide the number of periods T that minimizes the Expected Total Relevant Cost per Unit Time ($ETRCUT$). If T = 1:

$$ETRCUT(1) = \frac{A}{1} \tag{6}$$

there are not carrying costs, because we only replenish enough to cover the actual requirements of the first next period.

With T = 2 the expected carrying costs are $vrF_{t,1}$, that is the cost of carrying the expected requirement $F_{t,1}$ for one period:

$$ETRCUT(2) = \frac{A + vrF_{t,1}}{2} \tag{7}$$

With T = 3 we still carry $F_{t,1}$ for one period, but now we also carry $F_{t,2}$ for two periods. Therefore,

$$ETRCUT(3) = \frac{A + vrF_{t,1} + 2vrF_{t,2}}{3} \tag{8}$$

$$ETRCUT(4) = \frac{A + vrF_{t,1} + 2vrF_{t,2} + 3vrF_{t,3}}{4} \tag{9}$$

As before, we will chose to cover a number of period T for which:

$$ETRCUT(T+1) > ETRCUT(T) \tag{10}$$

corresponding to an ordered quantity expressed by the (5). In our particular case, due to the short rolling horizon of 4 months and the limited number of items, all possible values of $ETRCUT$ (T = 1, 2, 3, 4) have been calculated through (6)(7)(8)(9) and the T value that gives the minimum have been chosen.

4. RESULTS

The algorithm has been implemented in Java with the compiler NetBeans IDE6.0.

Table 2 shows the outputs of the simulation study for item#1, consisting in production quantities during 2007 and the corresponding inventory level. Results are compared to what obtained in the same period by the company.

Figures 7 and Figures 8 shows graphically the same comparison for all the 5 items during year 2007. Starting from the same inventory level, the algorithm tends initially not to produce, because stocks on hand are still enough to cover the next periods requirements.

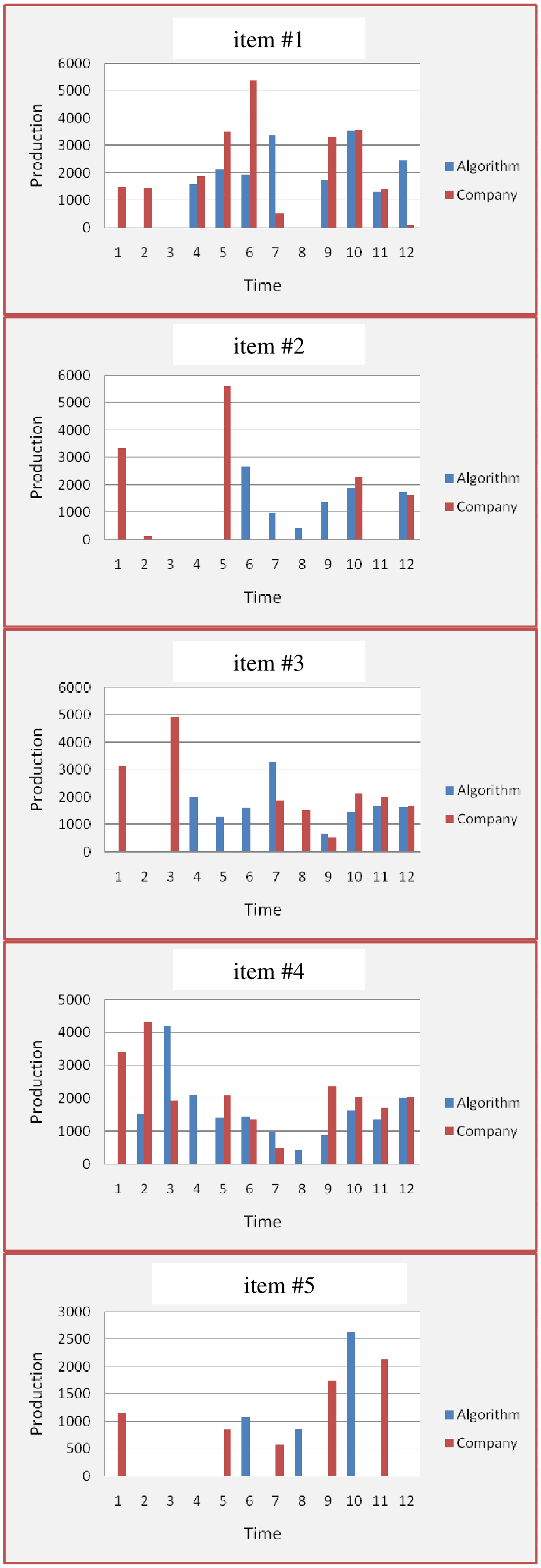

Figure 7. Production quantities: comparison between algorithm and company during year 2007

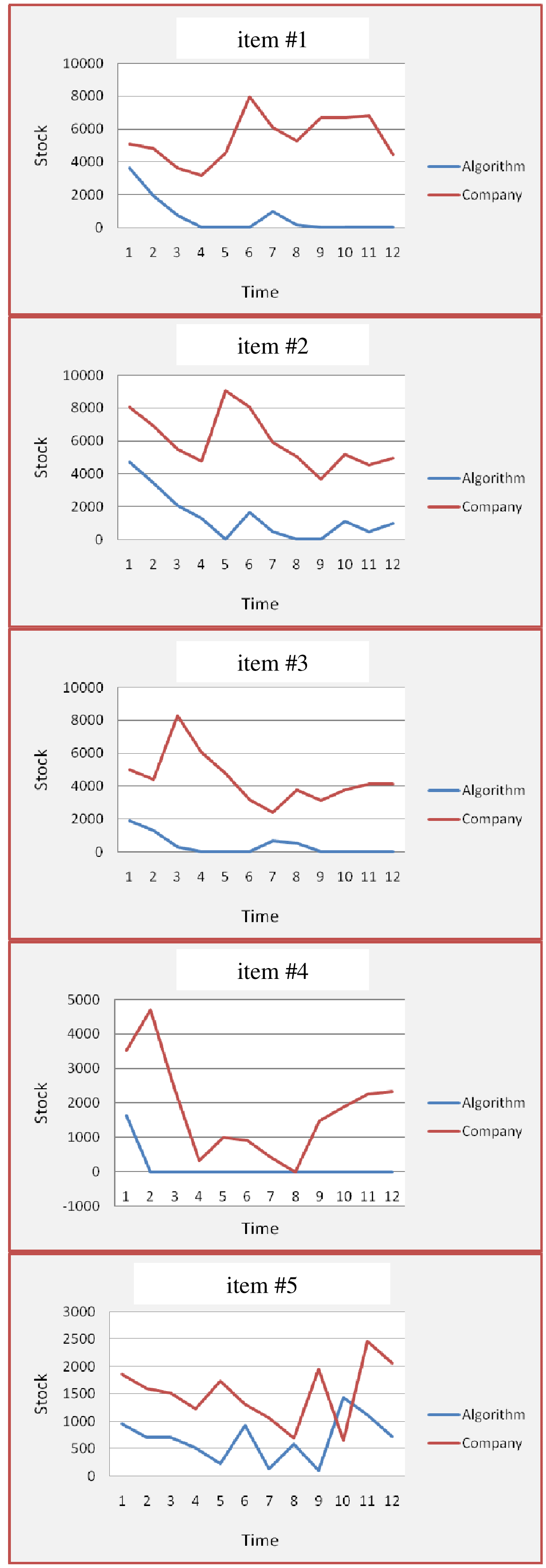

Figure 8: Inventory levels: comparison between algorithm and company during year 2007.

Table 2. Output of the simulation study for item#1

Algorithm		Company	
Production	Inventory	Production	Inventory
0	3633	1469	5102
0	1941	1427	4837
0	753	0	3649
1593	0	1874	3177
2111	0	3486	4552
1926	0	5360	7986
3345	976	493	6110
0	137	0	5271
1729	0	3286	6691
3541	0	3571	6721
1282	0	1391	6831
2437	0	88	4482
N Setup	Average Inv.	N Setup	Average Inv.
8	620	10	5451
Setup Cost	Holding Cost	Setup Cost	Holding Cost
2369	1984	2961	17443
Total Cost		Total Cost	
4352		20403	

On the contrary the company tends always to saturate its capacity, and produces even when it is not still required. Only when the inventory level is much lowered, the algorithm gives production quantities that are similar to the company's ones.

This implies that the number of setups performed by the algorithm is about the same of those performed by the company, but average inventory levels are much lower. The same behaviour is observable from data reported in Table 3 and Figure 9, that show the aggregate results for all 5 five products.

Table 3. Aggregate results of the simulation study.

Algorithm		Company	
Production	Inventory	Production	Inventory
0	15046	12492	25790
1503	9194	5850	24259
4195	5183	6814	22791
5646	2867	1874	16614
4774	1260	13088	23272
8684	3396	6716	23384
8572	2451	4583	18451
1662	1467	1503	17328
4919	301	7895	19129
11099	2554	9980	20254
4771	1871	7221	22022
8048	1890	5379	19398
N Setup	Average Inv.	N Setup	Average Inv.
36	3957	38	21058
Setup Cost	Holding Cost	Setup Cost	Holding Cost
13888	14804	14184	83877
Total Cost		Total Cost	
28692		98061	

The reduction of total costs is impressive (about 70%) and substantially due to the drastic decrease of inventories. It is noteworthy that the algorithm never generates peaks of production quantities higher than those planned by the company.

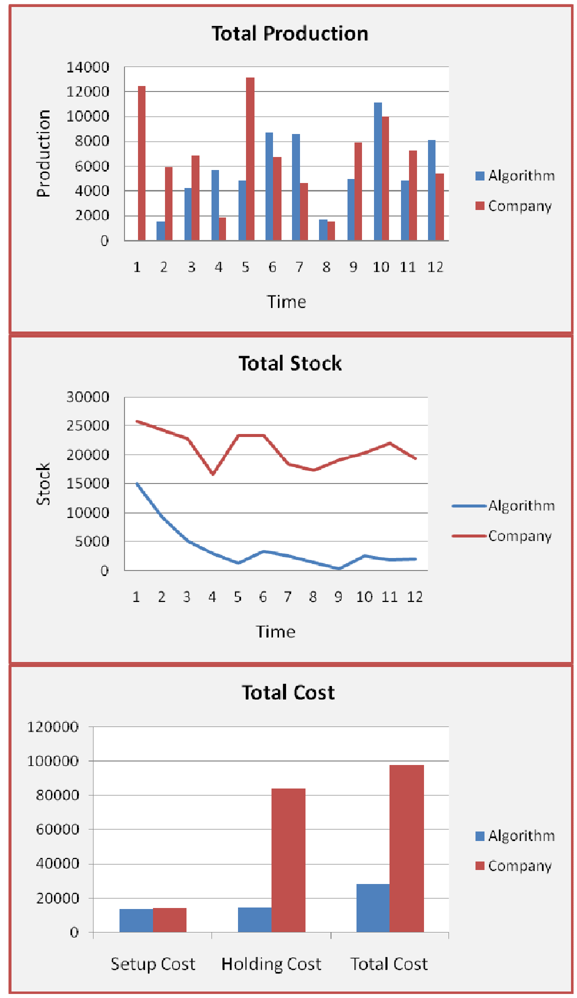

Figure 9. Aggregate results of the simulatio study.

4.1. The impact of uncertainty

In order to evaluate the impact of uncertainty, we evaluate the extra-cost associated to the use of forecasted demand (and the described algorithm) with respect to the optimal solution obtained by considering the actual demand.

In effect, if demand is deterministic, the lot sizing problem can be solved to optimality through the well known dynamic programming approach of Wagner-Whitin (Wagner and Whitin 1958), or through its equivalent formulation as Mixed Integer Programming model.

We consider the following MIP model for each of the five items:

$$\text{Minimize} \quad \sum_{t=1}^{12}\left(vrI_t + Ay_t\right) \qquad (11)$$

subject to:

$$I_{t-1} + Q_t - I_t = d_t \qquad \forall t \qquad (12)$$

$$Q_t \le M\, y_t \qquad \forall t \qquad (13)$$

$$y_t \in \{0,1\} \qquad \forall t \qquad (14)$$

$$Q_t \ge 0 \qquad \forall t \qquad (15)$$

$$I_t \ge 0 \qquad \forall t \qquad (16)$$

where: M is a large positive number (higher than the largest quantity that can be produced in one period), y_t is a binary decision variable equal to 1 if the item is produced for period t and equal to 0 otherwise; I_0 is the initial inventory level. Note that the planning horizon is equal to 12 months and that only actual demand data d_t are considered. The MIP problem has been coded and solved using Xpress® (Dash Optimization Xpress-MP).

Figure 10 shows the comparison between the solution obtained by solving the MIP problem (using actual demand data and a time horizon of 12 months), and the one, previously described, obtained by the algorithm (using forecasted demand data and a rolling horizon of 4 months).

Production quantities are very similar, and also inventory levels show the same behaviour (a consistent initial decrease due to very little production quantities). Nevertheless the gap is still consistent. This is due to fact that just a little difference between forecasted and actual demand causes the impossibility to place orders exactly when the items inventory level are equal to 0, forcing in this way to make a higher number of setups and to have always higher inventory levels. Furthermore, holding costs also increase, when using forecasted data, due to demand overestimate for some of the items.

Table 4 shows costs percentage deviation of optimal and algorithm solutions from the company solution. The difference in total saving between the 70.7% obtained by the algorithm and the 82.3% obtained in the deterministic case can thus be ascribed to the use of forecasted demand data. This gap could be taken as performance indicator of the forecasting method.

Table 4. Percentage deviation of algorithm and optimal solution costs from company costs in 2007

Costs and percentage deviation from Company solution						
	Setup		Holding		Total	
Comp.	14184	/	83877	/	98061	/
Alg.	13888	-2.1%	14804	-82.4%	28692	-70.7%
Opt.	7715	-45.6%	9607	-88.5%	17322	-82.3%

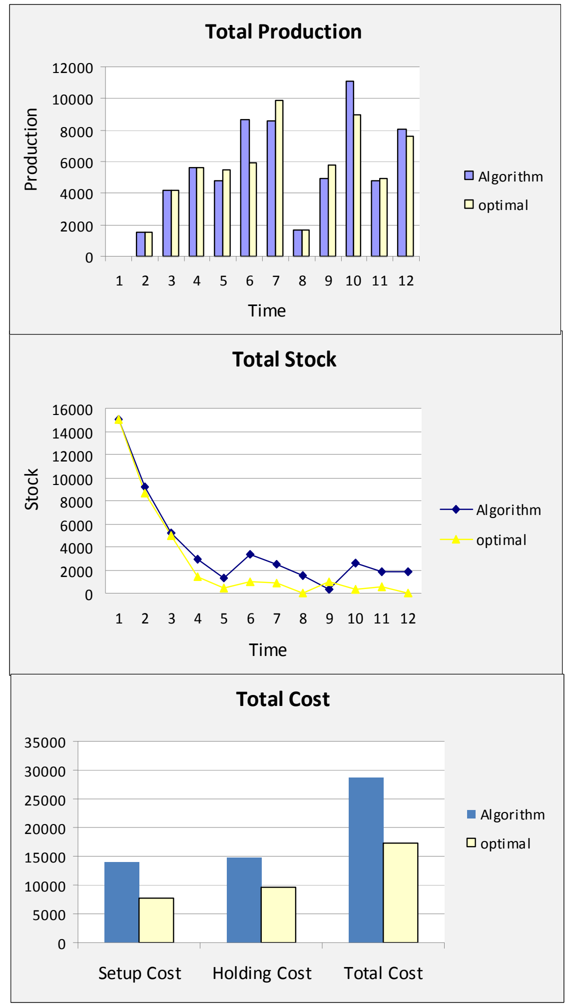

Figure 10. Comparison between optimal and algorithm's solutions.

5. CONSLUSIONS

The model that has been implemented allowed to simulate both the demand forecasting and the production planning activities of the company for one year.

To consider this two phases at the same time in a simulation study is the best way to quantify possible improvements deriving from the adoption of a planning algorithm that, in practice, receives forecasted demand data as inputs.

The procedure allows also to evaluate the goodness of the demand forecasting method not only through per period forecasts error indicators (as the Mean Absolute Deviation) but also through their direct impact on total costs.

Final results show that the application of the selected demand forecasting method and the production planning algorithm allows a total costs reduction, with respect to the actual company policy, up to the 70% in one year.

If actual demand data, instead of forecasts, are considered during the same period, then the planning problem can be solved to optimality. Thus, the impact of the demand forecasting method inaccuracy can be evaluated by comparing the total costs reduction obtained using forecasts (70%) and the one related to the optimal solution (82.3%).

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