# MODELLING AND CONTROLLER PROTOTYPING FOR UNMANNED VERTICAL TAKE OFF AND LANDING (UVTOL) VEHICLES

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## ABSTRACT

This paper describes a methodology to parameterize linear, time invariant (LTI) models which represent the dynamics of UVTOLs and that are appropriate for analytical development of controllers. The models' validity was tested against real telemetry from two vehicles, a mini-helicopter and a quad-rotor. The experiments show that despite its inherent limitations the LTI models are suitable for modeling the complex dynamics of aerial vehicles. Different LTI models for the mini-helicopter's stationary, lateral and longitudinal flights were obtained. Similarly, given the geometrical and dynamic characteristics of the quad-rotor no distinction is made between stationary, lateral and longitudinal flights, and only one LTI model was obtained, which represents the overall dynamic behavior of the vehicle. Because of their relative simplicity these models were used to design analytical controllers and to obtain different controller prototypes in a quick and simple way to evaluate the UVTOL's performance in different flight conditions.

Keywords: UVTOL, modelling, controller prototyping

## 1. INTRODUCTION

The modeling, guidance, navigation and control of Unmanned Vertical Take Off and Landing (UVTOL) vehicles is a research topic with great developments, and has achieved significant progresses in recent years. It is sufficient to mention some work on helicopters modeling and control to note the large number of approaches that has been given to this field of engineering. As for the modeling we can cite works as the La Civita (2002), who presents a nonlinear helicopter model, derived on the basis of aerodynamic principles, and its linearization, enabling the possibility of including several equilibrium points in the flight envelope. Avila et al (2003) who develop a nonlinear model and a nonlinear control strategy for a scale model helicopter. Cunha and Silvestre (2003), introduce a helicopter dynamic simulation model specially suited for the design, test, and evaluation of flight control systems for model-scale helicopters. Castillo et al (2004) present a controller design and its implementation on a quadrotor, whose dynamic model is obtained via a Lagrange approach. DelCerro et al

(2004) develop an analytical model of a small helicopter as well as a statistical study about the influence of its main parameters. The stability and response of the model is presented after a brief comparison of different techniques for helicopters dynamic modeling. Madani and Benallegue (2006) present a nonlinear dynamic model for a quadrotor helicopter in a form suited for backstepping control design. Vélez et al (2006) present a rapid software prototyping environment for the design, development and simulated test of a control system for an autonomous mini-helicopter. Amir and Abbass (2008) propose a non-linear mathematical model of quadrotor dynamics. Whit regard to control techniques used in helicopters and quadrotors, it include some as varied as robust control (Isidori 2003), backstepping (Bouabdallah and Siegwart 2005), slidingmode (McGeoch and McGookin 2005, Rong Xu and Ozguner 2006), PID/PD and fuzzy (Castillo 2005). As you can see, there is a continued interest in applying different techniques to control this type of aircraft and also the need of modeling VTOL in each desired case.

The UVTOL's complex dynamics and its variations related to flying altitude, weather conditions, changes in the vehicle's configuration (for example: weight, payload and fuel quantity), disrupt the modeling process and, consequently, the systematic development of control systems, resulting in tedious and critical heuristic adjustment procedures. This paper proposes a modeling methodology that leads to a set of LTI models, which allow representations of stationary, lateral and longitudinal phases of flight; these models are then used for the development of analytical controllers and, as a consequence, a systematic synthesis process of them.

# 2. DESCRIPTION OF USED VTOL AND PROPOSED MODEL

# 2.1 Description of aerial vehicles

In this section we briefly describe the VTOLs used and give a description of some details about the characteristics, variables and signals involved in their flight dynamics.

VTOLs are considered systems of six degrees of freedom, defined by three degrees to the position or location (X, Y, Z) and three other degrees to attitude

(Roll, Pitch, Yaw angles) as described in Figure 1 for the mini-helicopter.



Figure 1: Nomenclature Used for the Mini-helicopter's Variables

The mini-helicopter used is a Benzin Trainer of VARIO. Its main rotor measured 1.5 meters and has a 5 kg payload, thanks to its 26 cc engine.

On the other hand, the quadrotor used is a Draganflyer SAVS, with 0.8 m diameter, a payload of 85 g and energy autonomy for 12 to 15 minutes flights.

Thanks to the instrumentation on board the aircrafts and communications systems available, data from signals involved in the flight can be stored on land. Among the variables that can be obtained, are relevant to the model developed the following signals (as graphically represented in Figure 1):

X, Y, Z: Measured respect to an inertial system and only in the case of mini-helicopter, with a GPS.

Roll, Pitch: Measured with an IMU, with respect to the mobile axis of each VTOL.

Yaw: Measured with magnetic compass in the minihelicopter and with IMU in Quadrotor.

Vroll, Vpitch, Vyaw: Calculated using a Kalman filter from the signals measured into IMU.

Croll, Cpitch, Cyaw: Control Signals of the respective Roll, Pitch and Yaw angles, sent through of RC transmitter.

#### 2.2 Mini-helicopter's mathematical model

Dynamic features of VTOLs can be represented by complex models, where components such as nonlinearities or parameter uncertainty are not easy to determine. In contrast, the majority of controller design procedures require a relatively simple mathematical model to establish some characteristics and adjust the parameters of the controller to design.

A relatively complex mathematical model for the mini-helicopter used in this work was obtained in earlier work (Aguirre 1999, DelCerro 2007). This model offers a good input-output representation of the system, but its complexity does not allow to use it in analytical procedures for controllers design. From this model we obtained a structure of a simple mathematical model, which can be represented with linear, time invariant (LTI) state equations and use it to develop a systematic procedure of controllers design.

The mini-helicopter was considered a decoupled system respect to its three main movements: the movement in a horizontal plane, parallel to the ground, with varying angles roll, pitch and the resulting lateral and longitudinal displacement, the Yaw movement and vertical movement or Z variation.

#### 2.2.1 Model structure

The structure of LTI model proposed in this paper for the movement in a horizontal plane, with constant Z and based on zero Yaw angle assumption, is represented by the following state equations:

	Го	1	0	0	0	0	0	0]	Го	0		
	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$	0	0	0	0	$b_{21}$	$b_{22}$		
	0	0	0	1	0	0	0	0	0	0		
•	$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$	0	0	0	0	$b_{41}$	$b_{42}$	$[u_1]$	
<i>x</i> –	$a_{51}$	$a_{52}$	0	0	$a_{55}$	0	0	0	0	0	$u_2$	
	0	0	$a_{63}$	$a_{_{64}}$	0	$a_{66}$	0	0	0	0		
	0	0	0	0	0	$a_{76}$	0	0	0	0		
	0	0	0	0	$a_{85}$	0	0	o	o	• • _		(1)

Where:

$x_1$ : Roll	x <sub>5</sub> : Lateral linear speed
x <sub>2</sub> : Roll derivative	x <sub>6</sub> : Longitudinal linear speed
x <sub>3</sub> : Pitch	x7 : Frontal displacement
x <sub>4</sub> : Pitch derivative	x8 : Longitudinal displacement
u <sub>1</sub> : Roll control signal	u <sub>2</sub> : Pitch control signal

This model structure assumes the existence of coupling between the Pitch and Roll angles, and a dependence on the lateral and longitudinal movements only with variations in Pitch and Roll angles respectively. From various experiments, we were able to verify that this structure model does not depend on the value of Z, or different constant values that can take the angle Yaw.

The model structure represented in Equation 1 is a parametric model whose coefficients  $a_{ij}$ ,  $b_{kl}$ , should be defined for a particular aerial vehicle. For this, we developed an identification process on the real system, using genetic algorithms as computational tool.

During early experiments with the mini-helicopter and after data analysis aimed at obtaining a single model to represent its dynamics, we were able to show that its dynamic behavior when flying in a horizontal plane was sufficiently diverse from stationary, frontal and lateral flight. This raised the need to obtain a specific model for each of these types of flight.

#### 2.2.2 Parameters Identification

A genetic-algorithm-based tool, which has been successfully used to parameterize other models (DelCerro 2007), was used for the  $a_{ij}$  and  $b_{kl}$  parameter's identification process.

The selection criterion used was the minimization of an objective function given by the difference between real signals measured in the VTOL and signals provided by the model. This difference is represented by the value of the mean square error between those signals.

The implementation of the identification process in the case of mini-helicopter resulted in the parameters that are detailed in Table 1, for the three different types of flight.

Table 1. Model parameters for mini-helicopt	opter
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Parameter	Hover	Longitudinal	Lateral	
	flight	flight	flight	
a <sub>21</sub>	-0.0897	-4.9522	-9.4286	
a <sub>22</sub>	-1.0211	-0.1898	-2.8422	
a <sub>23</sub>	1.8807	1.5900	6.1920	
a <sub>24</sub>	0.7150	5.0240	-4.6793	
a <sub>41</sub>	-1.0785	-3.6885	-0.0641	
a <sub>42</sub>	-2.6725	-1.8418	0.4656	
a <sub>43</sub>	-1.1095	-4.4536	-8.1002	
a <sub>44</sub>	-2.0666	-3.4587	-5.5487	
a <sub>51</sub>	0.3604	-0.7884	0.2680	
a <sub>52</sub>	0.0213	0.8237	0.5478	
a <sub>55</sub>	-1.1681	-0.1264	-0.9287	
a <sub>63</sub>	1.2946	-0.4601	-1.3005	
a <sub>64</sub>	3.0741	0.5184	0.3482	
a <sub>66</sub>	-4.4571	-0.9718	-0.1548	
a <sub>76</sub>	-0.2156	-1.8407	-0.4067	
a <sub>85</sub>	0.3313	0.0961	1.1147	
b <sub>21</sub>	1.2969	-5.0342	-12.7259	
b <sub>22</sub>	0.8092	2.8049	-0.0557	
b <sub>41</sub>	-3.6516	-4.4360	1.8356	
b <sub>42</sub>	-1.3202	-4.0981	-3.6725	

#### 2.3. Quadrotor model

Similarly to mini-helicopter, was considered the quadrotor as a decoupled system respect to its three main movements: the movement in a horizontal plane parallel to the ground, with varying angles roll, pitch and the resulting lateral and longitudinal displacements, the Yaw movement and vertical movement or Z variation. However, given the geometrical and dynamic characteristics of quadrotor no distinction is made between stationary, lateral and longitudinal flights, and only one LTI model was obtained, which represents the overall dynamic behavior of the vehicle.

#### 2.3.1. Quadrotor model structure

Given the physical limitations of quadrotor's payload and the consequent difficult to obtain reliable measures of variables such as the position (X, Y, Z) a simple model to represent a smaller number of state variables with respect to mini-helicopter was chosen.

We propose a LTI, fourth order model (Equation 2) that will be useful to represent the Pitch and Roll angles, and then develop attitude controllers to provide stability to dynamics of quadrotor.

$$\begin{bmatrix} \mathbf{i} \\ x_{1} \\ \mathbf{i} \\ x_{2} \\ \mathbf{i} \\ \mathbf{i} \\ x_{4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \\ x_{4} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} u_{1} \\ u_{2} \end{bmatrix}$$
(2)

Where:

- x<sub>1</sub> : Roll
- x2: Roll derivative
- x<sub>3</sub> : Pitch
- $x_4$ : Pitch derivative
- u<sub>1</sub> : Roll control signal
- u2: Pitch control signal

### 2.3.2. Parameters Identification

Using the same computational tools of mini-helicopter case, the parameters of the LTI model for quadrotor were identified, whose values are summarized in Table 2.

	<u> </u>
Parameter	Value
a <sub>21</sub>	-1.1132
a <sub>22</sub>	-0.7118
a <sub>23</sub>	0.0001
a <sub>24</sub>	0.0000
$a_{41}$	7.2721
$a_{42}$	6.1794
a <sub>43</sub>	-7.9908
$a_{44}$	-5.2474
b <sub>21</sub>	-1.1045
b <sub>22</sub>	0.0000
b <sub>41</sub>	-4.5267
b <sub>42</sub>	-14.4146

#### 3. Models Validation

#### 3.1 Mini-helicopter's model.

Some graphics are shown to compare the results of the LTI model simulations and the real data obtained in different experimental flights. Figures 2, 4 and 6 show, on the left side, the comparison between the real data (solid lines) and those resulting from the model simulations (dotted lines) with the same set of data used in the parameters identification process. In the right side is reported the same comparison between real and simulated data, but with a different set of data called "control data".

Figures 2, 4 and 6 compare real values of Pitch and Roll angles, as well as lateral and longitudinal movements of mini-helicopter (which coincide with the X and Y axes, when Yaw angle is zero), with those resulting from the simulation of stationary, lateral and longitudinal flights, obtained from the respective models previously described.



Figure 2: Real and Simulated Data Comparison for Hover Flight.



Figure 3: Quadratic Error for Data Control in Hover Flight



Figure 4: Real and Simulated Data Comparison for Longitudinal Flight

The plots in Figures 2, 4 and 6 shows that real Roll and Pitch signals have a higher frequency component than those obtained from the model simulation. However, frequency analysis of signals measured were performed with the mini-helicopter in flight, with the mini-helicopter on the ground and engine turned on and then with the engine off. Such analysis could determine that these high-frequency signals are due to mechanical vibrations caused by the engine movement.



Figure 5: Quadratic Error for Control Data in Longitudinal Flight



Figure 6: Real and Simulated Data Comparison for Lateral Flight



Figure 7: Quadratic Error for Control Data in Lateral Flight

The quadratic error graphics (Figures 3, 5 and 7) show differences smaller than 0.5 degrees between real and simulated data, in Pitch and Roll angles. The resulting behavior on the position in X and Y also presents a negligible error, except for some specific samples where the error Pitch and Roll is a little higher (in all cases of flight). This error may be attributed to high frequency vibrations commented previously.

It should be noted that the latest samples from each data sets are part of a transition phase between the three different types of flight (stationary, front and side), which explains that towards the end of the quadratic error graphics, this error becomes bigger, because the actual behavior of the helicopter in these transitions does not correspond exactly with any type of flight consideration.

With the above can be concluded that model structure, with the corresponding sets of parameters obtained, is a suitable representation of mini-helicopter dynamic behavior. In other words, the model, with relevant parameters depending on type of flight, allows an approximate representation of mini-helicopter complex dynamic. For this reason, this model can serve as a practical and useful tool for driver design and development procedures, despite the limitations and approaches set out in the model definition (LTI model with relatively small order).

#### 3.2 Quadrotor model.

Figure 8 shows the same comparisons for the case of the quadrotor. On the left side real data (solid line) is compared to model simulation (dotted line) on the set of data used in the identification process. On the right side the same comparison using a different set of data (control data).



Figure 8: Comparison of Real and Simulated Data for the Quadrotor Model.



Figure 9: Quadratic Error of the Control Data in the Quadrotor Model

Figure 9 shows the difference between the measure and simulated Roll and Pitch values, expressed as quadratic error. As it can be noticed, the difference is

bigger than in the case of the helicopter. This is due to the fact that in the quadrotor case only one model has been used for the three flight modes. Moreover, due to the limited payload of this vehicle, an IMU with less precision had to be used.

# 4. Design procedure obtained

Once obtained the models for the two vehicles described above, it was possible to establish a standard procedure for the design of analytical controllers with many proven techniques, such as pole placement, linear quadratic regulators and sliding modes control. This is the main advantage of our modeling methodology, which was validated by the design of various controllers for both UVTOLs.



Figure 10: Comparison Between Two Different Control Techniques for the Mini-helicopter.

Figure 10 shows the comparison between X and Y position signals for the mini-helicopter, in response to small changes in the reference for maintaining it in stationary flight, controlled using sliding mode control (SMC) and pole placement control (PPC).

In the same way, Figure 11 shows simulations of quadrotor's Roll and Pitch regulated using pole placement control (PPC), sliding mode control (SMC) and LQR control over the quadrotor.



Figure 11: Comparison Between Simulations of Three Different Control Techniques for the Quadrotor.

Finally, in order to compare the goodness of the controller modelling and design process, we have realized some experimental flight with the quadrotor using the pole placement control technique for controlling the Roll and Pitch signals, achieving stationary attitude for the vehicle as shown in Figure 12. The third graph of Figure 12 shows the signal command for height (Z) to show how long the quadrotor was in the air.

Note that the Pitch values do not reach the value zero due to the position of the IMU that involve an offset for this variable. In general, the controller designed for this case allow a good behaviour, compare with manual control, including reject of perturbation that can be noticed at t = 8.5 s.



Figure 12: Results Obtained for Sliding Mode Control for the Quadrotor.

## 5. CONCLUSION

We have developed and tested a simple methodology that allows obtaining LTI models for small VTOL vehicles, as well as to design implement and test different kinds of controllers for them. This methodology can be summarized as follows:

- Perform manual flights with the VTOL to get their input-output data (Roll, Pitch, V<sub>Roll</sub>, V<sub>Pitch</sub>, V<sub>X</sub>, V<sub>Y</sub>, X, Y)
- Feed the measured data to software designed to get the LTI model parameters.
- Validate the model based on simulations, comparing it with actual data.
- Define the design criteria to use and, with the control technique desired, to obtain the controller design.
- Simulate the controlled system and tune the controller parameters, test the performance of UVTOL using Simulink®
- Proceed to rapid prototyping controller using Simulink®, getting the code in C or C++.
- Test the controller in the UVTOL architecture.

## ACKNOWLEDGMENTS

The authors would like to thank Ministerio Educación y Ciencia (Spain) by the FRACTAL project (DPI 2006\_03444) and Pontificia Universidad Javeriana Cali (Colombia) for founding part of this research work.

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