ASSESSING POOLING POLICIES IN MULTI-RETAILER INVENTORY SYSTEM WITH LOST SALES

Mounira Tlili [a], Mohamed Moalla [a], Jean-Pierre Campagne [b], Zied Bahroun[a]

[a]LIP2, Faculté des Sciences de Tunis, 2092 Manar 2, Tunis, Tunisie
[b] LIESP, INSA de Lyon 19, Avenue Jean Capelle, 69621 Villeurbanne Cedex, France

[a]mounira.tlili@fst.rnu.tn, mohamed.moalla@fst.rnu.tn, zied.bahroun@fst.rnu.tn [b]jean-pierre.campagne.insa-lyon.fr

ABSTRACT
In this paper, we address a multi-retailer inventory system with transshipment. Transshipment is used as recourse action occurring after an expected demand is realized. The remaining unsatisfied demand after transshipment is lost. We use various pooling policies for transshipment. We also develop a simulation model for such system that allows us to characterize the effects of pooling policies with/without constraints on system performance measures. The constraints are system lost sales per period and capacity per tranship.

Keywords: multi-retailer inventory system, transshipment, lost sales

1. INTRODUCTION
Collaboration between locations is a big challenge until today in a multi-echelon, multi-location inventory system. The lateral transshipment between locations at the same echelon is the most popular method of collaboration. The lateral transshipment means that the locations share the stocks (or stock pooling.) There are two approaches to share the stock. The first, if a location cannot satisfy an actual demand, other location(s) with stock on hand may ship stock to the location where the demand occurred. It is called emergency transshipment. The second, transshipment occurs as redistribution of stock before the realization of the demand, so it is called preventive transshipment. With both lateral transshipments, we can reduce costs and improve service level even if the same total stock still maintain. In general, it is assumed that the lead-time for transshipments is zero.

In multi-echelon systems, it is often assumed that customer demand appears only at the lowest echelon. The most common assumption is that the customer demand follows stationary stochastic process. Moreover, if a customer places an order it is often assumed that the customer will wait until the order arrives. This means that backorders are allowed in the model. However, if the market is competitive a lost sales model should be used. We shall also distinguish continuous review from periodic, since these models are quite different in the analysis.

The remainder of the paper is organized as follows. In section 2, we review the relevant literature in the transshipment domain. In section 3, we describe the multi-retailer transshipment problem and state the optimization problem. In section 4, after reviewing the classical expressions for computing the reorder point s and the order-up-to level S in single location inventory system, we present the simulation model. We then, propose a methodology to establish a simulation-optimization model for finding s and S that minimize the total cost. In section 5, we evaluate the benefits pooling policies over a wide range of parameter values. Finally, in section 6, we present our conclusion and some future researches.

2. THE TRANSSHIPMENT LITERATURE
Since the literature on transshipment problem is very large, we limit, in this section, to provide an overview of some works that have addressed (i) the emergency transshipment and (ii) the difficulties encountered in transshipment inventory system.

Robinson (1990) has formulated a multi-period (finite horizon), multi-location problem with emergency transshipment. In a backordered model, Robinson has proven the optimality of the order-up-to policy under the assumptions of instantaneous replenishment and transshipment lead-times. In fact, the order-up-to levels can be found analytically only when there are two retailers or the cost parameters are identical at all retailers. He has proposed a heuristic for the general case. Tagaras and Cohen (1992) have considered an inventory system consisting of a central warehouse and two retailers. They have assumed non-negligible replenishment lead-times and instantaneous transshipment times. They have allowed transshipment not only in cases of stock-outs but in also when the inventory position at a retailer falls below a critical level. The authors have defined four pooling policies that determine when and how much stock is transshipped from one retailer to the other; two policies are based on inventory level and the two other policies are based on inventory position. They have used simulation and grid-search to identify the optimal order-up-to levels. They have compared expected costs of the four policies by simulation and find that the best policy is complete pooling (the transshipped quantity is equal to the minimum of the surplus and the shortage.) Their analysis is extended by Tagaras (1999) to three retailers with identical costs. He has pointed out that the transshipment policy complicates significantly the problem and has investigated the random, risk balancing, and priority policies using optimization by simulation.

The continuous review policies are often applied in connection with spare parts (or repairable items.) Models for spare parts are commonly used in military application, and the relative literature is quite extensive. These models often assume that spare parts are characterized by high costs and low demand (Poisson process.) The one-for-one replenishments, i.e. (S-1, S) policy, is the most popular policy. The METRIC model (Multi Echelon Technique for Recoverable Item Control) is commonly used as a basic model (Sherbrooke 1968). Sherbrooke has approximated the real stochastic lead-time by its mean. Lee (1987) has extended the Sherbrooke's model by allowing lateral transshipment. In the case of identical locations, Lee has proposed three different lateral transshipment sourcing rules (random source rule and two priority rules.) Axsäter (1990) has extended the Lee's model by allowing non-identical locations.

The (s, Q) continuous review system is also used by some authors. In the inventory system with lost sales, Needham and Evers (1998) have shown that the penalty cost is the primary determinant on transshipment benefits. Evers (2001) has considered "all or nothing" policy (i.e., satisfy all the remaining demand or no) and variable transshipment cost per unit transshipped. In order to solve this problem, Evers has developed heuristic approaches to determine when transshipments should be made. Xu, Evers and Fu (2003) have dealt with the emergency transshipment in a multi-retailer inventory system with backorders. They have introduced to the classic (s, Q) policy a third parameter, hold-back level (H), which controls the level of outgoing transshipments. That is, if a retailer has only a few units on hand, it may choose not to share its inventory with the stocked-out retailer.

The literature relating to our model is not extensive. Hu, Watson and Schneider (2005) have examined a periodic review (s, S) policy in a multi-retailer system with centralized ordering and demand backordered. They have assumed that the lead-times of replenishment and transshipment are negligible, and the cost parameters at retailers are identical. Under these restrictive hypotheses, they have established a dynamic programming model in order to find the approximate optimal (s, S) policy of the system instead of each retailer. Kurkreja and Schmidt (2005) have considered a continuous review (s, S) policy in a multi-retailer inventory system with demand backordered. They have dealt with an optimization problem to obtain (s, S) policies for each retailer that minimize the total cost and satisfy the service requirement. In addition, they have proposed a research procedure based on a simulation methodology in which only the values of s are manipulated; S is obtained by adding the economic order quantity to s.

In both (s, Q) and (s, S) inventory systems, the commonly hypothesis used in all mentioned models is to consider at most one outstanding order per replenishment cycle. That is, no order arrives at the reordering period, i.e. Q>s or S-s>s.

In the literature, the analytical models can only be found under restrictive assumptions (instantaneous replenishment, two locations, and identical costs). In fact, relaxing the assumption of instantaneous replenishment complicates significantly the mathematical analysis, because of the interrelationships among demand, transshipment quantities and pipeline inventories, and consequently the state space should be expanded (Tagaras 1999). In addition, Tagaras (1999) has shown that the exact model becomes intractable even in the case of two locations. Minner, Silver and Robb (2003) have also indicated two other reasons: (i) transshipments have a secondary effect in changing the time and the size of replenishment orders at a location releasing the transshipment, and (ii) the establishment of safety stocks must take into account the transshipment possibilities instead of the constraint of desired service levels. For these reasons, many authors have resorted to simulation in order to study multi-retailer inventory system under relaxed hypothesis. It is in this direction that our work is pointing out.

We are interested to study a periodic review (s, S) with lost sales in a multi-retailer inventory system integrating transshipment. The simulation model is designed to search policy parameters at each retailer that reduce the total system cost (holding, ordering, penalty, and transshipment costs). In a recent work, Tlili *et al.* (2008) have examined the benefits of complete pooling and all or nothing policies in a two-retailer inventory system and have concerned to study the effects of transshipment and penalty costs. Tlili *et al.* have evaluated these transshipment policies under identical/non-identical replenishment lead-times. For this system, the authors have proposed an effective procedure based on a grid-search by coupling simulation and optimization. This procedure is very appealing to practitioners in inventory management. The most important findings can be summarized in: (i) the type of transshipment policies does not have a significant difference on the system performance measures, (ii) the transshipment is more efficient only when the transshipment cost is non expensive in comparison with the penalty cost, and (iii) it is preferable to design distribution systems with identical replenishment lead-times and demand parameters in order to achieve a desirable savings of transshipments. In this paper, we relax some assumptions and extend the results of this earlier study to more than two retailers. In addition, our contribution in this work is threefold. First, we study the transshipment problem using complete pooling under with/without the hypothesis that at most one outstanding order is possible per replenishment cycle. Second, we also examine these two systems by adding lost sale constraints per period at system level (all retailers). Third, we deal with a constraint of capacity per tranship instead of hold-back levels at retailers. Next, we present our transshipment problem in a multi-retailer inventory system.

3. MULTI-RETAILER TRANSSHIPMENT PROBLEM

3.1. Description of the problem

We study an inventory pooling system of multi-retailers, which are replenished by a warehouse. Each retailer i allows a periodic review (s_i, S_i) ordering policy and faces random demand, which is normally distributed (mean μ_i and standard deviation σ_i) and independent of the demand at the other retailers. We assume that the warehouse does not keep any stock; i.e., any ordering policy is adopted by the warehouse. The replenishment lead-time from the warehouse to retailers is L periods and a fixed cost (K) is associated with each replenishment order. If retailer i places an order at period t, it will arrive at the beginning of period t+L+1. The emergency transshipment is allowed from retailers having excess stock to the retailers having shortage stock with zero time of transshipment. The quantity transshipment from j to i in period t is denoted by $X_{j,i,t}$ and a fixed transshipment cost (ct) is associated with this activity. The transshipment cost is independent of the number of units transshipped as well as the number of transshipment requests. This can be justified only when all units to be transshipped from one retailer to another can be transshipped by a single shipment. If the transshipment is not possible or the demand is partially satisfied via transshipment, the remaining unmet demand is lost and a penalty cost (cp) per unit lost occurs. Furthermore, at the end of the review period, the remaining stock is subject to holding cost (ch) per unit and period unit.

We define a system as a group of retailers. The α-service (no stock-out probability) and β-service (fraction of satisfied demand) levels are measured at the system level.

3.2. Notation

N	Number of retailers
$\overline{\beta}$	Pre-assigned fraction for the number of lost sales per period at the system level
DS	Desired system service level
Q_{max}	Capacity maximum per tranship
k_i	Safety factor for retailer i
$x_{i,t}$	Demand at retailer i in period t
$y_{i,t}$	Inventory level at retailer i in period t
$G_u(k_i)$	The unit normal function (mean 0, standard deviation 1)
$\delta_{i,t}$	1 if retailer i in period t is still in a shortage situation and 0 otherwise
$TX_{i,t}$	Total transshipment quantity towards retailer i in period t
EOQ	Economic order quantity

$$EOQ = \sqrt{\frac{2K\mu_i}{ch}} \tag{1}$$

3.3. Transshipment policies

3.3.1. Complete pooling

In a multi-retailer system, if one retailer could not satisfy its local demand from its own inventory on hand, it could be place a transshipment request to all the other retailers which are capable of providing such excess stock. Thus, the total transshipment quantity into retailer i from the other retailers is:

$$TX_{i,t} = \sum_{j=1, j\neq i}^{N} X_{j,i,t} * \delta_{i,t} = \min\left\{\left[x_{i,t} - y_{i,t}\right]^{+}, y_{j,t} * \delta_{i,t}\right\} \tag{2}$$

Where $[x]^{+} = \max(0, x)$

The principal rule of complete pooling is to satisfy the unmet demand as long as there are available stocks at the other retailers and there is still shortage at retailer i ($\delta_{i,t}$=1).

3.3.2. Partial pooling

We consider a partial pooling policy that uses a capacity constraint per tranship (Q_{max}). In this situation, if a retailer has stocked out upon the unexpected of a customer demand, it places a transshipment request to the other (requested) retailer. The requested retailer is allowed to tranship any inventory lower than the quantity maximum (Q_{max}) to be transshipped. The capacity constraint can be interpreted as the capacity of the cargo that can be transported. Thus, the partial pooling policy under capacity constraint for multi retailers can be formulated as below:

$$\begin{aligned} &\text{if } \; x_{i,t} - y_{i,t} \leq \sum_{j=1, j\neq i}^{N} y_{j,t} * \delta_{i,t} \;\; \text{then } \; X_{j,i,t} = \min\left(\left[x_{i,t} - y_{i,t}\right]^{+}, Q_{\max}\right) \\ &\text{else } \; X_{j,i,t} = \min\left(\sum_{j=1, j\neq i}^{N} y_{j,t} * \delta_{i,t}, Q_{\max}\right) \end{aligned} \tag{3}$$

3.3.3. Sequential procedure of transshipment

For both complete pooling and partial pooling policies, we should specify the transshipment decisions. For simplicity, we consider a sequential procedure to determine from which retailer the transshipment will be requested first. For instance, in the case of N=4, the retailers are numbered 1, 2, 3 and 4. While retailer 1 is in shortage situation at period t (i.e., $\delta_{1,t}$=1), the transshipments follow the sequence of retailers 2, 3 and 4. Nevertheless, transshipments depend on the available stock at these retailers:

Condition 1: if retailer 2 has sufficient stock to fill the requested quantity hence the transshipment is realized, so $\delta_{1,t}$=0.

Condition 2: if retailer 2 has an available stock but it is lower than the requested quantity then we satisfy partially the request, so $\delta_{1,t}$=1. The request corresponding to the remaining quantity is transmitted to retailer 3. If there is still shortage at retailer 1 ($\delta_{1,t}$=1), the transshipment request is sent to retailer 4.

Condition 3: if retailer 2 is in shortage situation the transshipments follow the sequence of retailers 3, 4 and 1. Hence, retailer 1 will be satisfied from retailer 3 and 4 after the transshipment decisions of retailer 2.

It is worth noting that retailers 3 and 4 should also verify the conditions 1, 2 and 3.

3.4. The optimization problem

We are concerned with finding (s, S) policies for each retailer that minimize the total system cost E(TC). The total system cost consists of ordering, holding, penalty, and transshipment costs. We want to study on assessing the benefits of pooling policies (complete and partial pooling) under with/without the hypothesis that at most one outstanding order per replenishment cycle (S-s>s) and under the system lost sales constraint per period ($\overline{\beta}$). Thus, we have the following models:

Model 1: Complete pooling without S-s>s constraint

$$\begin{aligned} &\min_{(s_i,S_i);\, i=1...N} E(TC) \\ &s.t.\ \ s_i \geq 0, S_i \geq 0,\ and\ Integers; i=1...N \end{aligned}$$

Model 2: Complete pooling with S-s>s constraint

$$\begin{aligned} &\min_{(s_i,S_i);\, i=1...N} E(TC) \\ &s.t.\ \ s_i \geq 0, S_i \geq 0,\ and\ Integers; i=1...N \\ &\quad S_i - s_i \geq s_i;\ i=1...N \end{aligned}$$

Model 3: Complete pooling without S-s>s constraint

$$\begin{aligned} &\min_{(s_i,S_i);\, i=1...N} E(TC) \\ &s.t.\ \ s_i \geq 0, S_i \geq 0,\ and\ Integers; i=1...N \\ &\quad system\ lost\ sales\ per\ period \leq \overline{\beta} \end{aligned}$$

Model 4: Complete pooling with S-s>s constraint

$$\begin{aligned} &\min_{(s_i,S_i);\, i=1...N} E(TC) \\ &s.t.\ \ s_i \geq 0, S_i \geq 0,\ and\ Integers; i=1...N \\ &\quad S_i - s_i \geq s_i;\ i=1...N \\ &\quad system\ lost\ sales\ per\ period \leq \overline{\beta} \end{aligned}$$

Model 5: Partial pooling without S-s>s constraint

$$\begin{aligned} &\min_{(s_i,S_i);\, i=1...N} E(TC) \\ &s.t.\ \ s_i \geq 0, S_i \geq 0,\ and\ Integers; i=1...N \\ &\quad X_{j,i,t} \leq Q_{\max};\ i,j=1...N\ and\ j \neq i \end{aligned}$$

Model 6: Partial pooling with S-s>s constraint

$$\begin{aligned} &\min_{(s_i,S_i);\, i=1...N} E(TC) \\ &s.t.\ \ s_i \geq 0, S_i \geq 0,\ and\ Integers; i=1...N \\ &\quad S_i - s_i \geq s_i;\ i=1...N \\ &\quad X_{j,i,t} \leq Q_{\max};\ i,j=1...N\ and\ j \neq i \end{aligned}$$

In fact, our transshipment problem with lost sales is a complex problem and hard to solve it with analytic method, so we have recourse to solve this optimization problem by integrating the simulation with the optimization.

4. RESOLUTION METHOD

4.1. Initial phase

Before we begin the simulation, we should have an initial phase. In the inventory literature, many research works are interested to determine the policy parameters under various assumptions for a single location/single product inventory system. Since our model deals with a normal distribution demand, an exact approximation of the reorder point based on service level constraint is derived by Schneider and Ringuest (1990). Let (s_i^0, S_i^0) denotes the initial values for each retailer i.

$$s_i^0 = \mu_i (L+1) + k_i \sigma_i \sqrt{L+1} \tag{4}$$

In lost sales systems, the safety factor k_i is a solution of the equation (5). The numerical results are given in (Silver, Pyke and Peterson, 1998) p. 725-734.

$$G_u(k_i) = \frac{EOQ}{\sigma_i \sqrt{L+1}} \left(\frac{1-DS}{DS} \right) \tag{5}$$

Finally, the order-up-to-level is determined as:

$$S_i^0 = s_i^0 + EOQ \tag{6}$$

Wagner, O'hagan and Lundh (1965) have indicated that the economic order quantity Q is a good approximation for the optimal reorder quantity under the condition that the ratio K/ch is relatively high to μ_i, which is the case in our study.

4.2. Simulation-optimization model

The simulation-optimization models corresponding to the six models (section 3.4) are implemented on a spreadsheet and run by both Monte Carlo simulation and OptQuest of Crystal Ball 7.2®.

The simulation model is operated on a basic time period of one period; i.e., the inventory is reviewed once each period. The demand per period is considered as the input of the model. We use Monte Carlo techniques to generate the demands. The rule of Monte Carlo simulation is to select randomly the customer demands according a normal distribution, which is specified by a mean and a standard deviation. In order to confirm the hypothesis of i.i.d distribution we should disable the correlation between demands over time and among retailers. As output, the simulation model saves a variety of system performance measures: the total cost, the system service levels (α-service and β-service), transshipment rate (total transshipment quantity/total demand), and lost sale rate (total lost sales/total demand). The simulation model is validated by using a 95% confidence interval for all the performance measures. In addition, the simulation model is run over a planning horizon of 100 period units and each period is simulated 1000 times, so that the system performance measures are more accurate; i.e., low mean standard error and low coefficient of variability.

The optimization phase is based on a wide grid-search for (s_i, S_i), i=1…N. In order to construct this grid, we set the values of s_i and S_i within two given intervals; $s_i \in [LB_{1,i}, UB_{1,i}]$ and $S_i \in [LB_{2,i}, UB_{2,i}]$. The bounds of these intervals are chosen based on the following idea:

- The lower bounds for (s_i, S_i) are computed as follows: $LB_{1,i}$ is equal to demand during L periods and $LB_{2,i}$ is equal to demand during the replenishment cycle (L+1 periods);
- The upper bounds for (s_i, S_i) are the initial values of the reorder points and the order-up-to levels (determined in section 3.3).

Since the minimization criterion is not a convex function for the (s, S) inventory system; i.e. present several distinct relative minima (Wagner, O'hagan and Lundh 1965), we have to search the best combination (s_i, S_i), i=1…N, through this grid. However, the number of all possible combinations within this grid is combinatory. So, we run the OptQuest optimization tool of Crystal Ball 7.2® by using two different step sizes (five units and one unit). For each step size, we run OptQuest for 1000 simulations.

By starting with the initial solution (s_i^0, S_i^0), the research for (s_i, S_i) is carried out in $[LB_{1,i}, UB_{1,i}]$ and $[LB_{2,i}, UB_{2,i}]$ with a step size of five units. At the end of this phase, OptQuest of Crystal Ball provides the best solution among all the evaluated solutions which minimize the total cost and satisfy the constraints. We denote this solution by (s'_i, S'_i) for each retailer i, then, a new research is made closely around this solution by using a one step size and restricting the intervals as below:

$LB_{1,i} = s'_i-5$; $UB_{1,i} = s'_i+5$

$LB_{2,i}= S'_i-5$; $UB_{2,i}= S'_i -5$

Finally, we obtain the best values (s_i, S_i) for each retailer i achieving our objective.

We firstly run the simulation-optimization model for one retailer by using the initial values s_i^0 and S_i^0 as starting points. This phase gives us the optimal inventory control parameters (s, S) for the no pooling system. These optimal values will be served as starting points for the multi-retailer pooling inventory system.

5. NUMERICAL RESULTS

The numerical analysis are reported to show the sensitivity of the constraints made upon the mean lost sales and capacity per tranship on the performance measures. The experiments are evaluated via large combination of the input parameters: DS=98%, K=100, ch=1, cp=100, ct=20, EOQ=89, the other parameters are given in Table 1 and the initial solutions for the corresponding parameters are presented in Table 2. According to this combination, we have in total 360 problems to be evaluated.

Table 1: Input Parameters for Numerical Examples

Parameter	Levels	Values
N	3	2; 4; 8
L	2	1; 3
σ_i	2	10; 20
$\overline{\beta}$	4	0.1; 0.05; 0.02; 0.01
Q_{max}	3	5; 10; 15

Table 2: Initial Solutions

L	μ_i, σ_i	k_i	s_i^0	S_i^0
1	40, 20	1,13	112	201
3	40, 20	1,3	212	301
1	40, 10	0,76	91	180
3	40, 10	0,95	179	268

5.1. Evaluation of models 1 and 2

An examination of Table 3 leads to several interesting observations. In all cases, the total cost in model 2 is higher than that in model 1. This can be explained by the increase/decrease of both lost sales and transshipment rates (indicate by %LS and %TR in Table 3.) We can also observe that the difference between these two models declines with the number of retailers and becomes not significant (lower than 1%) in the case of shorter replenishment lead-time (L=1) and large number of retailers. However, the α-service in model 2 is lower than that in model 1 and the difference between two models goes up 2.11%. This can be interpreted by the increase of lost sale occurrences in the model 2 due to the severe constraint S-s>s and the partial satisfaction via transshipments. Moreover, the difference between two models in β-service is negligible and does not exceed 0.19%. This is due to the transshipments, which tend to satisfy immediately customer demand from stocks on hand at all retailers and alleviate partially or entirely the shortages.

We can also draw another conclusion, which concerns the stock levels. In most cases, the reorder points (s) in model 2 are more reduced than those in model 1 and, in all cases, the order-up-to levels (S) in model 2 are strongly higher than those in model 1. Indeed, in model 1, each retailer tends to have a higher reorder point, and so takes earlier precaution for lost sales by placing frequently orders with small quantity. While in model 2, the higher order-up-to levels are due to the constraint S-s>s which leads to place less frequently orders with large order quantity. Hence, the number of replenishment cycles in model 1 is about two times than that in model 2. That is, an order is placed in the middle of a replenishment cycle. This leads to have a higher reorder point and a lower order-up-to level and consequently the holding and penalty costs are more reduced in model 1 than that in model 2.

Table 3: Difference in Performance Measures of Model 1 vs. Model 2

N	σ_i	L	%α	%β	%E(TC)	%TR	%LS	%s	%S
2	10	1	0,00	0,02	0,91	-0,79	-20,01	0,62	14,41
	10	3	-1,55	-0,04	12,92	33,01	19,55	-5,79	26,36
	20	1	0,00	0,03	1,46	-9,50	-18,16	-1,63	10,03
	20	3	-2,11	-0,07	11,40	29,48	18,08	-9,44	21,68
4	10	1	-0,80	-0,03	0,20	11,68	19,88	-2,42	1,25
	10	3	-1,23	0,00	8,11	24,96	-2,71	-6,75	23,60
	20	1	-0,13	-0,02	0,10	5,19	9,02	-2,82	1,58
	20	3	-1,08	-0,06	5,17	38,37	23,28	-11,45	18,46
8	10	1	0,04	0,01	0,26	-2,11	-7,96	1,08	0,00
	10	3	-1,67	-0,19	5,90	17,25	71,95	-9,38	19,04
	20	1	-0,13	-0,02	0,10	5,19	9,02	-2,82	1,58
	20	3	-0,92	-0,12	1,90	23,30	47,04	-22,92	14,52

Next, we focus on accessing the benefits of transshipments according to the variation of replenishment lead-times and standard deviation of the demand for models with/without the constraint S-s>s. The cost increases with the lead-time as well as the standard deviation. For instance, in the case of two-retailer inventory system in model 2, the total cost, the transshipment rate, the reorder point, and the order-up-to level increase, respectively, in average about 19.73%, 47.59%, 46.81%, and 48.23% when the lead-time increases from one to three for σ_i=10 and these performance measures also increase, respectively, about 16.71%, 8.83%, 18.05%, and 14.18% when the standard deviation ranges from 10 to 20 for L=1. In addition, the increase of the performance measures declines with the number of retailers. That is, when the transshipment cost is non-expensive in comparison with the penalty cost, the transshipment leads to cost savings and reduction in inventories for large distribution systems. These results hold in the models 1, 3, 4, 5 and 6.

5.2. Effects of lost sales constraint

The observations that emerge from Figures 1 and 2 concerning the variation of the total cost and the transshipment rate are very interesting. For instance, in model 3, when the rate $\overline{\beta}$ becomes very tight (0.01), the total cost is lower (Figure 1). This can be explained by the decrease of the total number of lost sales due to the severe constraint $\overline{\beta}$ made upon each period. When the rate $\overline{\beta}$ is large (0.1), we obtain higher total cost even if the transshipment rate is also higher (Figure 2). Indeed, the increase of the total cost is due primary to the expensive penalty cost which dominates the transshipment cost. These results hold in the model 4.

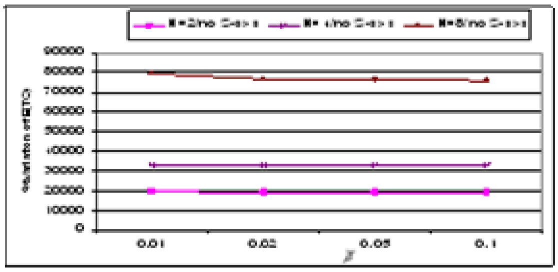

Figure 1: Variation of Total Cost for Model 3, where σ_i =10 and L= 1

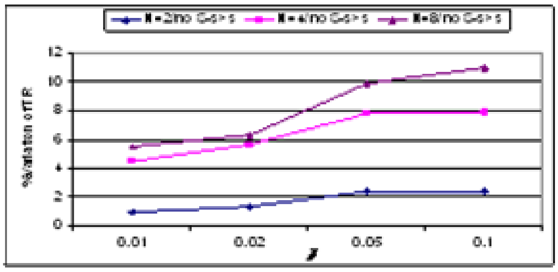

Figure 2: Variation of Transshipment Rate for Model 3 where σ_i =10 and L=1

A study of sensitivity of the constraint made upon the rate $\overline{\beta}$ indicates that the performance measures and stock levels depend on the variation of σ_i, L, and N. Figure 3 shows that, when the lead time ranges from one period to three periods, the total cost increases with the constraint $\overline{\beta}$ as well as with the number of retailers. In fact, during a longer lead time, the retailers tend to keep a large quantity of stock in order to protect against the shortage. In addition, the total cost also increases with the standard deviation (σ_i varies from 10 to 20) and the number of retailers (Figure 4). We study now the sensitivity of inventory levels to the constraint $\overline{\beta}$. In model 4, the variation of $\overline{\beta}$ from 0.01 to 0.1 leads to an increase about of 8.61% in s and about of 8.35% in S for N=2, σ_i=10 and L=3. For N=4, the increase in s is of 7.97% and in S is in average about 4.81%. For N=8, the increase in s is about 5.22% and in S is about of 3.30%. We observe that the increase of stock levels declines with the number of retailers. This can be explained by the fact that, in a large distribution system, transshipment tends to alleviate completely the shortages (i.e., the total number of lost sales is nearly zero), so the inventory levels are less sensitive to the constraint $\overline{\beta}$.

The difference between the models 3 and 4 is more significant in longer lead time and higher standard deviation. This is substantiated in Figures 3 and 4. However, the difference between these models in α-service and β-service levels does not exceed 1% in all cases. This is due to the lower effect of $\overline{\beta}$, resulting from its small value.

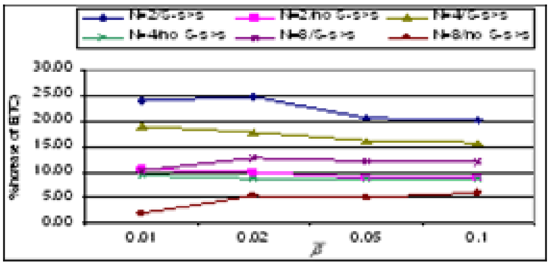

Figure 3: Sensitivity of Total Cost to the Lead Time L (from 1 to 3) where σ_i =10 for Models 3 and 4

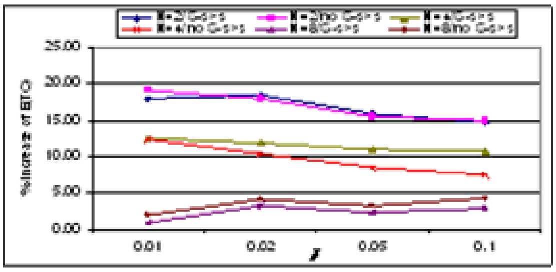

Figure 4: Sensitivity of Total Cost to the Standard Deviation σ_i (from 10 to 20) where L=1 for Models 3 and 4

5.3. Impact of capacity per tranship

We turn now our attention to examine the effects of the partial pooling on system performance measures. The degree of transshipment benefits depends on the capacity constraint per tranship. The α-service and β-service levels achieved by the models 5 and 6 are not much different in the whole range of the values. To be more specific, the system's α-service and β-service levels go higher as the capacity per tranship becomes large. Moreover, a reduction in the capacity per tranship will decline the outgoing transshipment in the retailers having stock on hand (Figure 6) and consequently will increase the lost sales (i.e., increase the penalty cost) at each retailer (Figure 5). In addition, the lower the capacity in all retailers is, the higher the stock levels at each retailer are (Figures 7 and 8). The change of the total cost due to the change of capacity per tranship appears to be as a linear behaviour (Figure 5). The models 5 and 6 show a considerable difference in the performance measures, in particular, in longer lead times. We also observe that this difference is very sensitive to σ_i, L, and N.

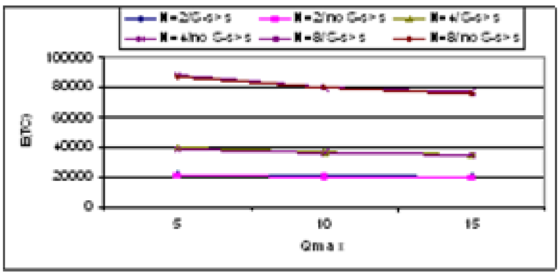

Figure 5: Variation of Total Cost for Models 5 and 6 where σ_i =10, L=1

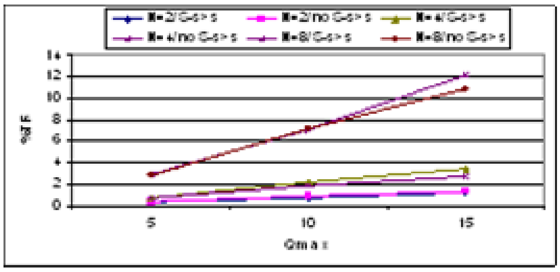

Figure 6: Variation of Transshipment Rate for Models 5 and 6 where σ_i =10, L=1

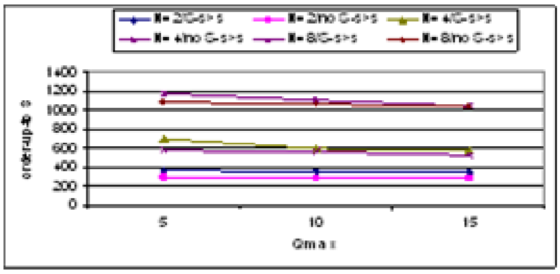

Figure 7: Variation of Order-up-to Levels in Function of Q_{max} and N where σ_i =10, L=1

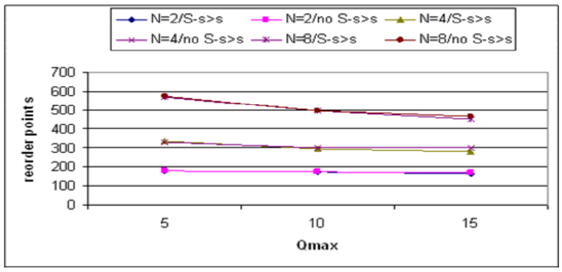

Figure 8: Variation of Reorder Points in Function of Q_{max} and N where σ_i =10, L=1

The experiments also indicate that the difference between the complete pooling and the partial pooling under large capacity (Q_{max}=15) for N=2, σ_i =10, L=1 is about 4.29% in E(TC), 3.59% in s, 5.93% in S, 0.81% in α-service, and 0.01% in β-service. Thus, for Q_{max} more than 15, the benefits with partial pooling under a large capacity per tranship tends to be similar to that of the complete pooling policy. This result holds for a large number of retailers as shown in Figures 9 and 10. In practice, a retailer would like keep some units of stock for its future needs instead of sharing all its available stock with the other retailers. In this situation, a partial pooling policy can be considered as an attractive policy to manage the stocks at retailers.

5.4. Efficiency of the EOQ

It is worth comparing the EOQ to the reorder quantity in all transshipment models. Our results indicate that the EOQ appears to be a good initial solution in models without S-s>s constraint. In all cases of models 1, 3 and 5, the reorder quantity is reduced in comparison with the EOQ. This reduction goes higher as the number of retailers becomes large and goes lower with the increase of lead-time/standard deviation. However, the EOQ is not recommended in the models with S-s>s constraint. In some cases, the reorder quantity in transsphipment model is about two times than the EOQ. To be more specific, a longer lead-time and/or a higher standard deviation lead to a large gap between the EOQ and the reorder quantity in transsphipment model. Nevertheless, in the model 2, the EOQ can be served as an initial solution only in the case of shorter lead-time (L=1) and lower standard deviation (σ_i=10).

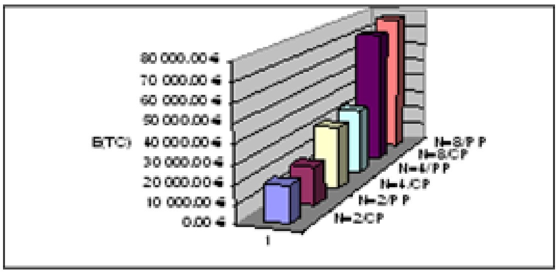

Figure 9: Complete Pooling (CP) vs. Partial Pooling (PP) where Q_{max}=15, for σ_i =10, L=1 in Model 6

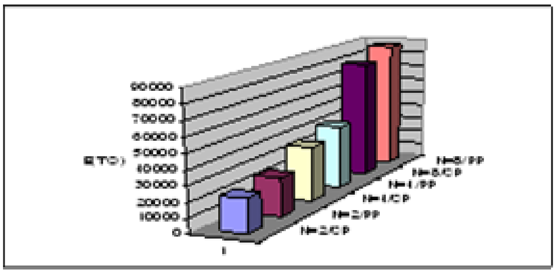

Figure 10: Complete Pooling (CP) vs. Partial Pooling (PP) where Q_{max}=15, for σ_i =20, L=1 in Model 6

6. CONCLUSION

This paper has examined the effectiveness of two transshipment policies in two-echelon distribution system, characterized by a single warehouse at the higher echelon and multiple retailers at the lower. One of these transshipment policies is a complete pooling policy, while the other is the more realistic, partial pooling policy. Our results indicate that the benefits appear to increase, as the number of retailers becomes large. Moreover, under some circumstances, either of complete or partial pooling policies, may be the most desirable policy.

Future works should focus on adding a fixed cost for each transshipment request in models 5 and 6. Another extension involves consideration of ordering policy at the warehouse and investigation of transshipment benefits.

REFERENCES

Axsäter, S., 1990. Modelling emergency lateral transshipments in inventory systems. *Management Science*, 36(11), 1329-1338.

Evers, P., 2001. Heuristics for assessing emergency transhipment. *European Journal of Operational Research*, 129, 311-316.

Hu, J., Watson E., Schneider H., 2005. Approximate solutions for multi-location inventory systems with transshipments. *Int. J. Production Economics*, 97, 31–43.

Kukreja, A., Schmidt C., 2005. A model for lumpy demand parts in a multi-location inventory system with transshipments. *Computers & Operations Research*, 32, 2059-2075.

Lee, H.L., 1987. A multi-echelon inventory model for repairable items with emergency lateral transshipments. *Management Science*, 33, 1302–1316.

Minner, S., Silver E., Robb D., 2003. An improved heuristic for deciding on emergency transhipments. *European Journal of Operational Research*, 148, 384-400.

Needham, P.M., Evers P.T., 1998. The influence of individual cost factors on the use of emergency transhipments. *Transpn Res.-E*, 34(2), 149-160.

Robinson, L.W., 1990. Optimal and approximate policies in multiperiod, multilocation inventory models with transshipments. *Operations Research*, 38 (2), 278–295.

Schneider, H., Ringuest J.L., 1990. Power approximation for computing (s; S) policies using service level."*Management Science*, 36 (7), 822–834.

Sherbrooke, C.C., 1968. METRIC: Multi Echelon Technique for Recoverable Item Control. *Management Science*, 16 (1), 122–141.

Silver, E.A., Pyke D.F., Peterson R., 1998. *Inventory management and production planning and scheduling*. John Wiley & Sons.

Tagaras, G., Cohen M., 1992. Pooling in two-location inventory systems with non-negligible replenishment lead-times. *Management Science*, 38(8), 1067–1083.

Tagaras, G., 1999. Polling in multi-location periodic inventory distribution systems. *Omega*, 27 (1), 39–59.

Tlili, M., Moalla M., Campagne J.P., Bahroun Z., 2008. Un modèle basé simulation-optimisation pour un système de distribution à deux stocks intégrant le transshipment. Proceedings 7e Conférence Internationale de MOdélisation et SIMulation (MOSIM'08), pp. 1240-1248, March 3 - April 2, Paris, France.

Xu, K., Evers P.T. and Fu M.C., 2003. Estimating customer service in a two-location continuous review inventory model with emergency transshipments. *European Journal of Operational Research*, 145(3), 569–584.

Wagner, H.M., O'hagan M. and Lundh B., 1965. An empirical study of exact and approximately optimal inventory policies. *Management Science*, 11(7), 690-723.