HIGHWAY TRAFFIC MODEL BASED ON CELLULAR AUTOMATA: PRELIMINARY SIMULATION RESULTS WITH CONGESTION PRICING CONSIDERATIONS

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ABSTRACT

Cellular Automata are a reputable formal support for traffic modelling and simulation. STRATUNA is a Cellular Automata model for simulating the evolution of two/three lane highways. It encodes the wide specification of driver's response to the events in his sight range. Encouraging comparison between simulated events and their corresponding in the reality bring to the specification of a theoretical general model characterized by an increased expression power and a significantly deeper forecasting potential, whose application fields are numerous and varied. Fair results in flow forecasting lead to the implementation of an established cost system in which simulation directly provides cost forecasting in terms of congestion toll.

Keywords: modelling, simulation, cellular automata, highway traffic

1. INTRODUCTION

In the field of modelling high complexity systems, Cellular Automata (CA) (von Neumann 1967) are a widely used computational paradigm to simulate those systems which evolve mostly according to an a-centrism schema through local interactions of their constituent parts. Basically, a CA can be seen as a *d*-dimensional space partitioned into uniform cells, each one embedding a computational tool, namely the elementary automaton (EA). If we consider this computational device as a black-box, the output of each cell is its internal state. On the other hand, input for these EAs is given by states of other neighbouring EAs. The neighbouring conditions are determined by a pattern invariant in time and in space. At the beginning, each EA is in an arbitrary state that defines the initial condition; afterwards, the CA evolves by changing concurrently states to all of the EAs at the same discrete time steps, according to EA transition function (parallelism property).

In the field of highway modelling, thanks to a-centric and parallel properties of such a system, CA are intensively used (Schadschneider 2006) for analysis of traffic emerging phenomena. In fact, when a highway exhibits fixed structural characteristics and there are no external interferences out of the vehicular interactions (normal conditions), the traffic evolution emerges by the mutual influences among next vehicles.

At our knowledge, the main CA models for highway traffic (Nagel and Schreckenberg 1992; Wolf 1999; Knospe, Santen, Schadschneider, and Schreckenberg 2000; Lárraga, Del Ríob and Alvarez-Icaza 2005) simulation can be considered "simple": external inputs to each EA and corresponding feedbacks are very regular and easy. However, these simple models reproduce the basic three different phases of traffic flow ("free flow", "wide moving jams" and "synchronized flow") and simulations are in general compared with single vehicle data (i.e. automatically collected data gathered by stationary inductive loops placed above highway lanes).

In this paper we present STRATUNA (Simulation of highway TRAffic TUNed-up by cellular Automata), a new model for highway traffic modelling based on CA. The first aim of our model is to express more accurately driver nearby conditions and reactions; a guide to easily interpret our model is that a vehicle and its driver are strongly coupled (and generally created according to a *normal* distribution or field data) and make a whole. We have based a $\beta 4$ implementation on an improvement of previous CA model (Di Gregorio and Festa 1981; Di Gregorio, Festa, Rongo, Spataro, Spezzano and Talia 1996), that was adequately rewarding in the past, but now it is out-of-date for the different industrial situations (for instance, the classification of vehicles on the base of mere acceleration/deceleration capabilities is now unrealistic). Field data used to feed this early model come from Italian highway A4 and are composed by timed highway entrance-exit specification coupled to vehicle type and approximated route length. These data make it possible the comparison between real event and simulated ones in terms of average speed.

The analysis of problems that come out from the adoption of the implemented model, together with a deep theoretical study, lead to the general STRATUNA model, whose specification is one of the objects of this paper and promises profound forecasting capabilities.

Finally, a cost system is proposed in which the simulation model finds application and fits well in cost forecasting. Conclusions are reported at the end of the paper.

2. STRATUNA $\beta 4$

STRATUNA $\beta 4$ is, at present, a representative implementation and is based on an extend CA definition, namely "macroscopic" (Di Gregorio and Serra 1999). The extension in fact introduces "substates" and "external influences"; a substate defines an attribute of a cell and can be either self-sufficient or composed by other sub-substates and so on, as attributes in reality. On the other hand, the only external influence is represented by the entrance of vehicles at tollgates, measured out according to field data or probabilistic function.

We use a one-dimensional CA (since the y-position of a vehicle in a cell can be stored as substate) where each cell corresponds to 5m of highway in the reality. Inside this one-dimensional CA, in order to manage highway zones with respect to drivers, we introduce the notion of "free zone": each vehicle controls a disjunction of zones that are free of vehicles and cannot be reached by surrounding vehicles in the next step. To refine this concurrency control set up with "free zones", the indicator light is introduced, by means of which vehicles reveal their intentions planned for the next step. Moreover, as just anticipated, we introduce actions that last more than one step and optionally provide a kind of synchronization between drivers (e.g. the overtake action is done in more consecutive steps).

In order to complete given intuitions, we formally state now the implemented model:

$$STRATUNA_{\beta 4} = \left\langle R,E,X',P',S',\gamma_{\beta 4},\tau_{\beta 4} \right\rangle \qquad (1)$$

$R = \{x \mid x \in N,\ 1 \le x \le n\}$ is the set of *n* cells, forming the highway, where *N* is set of natural numbers.

$E \subset R$ is the set of entrance-exit, special cells in *R* where vehicles can be introduced or eliminated.

$X' = \langle -r,\ -r+1,\ \ldots,\ 0,\ 1,\ \ldots,\ r \rangle$ defines the EA neighbouring, where *r* is a radius defining the sight range of the average driver, with $\#X = 2r+1$.

$P' = \{length,\ width,\ clock,\ lanes\}$ defines global parameters: *length* and *width* define the geometry of the cell, over x and y axis respectively; *clock* is the CA clock, *lanes* is the number of highway lanes (numbered 1, 2 .. from right to left) including additional lane 0, representing, from time to time, the entrance or exit or emergency lane.

$S' = \left\{ Static' \times \left(Vehicle' \times Driver' \right)^{lanes} \right\}$ represents the structure of substates and sub-substates that characterize every cell state. To be more precise, is proposed this hierarchy of substates in Table 1.

CellNO is a kind of ID that uniquely identifies a cell in the whole CA. *SpeedLimit* is the speed limit imposed and is specified lane by lane. After these static characteristics of a cell, we have the characteristics of a vehicle: *Type* is one in {motorcycle, car, bus/lorries/vans, semi-trailers/articulated}, while *Length* is the vehicle extension over the x-axis direction. Then we have the substates that code characteristics related to the speed and the acceleration of a vehicle. *Indicator'* can assume a value in {0,-1,1} respectively showing that {no, right, left} indicator is turned on. Finally, *Xposition* and *Yposition* collocate the middle point of the front side of the vehicle inside the cell. Going forward, we define driver's substates: the trip information (*Origin*, *Destination* $\in E \times E$), the desired speed and the suitability of every lane from driver's point of view.

```
BEGIN: TransitionFunction()
    FindNeighbours(); ComputeSpeedLimits();
    ComputeTargetSpeed(); DefineFreeZones();
    AssignLowSuitabilityWhereAFreeZoneIsCutted();
    if(ManouvreInProgress())
     continueTheManouvre(); return;
    if(myLane==0) //I'm on a ramp
     if(IWantToGetIn())
      if(TheRampEnded())
       if(ICanEnter()) enter(); return;
       else if(IHaveSpaceProblemsForward())
          slowDown(); return;
        else followTheQueue(); return;
      else //the ramp is not ended yet
       if(IHaveSpaceProblemsForward())
        followTheQueue(); return;
       else keepConstantSpeed(); return;
     else //I want to get out
      if(TheRampEnded()) deleteVehicle(); return;
      else if(IHaveSpaceProblemsForward())
        followTheQueue(); return;
       else keepConstantSpeed(); return;
    //end lane==0
    else if(myLane==1)
     if(MyDestinationIsNear())  slowDown();
     if(MyDestinationIsHere())  goInLowerLane();
    else //myLane==2 or more
     if(ICanGoInLowerLane())
      if(GoingInLowerLaneIsForcedOrConvenient())
        goInLowerLane();
     else //I cannot go in lower lane
      if(MyDestinationIsNear())
       slowDown();  goInLowerLane();
    if(!IHaveSpaceProblemsForward()) //every lane
     if(TakeoverIsPossibleAndMyDestinationIsFar())
      if(TakeOverIsDesired())  takeover();
      else followTheQueue();
     else followTheQueue();
    else //I have space problems forward
     if(TheTakeoverIsForced())  takeover();
    return;
END;
```

Table 1: How Sub-substates Compose Substates

Substate	Composing sub-substates
Static'	*CellNO, SpeedLimit*^{lanes}
Vehicle'	*Type, Length, MaxSpeed, MaxAcceleration, MaxDeceleration; CurrentSpeed, Indicator', Xposition, Yposition,*
Driver'	*Origin, Destination, DesiredSpeed, Suitability*^{lanes}

$\gamma_{\beta 4}$:$N \times E \rightarrow Vehicle \times Driver$ is the *normal* generation function that introduces new pairs <*Vehicle, Driver*> into the highway at discrete steps.

$\tau_{\beta 4}$:$S'^{\#X'} \rightarrow S'$ is the EA transition function. Since this function is the core of the evolving model, a pseudo-code block is proposed before Table 1 with the aim of stating it; in that context, (1) "return" ends the evolution of single EA at each evolution step; (2) functions starting in lowercase are actions en-queued to be performed in further steps; (3) underlined functions represent the beginning of a synchronized protocol actuated over consecutive steps (e.g. actions in consecutive steps of takeover-protocol are: control a "free zone" on the left, light on the left indicator, start changing *Yposition*, and so on).

2.1. Field data composition and treatment

A clear and unambiguous way to understand how our model is fed, is to describe data used to define what we consider "reality" and what we consider a "real event".

Entrance-destination matrices referring to 5 non-contiguous weeks are evaluated in order to feed the generation function $\gamma_{\beta 4}$. Data are given by about 1 million digitalized toll tickets, pertain to Italian highway A4 Venezia-est/Lisert. This highway is characterized by twelve fundamental entrances/exits, two base lanes, and is long approximately 120km. Toll ticket data are, contextually with emission, grouped in five categories depending on number of axles and vehicle height at first axle (Cfr. Table 2; however, this classification is reducible to our vehicle classification as defined in substate *Type*). Due to some inconveniences of synchronization between tollgates, these datasets require a data cleaning step that involves the following groups of tickets: (i) missed tickets: transits without entrance or starting time; (ii) transits across two or more days; (iii) trips that end before they begin; (iv) vehicles too fast to be true (exceeding 200 km/h as average speed). In the aggregate, the cleaning interested about 10% of total tickets.

Subsequently, each one of the 34 days, is analyzed with respect to the average speed and the total flow; these measurements are compared to averages over all days. The result of this quantitative study is fixed in Figure 1: each dot represents one of the 34 analyzed days: a shift over x-axis and y-axis is a variation respectively of "total flow" and "average speed" from the value of the mean day. *DesiredSpeed* distributions for each vehicle *Type*, as presented in Table 2, are easily deduced by highway data for vehicles covering short distance in optimal conditions of flow and weather; since trips taken into account are really short, the probability of a rest or a need of refuel is minimal. However, in general data scenarios, vehicles that parked for a while in the rest and service areas cannot be detected by data and introduce errors; these kinds of errors justify the slightly higher values of average speed, obtained in the simulated cases, when compared with values corresponding to real events.

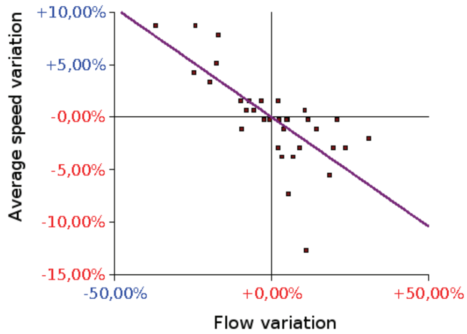

Figure 1: Daily Flow and Speed Fluctuation from the Average

Table 2: Share and Desired Speed for each Type of Vehicle in Selected Case of Free Flow

Vehicle type	Desired speed	Flow share
I	122,80 km/h	93,4%
II	112,77 km/h	4,6%
III	113,29 km/h	0,5%
IV	102,61 km/h	0,1%
V	93,90 km/h	1,4%

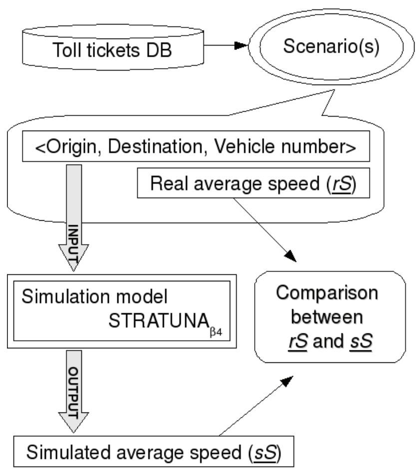

Figure 2: Base Workflow, from Field Data to Simulations and Comparison with Reality

Finally, a statistical sampling treatment is performed to select meaningful subsets and three groups of possible scenarios are singled out: free flow situations, moderated flow cases, and cases in which the flow is higher than moderated and concentrated in a small region of the highway. These scenarios are what we refer to as "real events" and are used as touchstone in simulations presented in the following. To be more precise, each scenario provides a number of vehicles (each one specified by the couple <Origin,Destination>) and the average real speed (*rS*) over all its vehicles and over all the event, as shown in Figure 2.

2.2. Simulation results and comparison with reality

Despite the capabilities of model $\beta 4$, which can take into account different types of vehicle, generated vehicles are all cars, which represent, in fact, *95%* of real traffic. Starting from typical highway conditions, we report five representative simulations: one for free flow (Figure 3), two for moderated-flow next to congestion (Figure 4 and Figure 5) and two for locally congested situations (Figure 6). The following information are presented in each figure: corresponding real average speed *rS* (painted as a line), step-by-step average simulated speed *sS* (represented as a fluctuating curve), and the desired speed *sDS* (represented as an invariant notch compiled with data presented in Table 2).

At the beginning of each simulation, the modelled highway is empty; then, it is fed, entrance-by-entrance, with vehicles according to the generation function $\gamma_{\beta 4}$. Because of this, and taking into account the fact that each generated vehicle starts from null speed, the initial *sS* value is very low. After this, we have a pump transient where a portion of the intake process takes place; during this transient, *sS* grows pointing the *sDS* value since each vehicle can move forward its desired speed. An increasing number of vehicles reverses this trend and adjusts the *sS* value according to emerged situations. When this "pump phase" ends (this is after about 500 simulated seconds), an "evolving phase" begins: the *sS* measure starts evolving according to emerged events, triggered by transition function through local interaction, synchronization events and driver behaviours induced by environment. This evolving phase and the whole simulated event are compared with real event by means of two error measurements, respectively e_1 and e_2. The e_1 value is calculated as the average relative error (over all CA steps) between *sS* and *rS*, while e_2 is the same as e_1 but calculated excluding the "pump phase". Now we present simulation results of the free flow scenario.

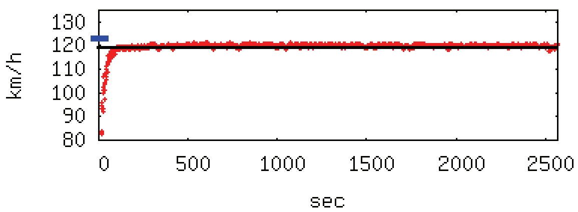

Figure 3: The Free Flow Case

In the free flow scenario presented above, the average simulated speed (*sS*) matches its equivalent in the reality (*rS*) during the whole simulation, outstanding slightly higher than field data, with very short oscillations (Figure 3). Indeed, we have recorded e_1*=1,29%* and e_2*=0,86%*. Then we simulate another scenario, previously referred to as the moderated flow, in which the increased number of vehicles proposes circumstances next to congested situations.

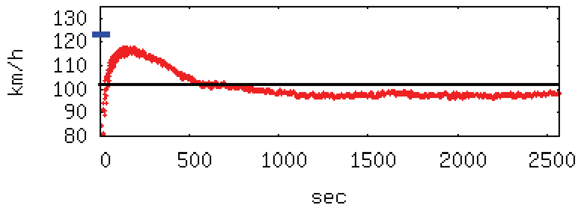

Figure 4: The Moderated Flow Case

In this scenario (Figure 4), after the same initial phase, *sS* becomes significantly lower than *rS* with moderate oscillations. Such a behaviour is unrealistic, despite of its low error rate: the cars in the simulation have to be faster than corresponding real cars, because they are "simpler" (they neither stop to give driver a rest neither need to refuel). This "slow moving" clearly depends on the fact that the simulated driver makes his subjective evaluation in a too much cautiously way. This cautious evaluation comes out because of the partial implementation of the transition function, which reduced the moving potentiality (reaction rigidity). Since a solution could be obtained by setting a shorter time step, that is equivalent to a shorter reaction time, only for next simulation, the value of parameter *clock* is changed from 1s (Figure 4) to 0,75s (Figure 5), without altering any other input to the simulation. The result of this tuning in the reactivity of drivers leads to a more realistic simulation, with error values e_1*=6,47%* and e_2*=5,83%*.

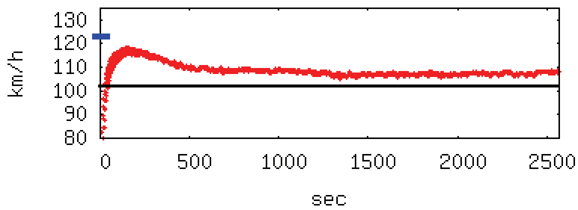

Figure 5: The Moderated Flow Case, with Tuned Reactivity Value

We present now two more simulations, with an implementation performance that is lower than the previous ones (Figure 6). Both simulations (lighter and darker oscillating curve) consider the particular, real, situation when a huge vehicle flow occurs only from one entrance and a climate issue alters driver behaviour (data refer to a really hot day in July concomitant with a migration from the city to the sea area). Both cases run on the same specifications of previous simulations, but *sS* becomes quickly significantly higher than *rS*. This means that, in this particular case, the reaction rigidity of the driver is rewarded by a higher speed due to a synchronization created by the forced filtering at the entrance. Corresponding error rates for lighter simulation are: e_1=*14,79%*, e_2=*13,94%*, while for the darker simulation we have e_1=*17,27%* and e_2=*17,12%*.

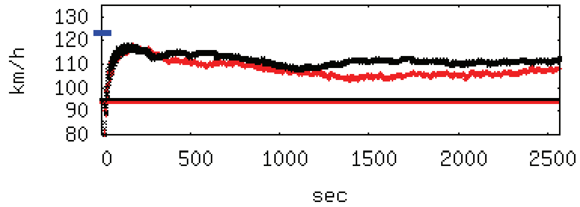

Figure 6: Locally Congested Situation

A chart summarizing the goodness of the model in speed forecasting is presented in Figure 7. Classical patterns of highway traffic (i.e. wide moving jams and synchronized flow) have been observed in simulations fed with synthetic (high) flow data. However the lack of single vehicle data (or data collected automatically by stationary inductive loops, as cited and presented by Schadschneider, 2006) does not enable a serious comparison between simulated patterns and real ones.

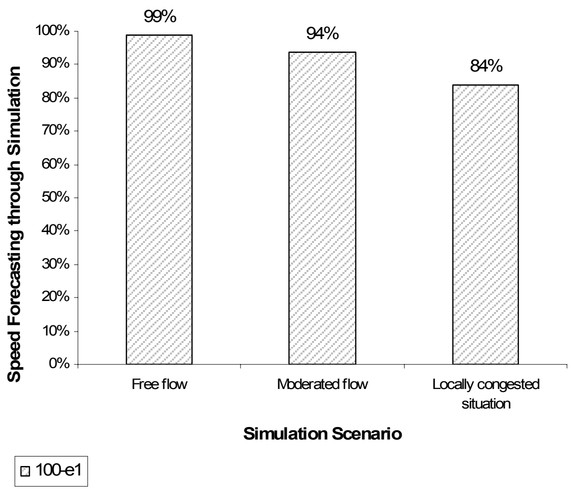

Figure 7: Goodness of Model in Speed Forecasting Task on Considered Scenarios

3. THE STRATUNA GENERAL MODEL

Encouraging results obtained with model $\beta 4$, along with the accurate analysis of its weakness, lead to the design of the extended mathematical model that is presented in this section.

Assuming that all the theoretical and practical structure previously detailed remain valid, we can go directly to the formal definition of the STRATUNA general model, stated by the following 8-tuple:

$$STRATUNA = \langle R,E,X,P,S,\mu,\gamma,\tau \rangle \quad (2)$$

Here, besides *R* and *E*, some components are extended to provide more expressiveness of the model in order to tackle rigidity limitations discussed in the previous section.

$X = \langle -b, -b+1, \ldots, 0, 1, \ldots, f \rangle$ defines the neighbouring pattern of every EA, limited by furthest cells, forward (*f*) and backward (*b*), according to average driver's sight when visibility is optimal (no fog, no clouds, sunlight, etc.).

$P = \{ length, width, clock, lanes, \overline{weights} \}$ is the same set of global parameters as presented in (1), with the addition of a vector of weights that is responsible for outlining the behaviour of the average driver in different situations. This vector will be detailed with the transition function of the model.

$S = \{Static \times Dynamic \times$
$\times (Vehicle \times Driver)^{lanes} \}$
specifies the high level EA substates, grouped by typologies. To be more precise, the first and the second group of substates model the statical and the dynamical features of the highway sector where the cell is located, respectively. Then, we have substates that model couples vehicle-driver, which are present in the quantity of at most one couple for each lane in the same cell. Each element of the set *S* is composed by sub-substates, as detailed in Table 3.

Table 3: How Sub-substates Compose Substates

Substate	Sub-substates self-explanatory names
Static	*CellNO, Slope, CurvatureRadius, SurfaceType, SpeedLimit, Lane1SpeedLimit, Lane0SpeedLimit*
Dynamic	*BackwardVisibility, ForwardVisibility, Temperature, SurfaceWetness, WindDirection, WindSpeed*
Vehicle	*Type, Length, MaxSpeed, MaxAcceleration, MaxDeceleration; CurrentSpeed, CurrentAcceleration, Xposition, Yposition, Indicator, StopLights, WarningSignal*
Driver	*Origin, Destination, DesiredSpeed, PerceptionLevel, Reactivity, Aggressiveness*

In addition to already detailed sub-substated and self-explanatory ones, we consider also *PerceptionLevel*, *Reactivity* and *Aggressiveness*, but we postpone argument of details where transition function is discussed. Evolution of the model is demanded to the external influences functions μ and γ, while interactions among vehicles are delegated to the transition function τ.

$\mu : N \times R \rightarrow Dynamic$ is the "weather evolution function", that can change values of substates in *Dynamic* substates if appropriated., for each step $s \in N$ and for each cell $c \in R$,

$\gamma : N \times E \rightarrow Vehicle \times Driver$ is the generation function that places pairs vehicle-driver, at certain steps, in cell in *E* (corresponding to tollgates). This generation function has been preferred to uniform intake rates in order to serve both field-data-driven input and *normal* distribution.

$\tau: S^{b+1+f} \rightarrow S$ is the EA transition function. It is important to notice that *b* and *f* are generally restricted in *b′* cells backward and to *f′* cells forward, according to visibility: cells out of driver's sight are considered without information.

The transition function τ, which represents the core of model evolution, is now widely detailed and exhaustively treated and can reveal the principal design choices concerning STRATUNA.

3.1. General model transition function

While highway characteristics are already partitioned in *Static* and *Dynamic* substates, both *Vehicle* and *Driver* contain constant and variable substates. It is important to separate statical and dynamical parts in the couple vehicle-driver because the main mechanism of traffic evolution is related to the assignment of new values to variable substates of this couple. What can change at each step in variable substates of *Vehicle* is: *Xposition* (individuating x coordinate of the middle point of vehicle front side, inside the cell), *Yposition* (same point as just cited, but in orthogonal axis and where a fraction value corresponds to occupy two lanes), *CurrentSpeed*, *CurrentAcceleration*, *Indicator* (which can assume one value from: null, left, right, hazard lights), *StopLights* (that can be only turned on or off according to a breaking process) and *WarningSignal* (on or off depending on the attention a driver needs). Moreover, we have driver's dynamical substates: *DesiredSpeed*, *PerceptionLevel*, *Reactivity* and *Aggressiveness*. A case in which a driver changes his desired speed is the accident case: he turns on hazards lights and his desired speed becomes null, without being immediately removed from highway. Other sub-substates in *Driver* or *Vehicle* are considered constant characteristics (e.g. the sub-substate *Type* models the macro category a vehicle belongs to: motorcycle, car, bus / lorries / vans, semi-trailers / articulated). By changing the value of *Indicator* or *StopLights* or *WarningSignal*, a vehicle can communicate its intentions to next vehicles; then a communication protocol starts, which induces EA to "ask each others" before performing actions, in order to avoid dangerous circumstances or deadlocking situations (e.g. an overtake starts from lane 1 to lane 2 concomitant with the end of another overtake from lane 3 to lane 2). *WarningSignal* is activated when a driver needs the lane portion immediately ahead of his vehicle to be free; the mechanism provides that the driver who perceives this warning adjusts temporary his behaviour, to give way. The behaviour is the key of every driver: sub-substates *PerceptionLevel*, *Reactivity*, *Aggressiveness* respectively describe a how much a driver pays attention to others, how long does he take to react to a stimulus and how emphasized his reaction will be.

Each moving vehicle *V* performs a set of operations and evaluations with a precise order; performed computations can be grouped from driver's point of view as "reading environment state" and "decide what to do". The former process is divided in objective and subjective perceptions. Objective perceptions are basically temporary changes of *Static* substates via *Dynamic* inputs, causing alterations of *Vehicle* characteristic. For instance, highway *surface_slipperiness* is derived by SurfaceType, *SurfaceWetness* and *Temperature*, then, this data are related to the *Vehicle* sub-substates in order to calculate the temporary variable *maximum_safe_speed* that accounts for the vehicle stability, speed reduction by limited visibility and speed limits in the lane, occupied by the vehicle; then *desired_speed* is set as minimum between *max_speed* and *DesiredSpeed*. Likewise, *Slope* and *surface_slipperiness* determine the temporary variables *max_acceleration* and *max_deceleration*, correction to sub-substates *MaxAcceleration*, *MaxDeceleration.* Proceedings in objective perception, the next step consist in the identification of "free zones" for *V*, i.e. all the zones in the different lanes, that cannot be occupied by the vehicles next to *V*, considering the range of the speed potential variations and the lane change possibility, that is always anticipated by *Indicator*; note that the possible deceleration is computed on the value of *max_deceleration* in the case of active *StopLights*, otherwise a smaller value is considered.

The next computation stage involves the subjectivity of the driver and consist in the behaviour for next step. First of all, actual *CellNO* is compared with the cell number reported in *Destination* in order to evaluate if the exit is close enough to force a really calm way of driving, opportunely waiting the exit ramp. If the destination is far, driver's aspire is to reach and maintain his *desired_speed*. To do this, in general several alternatives are available; to measure goodness of each option, a cost is linked to it, according to standard cost functions fixed by the vector *weight* in parameter set. Among all the possible options, the driver chooses the one with minimal sum of the costs.

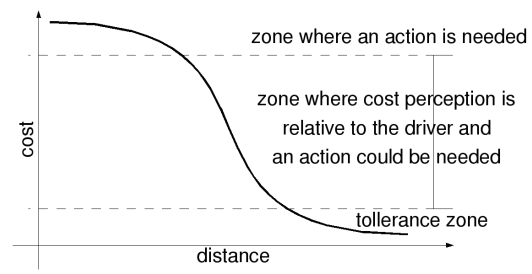

Figure 8: The Function that Connects the Distance from Front Vehicle with a Cost

An example of subjective process is now proposed. In Figure 8, a particular cost curve is presented, as individuated by *weight*, that connects the distance from front the vehicle with a penalty; in particular, this curve evaluates the gap between a vehicle and the one in front of it, which is supposed to suddenly break from time to time. Thus, the final choice is based on a driver subjective perception and evaluation of an objective situation by sub-substates *PerceptionLevel*, *Reactivity*, *Aggressiveness*.

Other examples of actions that lead to a cost are “remaining in a takeover lane”, “perceive a warning signal”, “staying far from *desired_speed”*, “breaking significantly”, “starting a takeover protocol”. P*erceptionLevel* concerns the perception of the free zones, whose length is reduced or (a little bit) increased by a percentage; however this “customized perceptions” are done in “customized security conditions”, considering the variable part of *Vehicle* substates, adding *max_speed*, *max_acceleration* and *max_deceleration*. *Aggressiveness* forces the deadlock that could rise during computation. For instance, when the entrance manoeuvre is prohibited due to a congested highway, since free zones are strongly reduced, the immobilised condition leads to a cost that increases significantly at each step; the variation of *Aggressiveness* value (a driver that is stressed by a cost) implies a proportional increase of the percentage value of *PerceptionLevel* until the free zone in a lane remains shorter than the distance between two consecutive vehicles. This leads to drivers that are temporary more aggressive, and can, therefore, perform the manoeuvre; when the manoeuvre is ended, and the generating cost too, *Aggressiveness* comes back to its original value.

As a result of illustrated features, the present general model has the needed expressivity to resolve problems that came out with model $\beta 4$. Indeed, the problem that the speed in the model is lower than the corresponding ones in the reality (Cfr. simulation in Figure 4) can be faced by *normally*-tuning the *Reactivity* value of simulated drivers, without altering the time step (*clock* parameter). Moreover, the possibility of reproducing locally congested situations got worse when climatic issues is introduced with the “weather function” μ. A serious problem that can rise with the adoption of sub-substates that describe average driver subjectivity, is the sure lack of field data. Without these data, that assign values of *PerceptionLevel* × *Reactivity* × *Aggressiveness*, the general model cannot lead to simulations. To overcome this obstacle, in the future we will employ the model inside a Genetic Algorithms (GA) (Holland 1975) to calibrate all included thresholds. The idea is that from the analysis of synthetic data and their comparison with the reality, we can infer those field data that characterize the real event, and use these “validated” deduced data to feed subsequent simulations (D'Ambrosio, Spataro and Iovine 2006).

3.2. Possible applications of the general model

Once the general model has been validated, a traffic forecasting analysis can be started, considering aspects that go beyond a simple comparison between simulated and real average speed. The expressivity taken into account from the general model suggests applications of the model in several fields related to traffic problems.

A promising use of STRATUNA model, probably through a GA, can be the study related to how highway owner should perform maintenance tasks in order to minimize road condition alterations during services. This can be summarized as “yard impact on road availability” and can be done at different levels: a starting level could be the yard planning such that the average simulated speed doesn’t decrease too much. A second level can be the design of better (that means safer or more reliable, etc..) highways through the simulation and the evaluation of different build options. Another field where the present model finds application is the evaluation of right price of a toll ticket, according to vehicular flow and private cost of each car. The basic idea is that a car which uses the highway reduces the time-gain derived from highway usage for other motorists; moreover, this travelling vehicle reduces the quality of life (by introducing pollution, noise, etc.) even of non-travelling people; Road pricing strategy searching is a widely diffused problem: a congestion toll system let motorists recognize the total cost they are imposing to other and has the good side effect of reducing vehicular flows. Since the STRATUNA general model exhibits forecasting features that can provide input information for a particular cost system, we now propose a preliminary cost study where our model can lead to a cost forecast.

4. PRELIMINARY STRATUNA-DRIVEN COST SYSTEM FOR CONGESTION TOLL

Theories on congestion pricing have been under research since the 1920’s and there are numerous references in literature about methods to estimate the costs for operating a car (fuel costs, maintenance, etc) in addition to the costs that each individual traveller imposes on other travellers due to the fact that each car increases the congestion of the highway. Road pricing has been implemented in various countries worldwide in order to reduce the traffic congestion problems in urban roads and highways. Here we propose an established cost system in which the simulation model can guide to quality of life and business advantages.

Assuming all vehicles are only cars, the principle of congestion pricing (Pigou 1920) provides a direct curve of correlation between traffic volume and its costs. In fact, every motorist making a trip introduces personal expenses in terms of private marginal costs, *MC*, (that are operating car costs plus the value of time spent in the highway) and takes a social cost (whose average will be denoted as *AC*). The difference between *MC* and *AC* represents the cost that a driver induced on his road neighbours (Li 2002): if *c* is the hourly average generalized travel cost (as above, it is composed by car operating costs plus value of travel time) and is supposed to be invariable, *dist* is the covered distance (assumed to be 1 km in the second part of Eq. 3), *V(q)* is a function of the flow *q* and represents the speed of vehicles, then *AC*, with respect to a certain flow value *q*, is given by:

$$AC(q) = c\frac{dist}{V(q)} = \frac{c}{V(q)} \quad (3)$$

Thus the total cost $T(q)$ of those vehicles is simply $T(q) = q\ AC(q) = (qc)/V(q)$. This means that for each new vehicle joining the flow q, we have the following marginal cost for the community:

$$MC(q) = \frac{d}{dq}\left(T(q)\right) =$$
$$= \frac{V(q)c - qc\frac{d}{dq}(V(q))}{V(q)^2} = \quad (4)$$
$$= AC(q) - \frac{qc}{V(q)^2} \cdot \frac{d}{dq}(V(q))$$

Assuming that MC increases much more rapidly than AC when congestion begins (i.e. a flow $q > q'$), the difference between these two values is the considered money that motorists have to pay if we want to charge the cost they are imposing to the society. This means that the "congestion toll" r is given by:

$$r = MC(q') - AC(q') =$$
$$= -\frac{qc}{V(q)^2} \cdot \frac{d}{dq}(V(q)) \quad (5)$$

This quantity could be equal to zero when there is no congestion (i.e. flow $q <= q'$. *Cfr. Fig. 9*), increases when the flow increases and subsequently decreases when $V(q)$ increases. Now we introduce a model that is widely used and empirically verified over several highway models to establish the correlation between the flow and the speed of vehicles composing it: the Drake model (Drake 1967). Let q_0 be the maximum flow capacity (vehicles per hour per lane), V_0 the corresponding speed at maximum flow capacity and V_f the speed in free flow condition, then in the framework of Drake model, q is given by:

$$q = V(q) \cdot \frac{q_0}{V_0} \cdot \sqrt[\delta]{\delta \cdot \ln(V_f / V(q))} \quad (6)$$

The speed-flow relationship given by Eq. 6 where δ is a parameter equal to *2* (Drake 1967), can be used inside Eqs. 3-5 to estimate the congestion toll when the flow is higher than q' (*Cfr. Fig. 9*) and the Drake model is an accepted approximation. As a result the congestion toll is given by:

$$r = \frac{c}{V(q)} \cdot \frac{\ln(V_f) - \ln(V(q))}{\ln(V(q)) - \ln(V_0)} \quad (7)$$

Assuming that European Euro/km rates (Theaa 2008) are also valid for Italy, we can take cost values reported in Table 3 as input and then derive the value of *c=1,08 €/km*. Moreover, in order to resolve Eq. 7, values for V_0 and V_f are needed; while the value of speed at free flow can be considered as the one presented in Table 1 (V_f*=122,8 km/h*), the inference of a proper value for V_0 needs more attention. The evaluation of a realistic V_0 value is where our STRATUNA model can help and, in fact, leads to cost forecasting through speed forecasting.

Table 3: Total of Standing Charges and Running Costs, Assuming 15000 km per Year

	Euro/km	Euro/hour
Petrol	0,112844	0,31090068
Tyres	0,00806588	0,0222226
Service labour costs	0,02184835	0,06019521
Replacement parts	0,01472219	0,04056164
Parking and tolls	0,01409571	0,03883562
Standing charges	0,21981479	0,60561986
Total	**0,39139093**	**1,07833562**

In fact, our model has the expressivity needed for speed forecasting and has exhibited a predicting reliability for different flow volumes even in its partially implemented version (detailed in Section 2). Therefore, it can be used, together with the cost system object of this section, to foresee how different highway designs influence the speed at maximum capacity (V_0). This enables a straightforward calculation of the corresponding income for the highway owners and for the society. We now present the curves of *AC* and *MC*, as stated by Eqs. 3,4, with the aim of fixing the cost system.

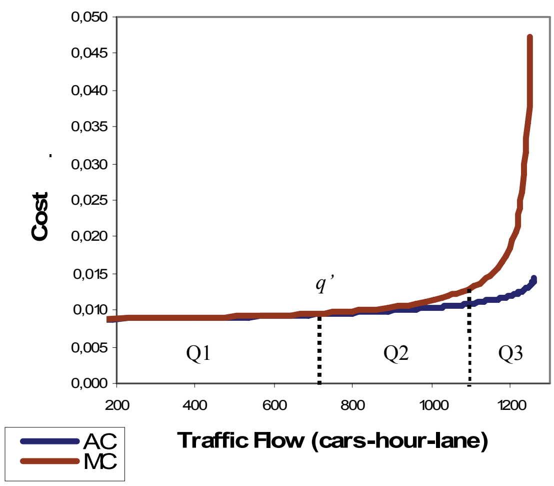

Figure 9: Cost of *AC* and *MC* in Relation to the Flow *q*

Up to a traffic volume of about 680 cars per hour per lane, the private cost of a motorist (*MC*) is, in fact, identical to the one that he imposes to others (*AC*). This, presented in Figure 9 as *Q1*, can be traced back to the free flow condition; same tracing is possible from *Q2* ad *Q3* (Cfr. Figure 9) to moderated flow and traffic jams, respectively. For quantity of cars $q > q'$ we have *AC* costs that increase more rapidly than *MC*: first linearly and then exponentially. This increasing cost, induced to others with heavier flow, can be represented by Figure 10: more cars means slower speed, that means more breaking/accelerating, low gears usage, higher petrol consumption and so on.

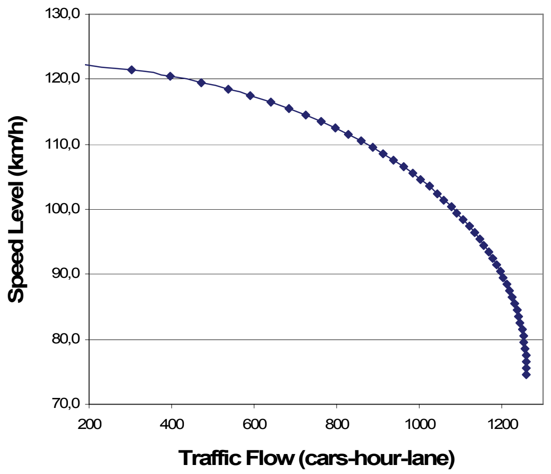

Figure 10: Speed-Flow Chart

Now that the cost system has been satisfactory detailed, we propose the congestion toll (€/km) evolution, in relation with the V_0 value deduced by our model when feed with scenarios detailed in section 2.

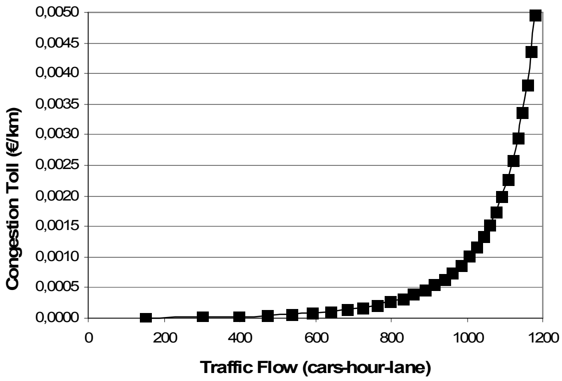

Figure 11: Congestion Toll with respect to Traffic Flow

Above results show clearly that, through a simulation model, the test of different highway designs is possible and then, to each design, is associable a simulated V_0 value, leading to the appropriate congestion toll. In other words, through the simulation of different highway design, differentiated V_0 values follow; then, the optimal congestion cost is derivable from it by means of the reported congestion toll system. As a result, we report in Table 4 the congestion toll that the price system of the simulated and analyzed highway could implement in relation to free flow, moderated flow and traffic jams.

Table 4: Congestion Toll and Different Traffic Flows

Flow type	Free flow	Moderated flow	Congestion
Corresponding minimum flow (cars per hour per lane)	0	681	1101
Corresponding maximum flow (cars per hour per lane)	680	1100	1260
Minimal congestion toll (€ per car per km)	+0,0	+0,0001	+0,0023
Maximal congestion toll (€ per car per km)	+0,0	+0,0023	+0,0339

5. CONCLUSIONS

A Cellular Automata approach for traffic flow modelling is proposed in this paper. The partial implementation $\beta 4$ presents interesting results in traffic forecasting task. Based on a validation on real data of the implemented model, the design has moved forward a more general CA simulator: the STRATUNA general model, created on the analysis of weak and strong points in implemented model along with the intent of an extended expressivity power. Moreover, when the general model implementation will be tackled, it will be possible to couple Genetic Algorithm to the simulator in order to fix information missing from the reality. As a result, the general model exhibits forecasting tools that, theoretically, outperform the mere speed prediction obtained with the preliminary implementation. The implemented model, used together with an established cost system, guides the interesting problem of the appraisal of the right price for a toll ticket. Indeed, the simulator shows the ability of associating to a simulated highway a value of average speed at maximum capacity. Thanks to this value, it is possible to establish a congestion toll mechanism. This mechanism, widely used worldwide, gives to motorists the perception of the costs they are imposing to others travelling and non-travelling people.

In conclusion, the CA approach demonstrates its validity and leads to interesting emerging phenomena, both from the traffic forecasting and from an economical point of view, where the model gives a feedback that straightforward links different highway designs to different congestion toll charges through an established cost system. In order to proceed to a further model improvement through validation, it is now important to get access to other types of data concerning highway traffic (i.e. single vehicle data), which authors are looking for at present.

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