SIMULATION AND OPTIMIZATION OF VEHICULAR FLOWS IN A HARBOUR

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ABSTRACT
This paper focuses on the simulation and optimization of car traffic, modelled through a fluid dynamic model, based on the conservation of the cars number. In particular, the case study of the harbour of Salerno, Italy, is presented. Simulations of vehicular flows are carried out by a graphical tool, that allows to reproduce the evolution of densities on the roads of the network through animated and coloured pictures. From the analysis of simulated densities on roads, obtained by a given input configuration for the network object of study, the tool is able to plan some strategies for the improvement of traffic conditions through an optimization routine.

Keywords: road networks, simulation, optimization.

1. INTRODUCTION
The aim of this paper is to present some simulations of the car traffic dynamics in the harbour of Salerno, Italy, and to consider an optimization study of vehicular flows in order to improve traffic conditions. The choice of the considered urban network, that belongs to the harbour of Salerno, is due to the fact that it is a critical point, since it separates the centre of the city from the highway. Hence, the harbour is interested either from a heavy car traffic, mainly coming from the highway, or from trucks, daily crossing around the harbour areas.

The simulations of urban traffic flows have been obtained using a numerical tool, based on a fluid dynamic model for car traffic introduced in Coclite et al. 2005. In last years, fluid dynamic approaches have been often used for several reasons. For example, they are evolutive, allowing the description of a given physical phenomenon in every instant of time; they are characterized by few parameters, and this permits simulations based on numerical schemes that are, at the same time, fast and accurate, with consequent study of networks of big dimensions. Moreover, as such models are based on conservation laws, according to which a given quantity has to be conserved, they have a wide range of applications: road and telecommunication networks, supply chains, gas pipelines, irrigation channels, etc. In particular, the model for road networks, that we consider, is based on the conservation laws formulation proposed by Lighthill and Whitham (Lighthill et al. 1955) and Richards (Richards 1956). On each single road, the traffic evolution satisfies the conservation law:

$$\partial_t \rho + \partial_x f(\rho) = 0, \tag{1}$$

where $\rho = \rho(t,x) \in [0, \rho_{\max}]$, $(t,x) \in \mathbb{R}^2$, is the density of cars, $\rho_{\max}$ is its maximal value, $f(\rho) = \rho v(\rho)$ is the flux and $v(\rho)$ the average velocity. In what follows, we assume that the velocity is a smooth decreasing function of the density, and that f is concave. A similar approach can be very useful for the description of phenomena, such as shock formation and their propagation (Bressan 2000, Dafermos 1999), although such formulation is limited for the reconstruction of some real traffic characteristics on highways, for which more rich models are necessary (Bellomo et al. 2005, Kerner 2004).

Recently, these approaches, of fluid dynamic type, have been extended to urban networks (for an exhaustive presentation, see Garavello et al. 2006b and for the relative numerical schemes Bretti et al. 2006): some are based on the LWR model (Cascone et al. 2007, Cascone et al. 2008, Godunov 1959, Helbing et al. 2005), other on the second order model by Aw – Rascle (Garavello et al. 2006a).

The greatest part of papers deals with results obtained by the LWR model because, if we consider the evolution of traffic in urban context, some particular situations (that are not present on highways, for example) can occur, for example short roads, traffic interruptions, reduced velocities, and so on, which can be captured by the LWR model.

Also optimization problems for traffic flows were addressed, in order to find the correct choice of traffic parameters for avoiding congestion phenomena and/or improving car traffic circulation: Helbing et al. 2005 is devoted to traffic light regulation, while the works of Gugat et al. 2005, Herty et al. 2003, are more related to our analysis but focus on the case of smooth solutions (not developing shocks) and boundary control.

In Cascone et al. 2007, Cascone et al. 2008, two cost functionals have been introduced to measure the traffic behaviour. The first functional J_1 measures the average velocity of drivers on the network, while the second J_2 measures the expected mean travelling time on the network. The optimization is done over right of way parameters and traffic distribution coefficients.

Starting from the model of networks with LWR and using the optimization procedure developed in Cascone et al. 2007, Cascone et al. 2008, it was possible to build a numerical software for the evolution of car traffic, which is useful in order to make some prevision of real conditions of vehicular behaviour and to indicate the best interventions on the network (in terms of traffic lights, and signals) in order to improve traffic conditions. The tool for urban traffic elaborates animated and coloured graphics to let the traffic analysis easier for users. The goodness of simulation and optimization algorithms is tested on the case study of the harbour of Salerno, via simulations.

The paper is organized as follows. First, we give some basics about the model for car traffic and the optimization techniques and numerical schemes useful to approximate the equations of the model. Then, we consider the structure of the graphical tool and present some simulation and optimization results for the harbour of the city of Salerno, Italy.

2. ROAD NETWORKS MODEL AND OPTIMIZATION TECHNIQUES

A road network consists of a finite set of roads, modelled by intervals $I_i = [a_i, b_i] \subset \mathbb{R}$, $i = 1,...,N$, $a_i < b_i$, with one of the endpoints that can be infinite. Roads are connected to some junctions, and each junction J has a finite number of incoming and outgoing roads; hence, the complete model is described by the couple (I, J), where $I = \{I_i \ : \ i = 1,...,N\}$ is the set of roads, while J is the set of junctions. On each road, the evolution is given by (1). We consider a linear decreasing velocity:

$$v(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right), \tag{2}$$

where $v_{\max}$ represent the maximal velocity of cars. The flux is given by:

$$f(\rho) = v_{\max}\rho\left(1 - \frac{\rho}{\rho_{\max}}\right). \tag{3}$$

Without loss of generality, from now on, we can consider $v_{\max} = \rho_{\max} = 1$. Consider now a junction J with n incoming roads and m outgoing roads. For simplicity, assume that I_i, $i = 1,...,n$, are the incoming roads and I_j, $j = n+1,...,n+m$, are the outgoing roads.

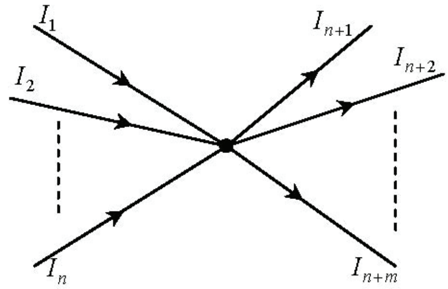

Figure 1: a road junction.

If $\rho = (\rho_1,...,\rho_{n+m})$, $\rho_i \in [0,+\infty] \times I_i$, is a weak solution (Bressan 2000) at the junction such that, for each $x \mapsto \rho_i(t,x)$ has bounded variation, then ρ satisfies the Rankine – Hugoniot relation at the junction J:

$$\sum_{i=1}^{n} f\left(\rho_i\left(t, b_i^+\right)\right) = \sum_{j=n+1}^{n+m} f\left(\rho_j\left(t, a_j^-\right)\right), \tag{4}$$

for almost $t > 0$. Notice that (4) expresses precisely the conservation of cars through the junction.

For a single conservation law (1), a Riemann Problem (shortly RP) is a Cauchy problem for a piecewise constant data with only one discontinuity. Solutions are either continuous waves, called rarefactions, or traveling discontinuities, called shocks. The velocity of waves is strictly connected to $f'(\rho)$. In a similar way, we call RP at the junction the Cauchy problem corresponding to a constant initial data on each road. As the condition (4) does not guarantee the uniqueness of solutions, further conditions are needed.

The aim is finding a way to solve RPs at junctions.

Definition. A Riemann solver for the junction J is a map $RS : [0,1]^n \times [0,1]^m \to [0,1]^n \times [0,1]^m$ that associates to a Riemann data $\rho_0 = (\rho_{1,0},...,\rho_{n+m,0})$ at J a vector $\hat{\rho} = (\hat{\rho}_1,...,\hat{\rho}_{n+m})$ and the solution on an incoming road $I_i, i = 1,...,n$, is given by the wave $(\rho_{i,0}, \hat{\rho}_i)$ and on an outgoing road I_j, $j = n+1,...,n+m$ is given by the wave $(\hat{\rho}_j, \rho_{j,0})$. The consistency condition is required: $RS(RS(\rho_0)) = RS(\rho_0)$.

For the traffic flow, in Coclite et al. 2005, in the case $m \ge n$ a Riemann solver, based on the following rules, was introduced:

(A) there are some fixed coefficients, that represent the preferences of drivers. Such coefficients indicate the distribution of traffic from incoming to outgoing roads, and they can be kept in a traffic distribution matrix:

$$A = \left\{ \alpha_{ji} \right\}_{j=n+1,\ldots,n+m,\ i=1,\ldots,n} \in \mathbb{R}^{m \times n}, \tag{5}$$

such that

$$0 < \alpha_{ji} < 1, \quad \sum_{j=n+1}^{n+m} \alpha_{ji} = 1, \tag{6}$$

for $i = 1,\ldots,n$ and $j = n+1,\ldots,n+m$, where α_{ji} is the percentage of drivers who, arriving from the i - th incoming road, direct to the j - th outgoing road.

Remark. If we refer to junctions with one incoming road ($n = 1$), *a*, and two outgoing roads ($m = 2$), *b* and *c*, respectively, then matrix *A* reduces to the column vector with components α and $1-\alpha$, where α (resp. $1-\alpha$) represents the probability that drivers could go the outgoing road *b* (resp. *c*), from the incoming road *a*.

(B) Respecting the rule (A), drivers behave such as the flux through J can be maximized.

If $m < n$, a yielding rule is needed. For example, if we consider a road junction with two incoming roads ($n = 2$), a and b, and one outgoing road ($m = 1$), *c*, we need a *right of way* parameter $p \in \left]0,1\right[$, and the yielding rule can be stated as follows:

(C) Assume that not all cars can enter the outgoing road and let C be the amount that can do it. Then, pC cars come from the first incoming road and $(1-p)C$ cars from the second one.

For further details, see Coclite et al. 2005 and Garavello et al. 2006.

In order to measure the efficiency of the network and hence obtain optimization results, in Cascone et al. 2007, Cascone et al. 2008, two cost functionals, measuring, respectively, the velocity at which cars travel through the network and the time taken by cars to travel on the network, were considered.

Since the model considers macroscopic quantities, the averages were estimated integrating over time and space the average velocity and the reciprocal of the average velocity, respectively. If ρ_i represents the density on road *i*, the following functionals were defined:

$$J_1(t) = \sum_i \int_{I_i} v\left(\rho_i(t,x)\right) dx, \tag{7}$$

$$J_2(t) = \sum_i \int_{I_i} \frac{1}{v\left(\rho_i(t,x)\right)} dx. \tag{8}$$

A fixed temporal interval $[0,T]$, for some $T > 0$, was considered.

For the regulation of traffic, the aim was to maximize J_1 and to minimize J_2. In this way, optimal parameters for regulating the behaviour of car traffic on networks were obtained.

Unfortunately, the analytical treatment of J_1 and J_2 for a complex network is very difficult, thus the following strategy, consisting of three steps, was adopted:

Step 1. Compute the optimal parameters for simple networks formed by a single junction and every initial data. For this, consider the asymptotic solution over the network (assuming infinite length roads so to avoid boundary data effects).

Step 2. For a complex network, use the (locally) optimal parameters at every junction, updating the value of the parameters at every time instant using the actual density on roads near the junction.

Step 3. Verify the performance of the (locally) optimal parameters via simulations.

The first step happens to be an hard task even for simple junctions. The reason for this is the hybrid nature of the problem, where continuous, time and space varying variables as ρ influence and are influenced by discrete variables as right of way parameters and traffic distribution coefficients. Thus, two special cases were treated: the 2×1 case with two entering and one exiting road; and the 1×2 case with one entering and two exiting roads. For the first type of junction, one has only one right of way parameter, called p. The second type of junction has no right of way parameter and only one traffic distribution coefficient α. It is possible to prove that, for the flux function that is considered here, $f(\rho) = \rho(1-\rho)$, the optimal solutions for p and α are the same either for J_1 or for J_2. For a systematic presentation of the optimization algorithms and obtained results for such cost functionals, see Cascone et al. 2007, Cascone et al. 2008.

The implementation of Step 2 is done for the case study of the harbour of Salerno. For Step 3, instead, we consider two different choices for right of way parameters and distribution coefficients: (locally) optimal (obtained through optimization algorithms), and fixed, assigned by the user on the basis of observations on the real network. Such choices are useful to test the goodness of the obtained optimization results.

3. A FINITE DIFFERENCE SCHEME FOR THE APPROXIMATION

The described mathematical model must be treated numerically in order to realize the tool for car traffic able to elaborate the densities for roads. We can refer to the finite difference method of Godunov (see Godunov 1959).

We define a numerical grid in $(0,T)\times\mathbb{R}^L$ according to the following notation:

- Δx is the space grid size;
- Δt is the time grid size;
- $(t_h, x_m) = (h\Delta t, m\Delta x)$ for $h \in \mathbb{N}$ and $m \in \mathbb{Z}$ are the grid points.

Consider the hyperbolic equation

$$\rho_t + f(\rho)_x = 0,\ \ x \in \mathbb{R},\ t \in [0,T], \tag{9}$$

with initial data

$$\rho(x,0) = \rho_0(x). \tag{10}$$

A solution of the problems is constructed taking a piecewise constant approximation of the initial data, v_0^Δ. We set

$$v_m^0 = \frac{1}{\Delta x}\int_{x_m}^{x_{m+1}} \rho_0(x)\,dx,\ m \in \mathbb{Z}, \tag{11}$$

and the scheme defines v_m^h recursively, starting from v_m^0.

Notice that waves in two neighbouring cells do not interact before time Δt if the CFL condition holds:

$$\Delta t \sup_{m,h}\left\{ \sup_{u\in I\left(u_m^h, u_{m+1}^h\right)} \left|f'(u)\right| \right\} \le \frac{1}{2}\Delta x. \tag{12}$$

Then, we define the projection of the exact solution on a piecewise constant function

$$v_m^{h+1} = \frac{1}{\Delta x}\int_{x_m}^{x_{m+1}} v^\Delta(x, t_{h+1})\,dx. \tag{13}$$

Under the CFL condition, the solutions are locally given by the Riemann Problems and, in particular, the flux in $x = x_m$ for $t \in (t_h, t_{h+1})$ is given by

$$f(\rho(t,x_m)) = f\left(W_R\left(0;\ v_{m-1}^h,\ v_m^{\ h}\right)\right), \tag{14}$$

where $W_R\left(\frac{x}{t};\ v_-,\ v_+\right)$ is the self – similar solution among v_- and v_+. As the flux is time invariant and continuous, setting $g^G(u,v) = f\left(W_R(0;\ u,v)\right)$ under the CFL condition, the scheme can be written as:

$$v_m^{h+1} = v_m^h - \frac{\Delta t}{\Delta x}\left(g^G\left(v_m^h, v_{m+1}^h\right) - g^G\left(v_{m-1}^h, v_m^h\right)\right). \tag{15}$$

The numerical flux g^G, for the flux we are considering, has the expression:

$$g^G(u,v) = \begin{cases} \min\left(f(u), f(v)\right), & if\ \ u \le v, \\ f(u), & if\ \ v < u < \sigma, \\ f(\sigma), & if\ \ v < \sigma < u, \\ f(v), & if\ \ \sigma < v < u, \end{cases} \tag{16}$$

where σ represents the value of ρ such that $f(\sigma) = \max_{\rho\in[0,\ \rho_{\max}]} f(\rho)$.

4. A TOOL FOR THE MANAGEMENT OF ROAD TRAFFIC

The tool for car traffic consists of three components:

- a web configuration component for the topological representation of the network, realized by a Java applet.
- The core of the application for the numerical methods, approximating conservation laws, that can be required by the Java applet.
- An intermediate layer, that allows the communication among the calculus core, the File System (necessary for saving informations) and web components.

We report the UML scheme of the application:

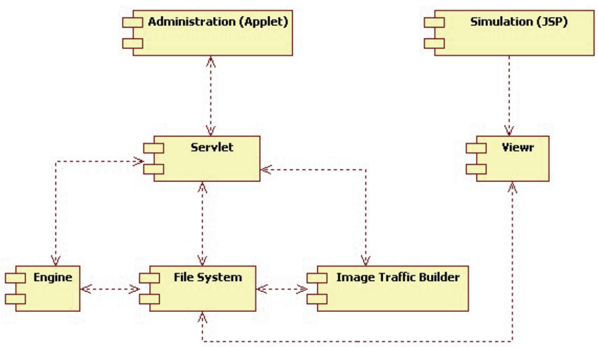

Figure 2: UML scheme.

We analyze the various components of the previous scheme.

File System
It is necessary for storing the various informations produced by the tool. In particular, inside the folder of the realized simulation project, three different folders are contained:

- the folder *InputFile* that contains .dat files, generated by the administration applet and used by the calculus engine for the elaboration of input informations.
- The folder *OutputFile* in which all .dbf files, that contain the numerical results of the simulation for every instant of time and produced as output of the engine, are stored.
- The folder *Images* with the pictures of the traffic state along the network as function of the corresponding time instant.

Administration Applet
It gives all the instruments to draw the topology of the network, with the insertion of all informations useful for the various arcs. Such information are stored in .dat files, contained in the File System in the folder *InputFile*. Such informations represent the input file for the calculus engine.

Engine
It is the numerical core for the application. It has access to the folder named *InputFile*, elaborates the .dat input files and produces .dbf files as output. Such files contain, for every instant of time, the densities for the various segments of each arc of the network to simulate. Such files are stored in the folder *OutputFile*.

Image Traffic Builder
It is the modulus that translates the numerical results (elaborated in .dbf files) of the folder *OutputFile* in images .jpg. Such images are stored in the folder *Images*.

Simulation JSP
It presents to the user all the saved projects for which animations are available (in such way that the user can refer to them if he wants).

Applet Viewer
The applet opens the files Images of the project associated to the simulation and executes in sequence the images of the simulation.

Servlet
This component allows to give to the applet all services available for building and simulating the network.

The tool for simulations is characterized by a simple user friendly interface. In particular, the use of mouse and some buttons allows to draw easily the network to analyze. For example, to draw arcs, it is necessary to click by the left button of the mouse at the center of the node and to direct the cursor toward the center of the node, that we want to connect (Figure 3).

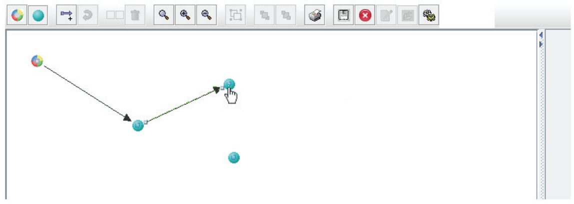

Figure 3: creation of an arc among two nodes.

For every arc and node, a menu is associated which is visualized clicking twice on them by the right button of the mouse. The menu for nodes consists of only one feature: *remove*, useful to remove the selected node. Instead, the menu for arcs is characterized by two features: *remove*, by which it is possible to clean the selected arc, and *properties*, by which it is possible to insert some characteristics for each arc (Figure 4). Such characteristics are the following:

- **Name:** name of the road;
- **Start Value:** initial density on the road;
- **Start Density:** incoming flux on the road (necessary as boundary data, if the node is virtual);
- **Precedence:** the right of way parameter (necessary if the road belongs to a junction with a number of incoming roads greater than the number of outgoing roads);
- **Distribution:** the distribution parameter, necessary to define traffic distribution matrix for the junction;
- **Length:** length of the road (in meters).

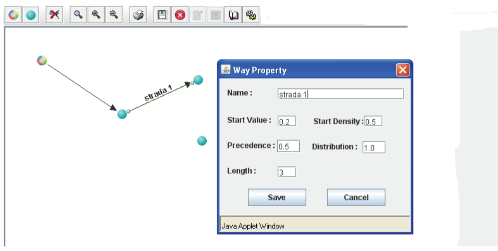

Figure 4: Edge property window.

After the simulation time has been chosen, the tool allows to reproduce the traffic evolution of the considered area. Different intervals of car densities are represented with different colours, as shown in the following table.

Table 1: Colours for densities.

Density interval	**Colour**
0.0 ≤ density < 0.1	
0.1 ≤ density < 0.2	

$0.2 \leq$ density < 0.4	
$0.4 \leq$ density < 0.6	
$0.6 \leq$ density < 0.8	
$0.8 \leq$ density ≤ 1.0	

An optimization routine allows to set the characteristic parameters in such a way to avoid congestion phenomena.

5. THE CASE STUDY OF SALERNO HARBOUR

We present some simulation results for the harbour of Salerno, Italy. The network, built through the numerical software, is the following (Figure 5):

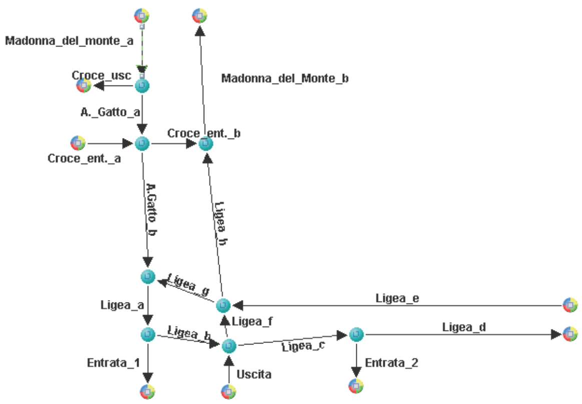

Figure 5: Network, that represents the harbour of Salerno.

In the following table, we show the input parameters for the network (notice that the incoming flux, that corresponds to the boundary data, is necessary only for roads connected to only one junction, which is to say roads with an infinite endpoint):

Table 2: Parameters for roads.

Name of the road	Initial density	Incoming flux	p	α
Madonna del monte a	0	0.6	1	1
Croce usc.	0	0	1	0.3
A. Gatto a	0	/	0.8	0.7
Croce ent. a	0	0.6	0.2	1
Croce ent. b	0	/	0.2	0.5
A.Gatto b	0	/	0.7	0.5
Ligea a	0	/	1	1
Ligea b	0	/	0.6	0.6
Entrata1	0	0	1	0.4
Uscita	0	0.3	0.4	1
Ligea c	0	/	1	0.6
Entrata 2	0	0	1	0.1
Ligea d	0	0	1	0.9
Ligea e	0	0.6	0.7	1
Ligea f	0	/	0.3	0.4
Ligea g	0	/	0.3	0.5
Ligea h	0	/	0.8	0.5
Madonna del monte b	0	0	1	1

These parameters are chosen by the user on the basis of the real networks observations and measures. We report the state of the considered vehicular flux in different instants of time. Such tool is very useful in order to detect queue formation on roads.

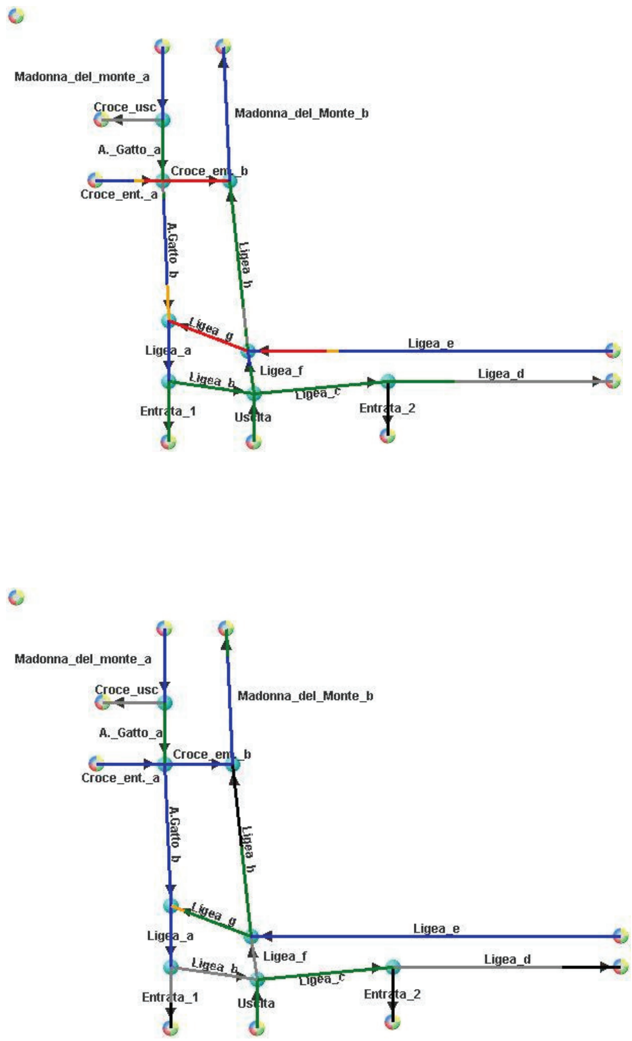

Figure 6: situation of the network at t = 40 (up) and t = 60 (down).

Let us refer to the Figure 6. At t = 40, we observe that, at the junction among *Via Gatto b* and *Via Ligea g* to *Ligea a*, there is a preliminary formation of queues along *Ligea g,* and this occurs because cars coming from these roads have not right of way with respect to cars coming from *A. Gatto b*.

It is interesting to notice how, as there are no cars that, from *via Ligea h*, have not reached the junction of *Croce ent b*, the flux coming from this last road is totally directed to *Madonna del monte b*.

At $t = 60$, the image underlines how the congestion along *Ligea g* is also propagating along *Ligea e*. In such instant, the flux of cars along *Ligea_h* has reached the junction of *Croce_ent_b*, and, on this last road, there is a queue formation due to a less right of way of the road itself with respect to *Ligea_h*.

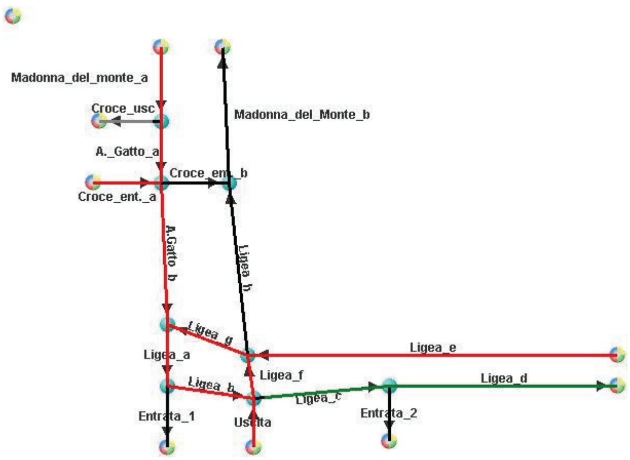

Figure 7: situation of the network at $t = 264$.

In the final instant of simulation (Figure 7, $t = 264$), there is a congestion phenomenon at the exit of the highway (*via Madonna del monte a*), and from the city center (*Ligea e*) to the junction among *via Ligea_e*, *Ligea f* and *Ligea g*. Moreover, the behaviour of the considered traffic is such that roads, that are directed to the highway, *Ligea h*, *Madonna del monte b* e *Croce ent b* are almost empty.

The simulative tool, on the basis of the network traffic characteristics, is able to suggest the user which are the more congested areas, on which it is suitable to make some interventions for the improvements of cars flows. The optimization procedure, once that a road junction, which presents congestions problems, is chosen, gives the optimal values of the characteristic parameters that can alleviate the congestion.

It is evident that queues and backward propagation occur on the road junction, characterized by two incoming roads, *A. Gatto b* and *Ligea g*, and one outgoing road, *Ligea a* (see Figure 8); hence, some optimization criteria are needed. In this case, the optimization of right of way parameters for the incoming roads is necessary.

Figure 8: junction to optimize.

Let us give a meaning to the optimization outputs in terms of interventions on traffic signals or on traffic lights.

The modification of traffic signals corresponds, in the fluid dynamic model, to a different choice of traffic parameters, which regulate the behaviour of traffic at a road junction. For a better comprehension, we can refer to the following figure:

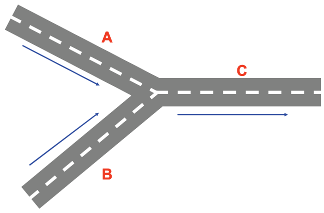

Figure 9: road junction with two incoming roads, *A* and *B*, and one outgoing road, *C*.

In this case, the regulation of traffic conditions for the road junction are the right of way parameters for the incoming roads *A* and *B*. In particular, if p is the right of way parameter for the road *A*, $1 - p$ is the corresponding one for the road *B*. Suppose that, for road *A*, a right of way parameter $p \in \left]0,\ 0.25\right[$ (and then for road *B* $q=1-p\in\left[0.75,\ 1\right[$) was obtained from the optimization procedure. The tool suggests that, to improve the traffic conditions, road *B* must have a higher right of way than road *A*. To obtain this, a stop signal for the road *A* can be introduced.

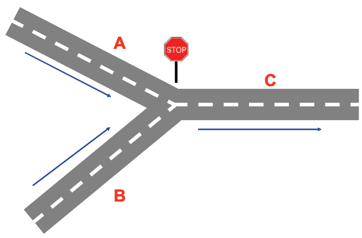

Figure 10: a STOP signal along road *A*.

Suppose that the optimal right of way parameter for road *A* is $p \in \left[0.25,\ 0.4\right]$. In this case, as road *B* has not a so high right of way as in the previous case, it is possible to insert a right of way signal for the road *A*, as in Figure:

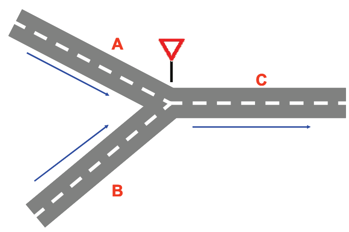

Figure 11: a yielding signal along road *A*.

Such traffic signals guarantee that cars, coming from the road *A*, will be able to stop if some cars are crossing the junction, coming from road *B*. Otherwise, they will be able to cross the junction, avoiding to necessarily stop, as in the previous case.

The optimization routine, applied to the chosen junction of Figure 8, indicates that a stop sign should be used on *Ligea g* and then some decongestion phenomena occur. The performances of the network improve, as indicated by Figure 12, that shows the behaviour of the functional J_1 in the optimal case and in the fixed case (the simulation defined by the user on the basis on real observations on the network). Notice that the primal aim, the maximization of the cars velocity, has been reached.

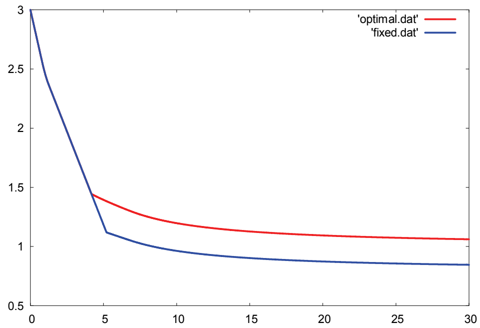

Figure 12: behaviour of the cost functional J_1 vs time in the optimal case (red) and in the fixed case (blue).

The insertion of traffic signals is surely the simplest intervention. However, in limit situations, determined by a high number of vehicles and frequent congestions, also in different hours of the day where it is necessary that drivers respect traffic rules, it is suitable to adopt traffic lights, which have to be adequately temporized.

Consider the road junction in Figure 13.

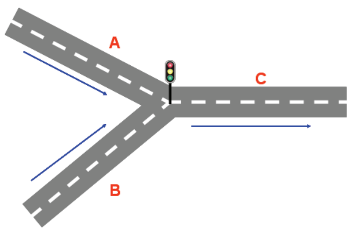

Figure 13: road junction with a traffic light.

The traffic light, in a generic instant of time, is red for a road and green for the other. In particular, if drivers for road *A* see the red phase, then drivers of road *B* can circulate. Hence, road *A* is characterized by a zero right of way parameter and, on the contrary, road *B* has a right of way parameter equal to 1. On the contrary, if drivers for road *A* see the green phase, they can circulate. Suppose that the optimization procedure establishes that, from the road *A*, a given percentage of traffic flux, *p*, should go the outgoing road. In this case, an adequate temporization of traffic light cycles is necessary. In particular, let Δ_v, Δ_r, and $T = \Delta_v + \Delta_r$, be, respectively, the green time, the red one, and the complete traffic light cycle. In this case, as *p* represents the percentage of drivers who, from road *A*, must cross the junction on average, then such parameter can be interpreted as the ratio among the green cycle and the total traffic light cycle. Hence, the road *B* is characterized by an averaged right of way parameter equal to $1-p$, or the ratio among the red cycle and the total cycle time. As a consequence, if *p* is the optimal right of way parameter, it is useful to design the traffic cycles such that $\Delta_v = pT$ e $\Delta_r = (1-p)T$.

In the previous example, for the optimization of car traffic either with traffic signals or with a traffic light, a road junction with two incoming roads and one outgoing road is considered. Such choice is not accidental. The typology of the presented junction is the most difficult to optimize, as it requires the modification of the parameter, that regulates the behaviour of car traffic at the junction, or the right of way parameter for a given road. The choice of such parameter is independent on the destination of drivers and, as a consequence, from paths that drivers choose to reach it.

The discussion made here for the optimization is valid for right of way parameters. As for traffic distribution coefficients, their optimization corresponds to some decisional criteria for the habits of drivers, that can change in cases of particular situations, as congestions. In such sense, distribution coefficients optimization allows the redirection of car flows to more free urban areas.

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