CRITICALITY ASSESSMENT VIA OPINION DYNAMICS

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ABSTRACT
In this paper a framework for merging clashing information is introduced based on the Hegelsmann and Krause opinion dynamics model, which represents the social behavior of humans taking decisions together. Such a model differs from traditional consensus models, since the group of agents tends to distribute the opinions into several clusters. With respect to the original model, where the agents were influenced by the estimations of the others provided that their difference in opinion was smaller than a global parameter, in this paper a different value of reliability is associated to each piece of information. In this way it is possible to implement an assessment framework for the criticality of the situation in a critical infrastructure or homeland security scenario based on several clashing information, taking also into account the reliability of the source.
The result is a framework able to suitably combine different pieces of information, each with a given reliability in order to derive the most likely value (i.e., the opinion that is reached by the greatest fraction of agents), by resorting to an analogy with human decision making dynamics.
Finally the possibility to apply the framework in a distributed fashion is investigated, analyzing different complex network topologies.

Keywords: Situation Assessment, Opinion Dynamics, Critical Infrastructures, Distributed Agent Based Systems.

1. INTRODUCTION
In the literature the characterization of the behavior of interacting agents that cooperate in order to reach an agreement has been widely investigated (Olfati-Saber et. al., 2007). The interaction among the agents is usually described by means of a fixed or varying network topology encoding the communication infrastructure; in this way it is possible to describe the interaction arising among entities such as in multi-robot systems or in sensor networks. While most of the studies in the literature (Olfati-Saber et. al., 2007) provide methodologies for the distributed averaging of several opinions or data, inspecting the conditions that lead to the actual average, in real contexts involving humans it is quite frequent to notice a clusterization of opinions (Groot, 1974; Leherer, 1975).

A similar behavior is expected when the criticality of a given critical infrastructure or homeland security scenario is evaluated, since there is the possibility to have spoofed/fake information, diffused by malicious attackers in order to underestimate/overestimate the actual ongoing situation. Recently, dynamic models representing such behavior, namely *Opinion Dynamics* models, gained momentum rapidly. Within such frameworks, as in consensus models, agents are assumed to interact in order to reach an agreement on their opinion. The peculiarity of these approaches is related to the topological structure of the agents interaction which is defined by the closeness in their points of view. Indeed, when studying this class of problems, it is reasonable to consider that an agreement between two agents may take place only if their opinions are sufficiently close to each other. The question arising is whether the agents will converge to a shared opinion or split into clusters. The group of agents can be very small, thus modeling the decision process of a judgment court or a team of experts, or can be very vast, thus representing the whole population of a country. In both cases it is interesting to model the process by which the opinion of the different individuals may converge to a common value or, conversely, these opinions may become fragmented, thus dividing the agents into clusters. In the literature several opinion dynamics models have been proposed (Groot, 1974; Leherer, 1975; French, 1956; Chatterjee and Seneta, 1977; Hegselmann and Krause, 2002). Among the others, the Hegelsman-Krause Model (HK), first introduced in (Hegselmann and Krause, 2002), and then further investigated in (Lorenz J., 2006; Mirtabatabaei and Bullo, 2011; Kurza and Rambaua, 2010; Constantin Morarescu and Girard, 2010;Blondel et al., 2009; Gasparri and Oliva , 2012; Oliva et. al., 2012; Oliva, 2012) is the most widely studied. It relies on the assumption that the opinion of each agent can be represented by means of a real value, thus modeling a scoring or a vote. In this paper, the Hegelsmann-Krause opinion dynamics model, introduced to model the social behavior of humans taking decisions together, is adopted as a framework for the merging of clashing information, each characterized by a reliability value. To this end a set of agents, each holding an initial opinion, is considered and it is assumed that the single agent is influenced by the values of the others, depending on how close their opinions are (e.g., depending on the reliability). In order to obtain such result, the original HK model is slightly modified, considering a reliability value for each agent, rather than a global parameter. Specifically, the modified model assumes that an agent will not influence another one if they have very distant opinions, unless its reliability is very high; conversely, agents with close opinions will be likely to influence each other unless their reliability is remarkably small. The result is a sophisticated framework for the composition of clashing opinions, where each agent is characterized by a reliability value; the opinions of the agents will split during the evolution of the system, leading to several clusters of opinion, each characterized by the number of agents composing the cluster. The idea, therefore, is to select the opinion of the cluster with higher cardinality as the most likely estimate of the group of agents (i.e., the resulting merged value). A crucial enhancement of such a methodology is how to let a sensor network perform a distributed agreement without resorting to a central processing unit (e.g. SCADA system). This would indeed contribute to increase the local awareness of smart equipment distributed within the field of critical infrastructures such as power grids or telecommunication networks. The underlying idea, in this case is to consider agents with local computational capability, able to interact with their neighbors according to a network topology and to exchange their opinions with such neighbors. Hence, in order to provide a distributed decision algorithm, the proposed model is endowed with a network topology, showing some simulation results depending on particular complex network topologies.

The paper is organized as follows: after an overview of the Hegelsman-Krause Opinion Dynamics Model, the proposed extension is discussed, as well as the application of the framework in a distributed fashion; finally some conclusive remarks are collected.

2. H-K OPINION DYNAMICS MODEL

Let a set of N agents, each with an initial opinion, that interact mutually, influencing their points of view. In the Hegelsman-Krause (HK) model the agent's opinion is represented by real numbers. This allows to model the behavior of a team of experts that have to reach consensus on the magnitude of a given phenomenon, e.g., the expected economic loss in a given nation, or to decide a scoring, e.g., for a project funded by an institution. The key idea of the HK model is that an agent is not completely influenced by the opinions of the others, nor completely indifferent. This behavior can be obtained by letting each agent take into account the other agents standpoint to a certain extent while forming its own opinion. This implies that agents with completely different opinions will not influence each other, while some sort of mediation will occur among agents whose opinions are close enough. This process, which is iterated several times (e.g., voting sessions), can be described by means of a discrete-time model. Let $z_i(k) \in \Re$ be the opinion of i-th agent at time step k and let $z(k) = [z_1(k), \cdots, z_n(k)]^T$ be the vector of the opinions of all the agents. The i-th agent will be influenced by opinions that differ from his own no more than a given confidence level $\varepsilon \geq 0$. Hence the *neighborhood* of an agent for each time step k can be defined as:

$$N_i(k) = \left\{ j \in \{1, \ldots, N\} : \quad | z_j(k) - z_i(k) | \leq \varepsilon \right\} \qquad (1)$$

Note that, at each step k, $N_i(k)$ contains the i-th agent itself. This models the fact that each agent takes into account also its current opinion to form a new one. The last ingredient of the model is the opinion influence mechanism, that is the average of the opinions in $N_i(k)$ for each agent i. Intuitively, the reader may expect that, iterating the average of the opinions, the agents will rapidly reach a consensus. Unfortunately, the HK model has a much more complex behavior. The HK dynamic model is in the form:

$$z(k+1) = A(z(k))z(k), \qquad z(0) = z_0 \qquad (2)$$

Where $A(z(k))$ is the time-varying (actually state-dependent) n x n adjacency matrix whose entries $a_{ij}(k) = 1/|N_i(k)|$ if $j \in N_i(k)$ and $a_{ij}(k) = 0$ otherwise, where $|N_i(k)|$ is the cardinality of $N_i(k)$. An important aspect of this model is the nature of the initial opinion profiles. Two different classes are considered in the literature (Hegselmann and Krause, 2002; Mirtabatabaei and Bullo, 2011), that is: the *equidistant profile*, where $z_i(0) = (i-1)/(n-1)$ with $z_i(0) \in [0,1]$; the *random profile*, where the opinions are uniformly distributed within $[0,1]$.

Several works can be found in the literature, which attempt to characterize the properties of the HK model. In (Hegselmann and Krause, 2002) it is conjectured that for every confidence level ε there must be a number of agents n such that the equidistant profile leads to consensus (i.e., a single shared opinion for all the agents), while in (Mirtabatabaei and Bullo, 2011) it is conjectured that, for any initial opinion profile, there exists a finite time after which the topology underlying the $A(z(k))$ matrix (i.e., the structure of the mutual influence among agents) remains fixed. In (Blondel et al., 2009) it is proven that, during the evolution of the system, the order of the opinions is preserved, that is $z_i(0) \leq z_j(0) \Rightarrow z_i(k) \leq z_j(k)$ for all k. Moreover it is proved that, if the initial opinion profile is sorted, the smallest opinion $z_1(k)$ is nondecreasing with time and the largest opinion $z_n(k)$ is non increasing with time. Clearly at any step k if $|z_i(k) - z_{i+1}(k)| > \varepsilon$ this remains true for any subsequent step, and the system splits into two independent subsystems. In (Dittmer, 2001; Lorenz, 2005) the stability of the dynamical model is investigated. In particular the fact the system converges to a steady opinion profile in finite time is proven in (Blondel et al., 2009). However, the fact the system might converge to a common opinion or split into clusters is still under investigation. Experimental results suggest that the number of clusters tends to increase linearly with ε, and indeed the inter-cluster distance appears to be bounded by ε (although with irregularities for a small subset of values of ε), although a formal proof of such behavior is yet to be provided.

Figure 1 shows an example of result of the HK model with n=100 agents, for different values of the parameter ε; clearly the number of clusters tends to decrease when grows.

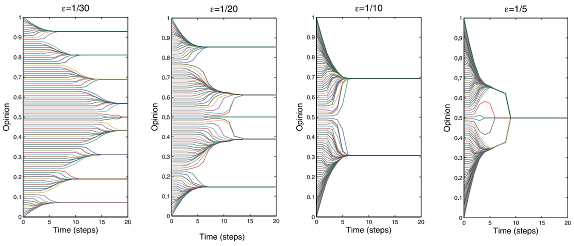

Figure 1 Simulation of Opinion Dynamics for n=100 agents and for different values of ε.

The origin of the complexity of this model is evidently the time-varying nature of the problem, and in particular the dependency on the state of the system. Note that, if the coefficients of the *A(z(k))* matrix are fixed (Groot, 1974; Leherer, 1975) the problem is significantly simplified. In order to better understand the complexity of the HK model, let us briefly discuss the difference (Gasparri and Oliva, 2012) between this model and the consensus model.

Let us consider a time-varying graph $G = \{V, E(k)\}$ with $V = \{1, \dots, n\}$ the set of nodes and $E(k) = \{e_{ij}(k)\} \in V \times V$ the set of links at time k, and let us recall that for a discrete-time first order average consensus over $G(k)$ the dynamics of an agent i is defined as follows:

$$z_i(k+1) = z_i(k) + \tau \sum_{j \in V} \gamma_{ij}(k)\left[z_j(k) - z_i(k)\right] \qquad (3)$$

where $\gamma_{ij}(k)$ is the entry of the time varying adjacency matrix $\Gamma(k)$ of the graph *G(k)* and $\gamma_{ij}(k) = 1$ if $e_{ij}(k) \in E(k)$ and zero otherwise. The parameter $\tau \in [0, 1/d_{\max}(k)]$ where $d_{\max}(k)$ is the maximum degree of the graph at time k and is given by:

$$d_{\max}(k) = \max_i \{\sum_{j=1}^{n} \gamma_{ij}(k)\} \qquad (4)$$

The dynamics of the overall system is:

$$z(k+1) = [I - \tau L] z(k) \qquad (5)$$

Where *L(K)* is the Laplacian matrix of the graph *G(k)* encoding the network topology at time , whose elements $\{l_{ij}\}$ are equal to the degree $d_i(k)$ of node *i* if *i=j* and is equal to $-\gamma_{ij}(k)$ else.

The following lemma (Gasparri and Oliva, 2012) points out a parallelism between the two problem formulations.

Lemma 1 *Let us consider the HK model given in eq.* (2). *Then, the dynamical matrix can be restated as* $A(k) = I - D(k)L(k)$ *where L(k) is a time-varying laplacian matrix and D(k) is a diagonal matrix whose elements are* $d_{ii}(k) = 1/|N_i(k)|$.

The above result emphasizes an interesting difference between consensus and opinion dynamics. For the consensus problem, the parameter τ is defined with respect to the maximum out-degree $d_{max}(k)$ of the network (over all the steps). Therefore, τ represents a global parameter common to all agents. Differently, for the HK opinion dynamics problem, there is a diagonal *D(k)* matrix of parameters where each entry $d_{ii}(k)$ is a local parameter inversely proportional to the neighborhood $N_i(k)$ of the i-th agent at time k. Finally, let us point out that for the (time-varying) consensus over a graph the convergence is related to the fact that the graph is *jointly connected* (Moshtagh and Jadbabaie, 2007) (i.e., the union of the graphs over a given time interval contains a directed spanning tree). Unfortunately, this assumption is not generally verified by the HK model due to the particular choice of the interaction policy which can lead to the isolation of some nodes, and thus to a Laplacian matrix with rows of zeros.

3. DISTRIBUTED OPINION DYNAMICS WITH HETEROGENEOUS RELIABILITY

In order to adopt the HK model as a framework for the composition and the filtering of several clashing opinions, an essential step is to modify the model in order to take into account for the reputation of each sensor. Specifically, let a reputation value $\varepsilon_i \in [0,1]$ be defined for each agent *i*, where $\varepsilon_i = 1$ means completely trustworthy information and $\varepsilon_i = 1$, means completely unreliable or fake information. Based on such reputation values, let us define the *neighborhood* of an agent for each time step k as follows:

$$N_i(k) = \{\ j \in 1, \dots, n : \ |z_j - z_i| \le \varepsilon_j\ \} \qquad (6)$$

in this way each agent *i*, for each time step, is influenced by the opinion of an agent *j* provided that the (absolute value of the) difference in their opinions is less than the reputation value of agent *j*, hence each agent *I* evaluates the reliability of the other agents. Figure 2 shows an example of application for *N=100* agents with equispaced initial opinion profile, each with a random reputation between 1/40 and 1/9. Note that in this case the cluster with greater cardinality (42 agents) has an opinion (0.257) that is very distant from the

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theoretical average (0.5) and from the weighted average considering the reliability values as weights (0.479), thus modeling a complex decision process where the reliability of the information provided by each agent is considered.

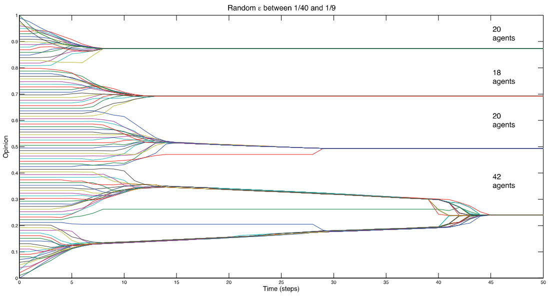

Figure 2: Simulation for n=100 agents with equispaced initial opinion profile, each with a random reputation between 1/40 and 1/9.

3.1. Distributed estimation via H-K model

The above framework is indeed a centralized framework, where the agents communicate with opinion-dependent neighborhoods, although theoretically the agents are free to communicate with each other. In order to provide a distributed framework, there is the need to endow the agents with a network topology, thus constraining the communications for an agent to the set of agents that are its neighbors according to both the network topology and the opinion differences. Let $\Gamma = \{\gamma_{ij}\}$ be the adjacency matrix that represents the topology of the network, and specifically a matrix whose coefficients $\gamma_{ij} = 1$ if there is a link between agent *i* and agent *j* (we assume each $\gamma_{ii} = 1$). Let us define the neighborhood for this case as
:

$$N_i(k) = \{ \; j \in 1,\dots,n: \quad |z_j - z_i| \le \varepsilon_j \quad \gamma_{ij} > 0 \quad \} \qquad (7)$$

The following figures show some simulations for n=100 agents with equispaced initial opinion profile, each with random $\varepsilon_i \in [\; 1/100, \;\; 1/10 \;]$, over different complex network topologies (considering the same reliabilities along the different simulations).

Figure 3 shows a simulation over an Erdos-Renyi random network with a maximum of *m=10* link per node; in this case the agents tend to spread in several opinions and the cluster with higher cardinality has 9 agents and has a value of 0.27.

Figures 4 and 5 show a simulation over a Scale-free network with a maximum of m=5 and m=10 links per node, respectively; in this case it is possible to notice few clusters of high cardinality and several clusters with small cardinality (indeed Scale-free networks have few highly connected hubs and many nodes with few links). In the first case (e.g., m=5, see Figure 4) the cluster with higher cardinality (31 agents) has a value of 0.14, while in the second case (e.g., m=10, see Figure 5) the value is 0.15 and the cardinality is 29.

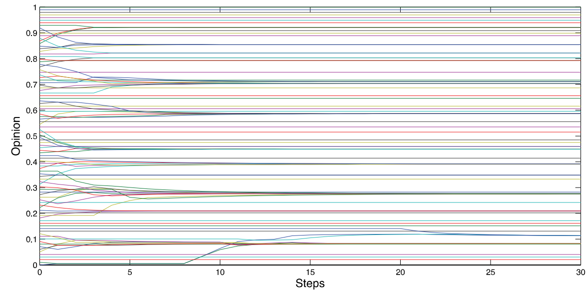

Figure 3 Simulation for n=100 agents with equispaced initial opinion profile, each with a random reputation between 1/00 and 1/10, over a random topology where a maximum of m=10 links were allowed.

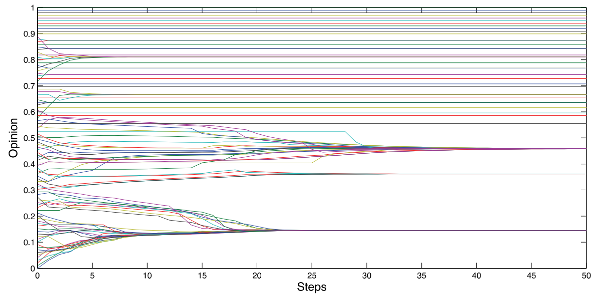

Figure 4: Simulation for n=100 agents with equispaced initial opinion profile, each with a random reputation between 1/100 and 1/10, over a Scale-free topology where a maximum of m=5 links were allowed.

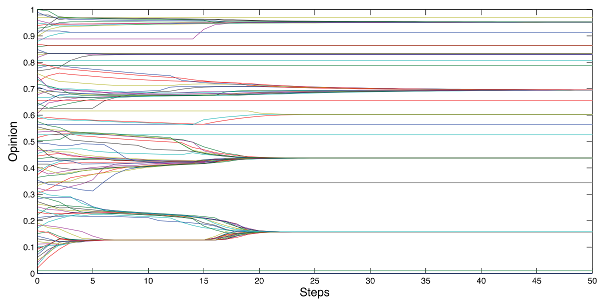

Figure 5: Simulation for n=100 agents with equispaced initial opinion profile, each with a random reputation between 1/100 and 1/10, over a Scale-free topology where a maximum of m=10 links were allowed.

As shown in Figure 6 and 7, the situation is different when a Small-world network is adopted (e.g., a lattice with rewiring probability p=0.3). Specifically Figure 6 depicts a scenario where a maximum of m=5 links is allowed for each agent, while in Figure 7 m=10 links are allowed. In this case few clusters are obtained, and the time required for obtaining a steady state is inversely dependent on the number m of maximum links per node. Notice that in this case the largest cluster coincides (the value obtained is 0.18 and the cardinality is 35 for both Figure 6 and 7).

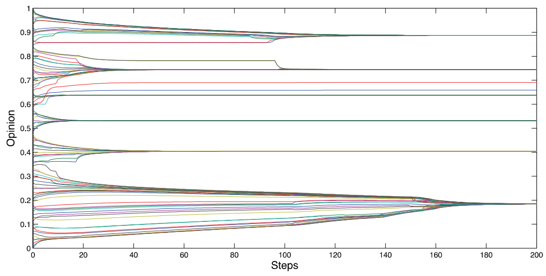

Figure 6: Simulation for n=100 agents with equispaced initial opinion profile, each with a random reputation between 1/100 and 1/10, over a Small-world topology (a lattice with rewiring probability p=0.3) where a maximum of m=5 links was allowed.

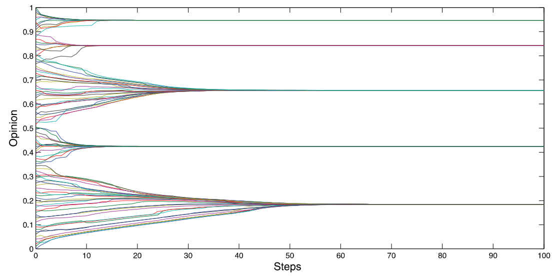

Figure 7: Simulation for n=100 agents with equispaced initial opinion profile, each with a random reputation between 1/100 and 1/10, over a Small-world topology (a lattice with rewiring probability p=0.3) where a maximum of m=10 links was allowed.

4. CONCLUSIONS

In this paper a framework for the composition of pieces of information with heterogeneous reliability that mimics the human decision-making process has been provided extending the Hegelsmann-Krause opinion dynamics model. The framework has been also extended in order to consider network topologies and therefore a distributed process. Future work will be devoted to compare such model with other approaches for sensor fusion and information merging existing in the literature. In particular, since each agent has a single initial opinion, the method is expected to be less descriptive than Dempster-Shafer Evidence Theory framework (Dempster, 1967), where each agent provides a belief in the power set of the possible hypotheses. To solve this issue future work will investigate the extension of the framework to interval opinions and to fuzzy opinions, adopting the framework in (Gasparri and Oliva, 2012; Oliva et. al., 2012; Oliva, 2012). Another issue yet to be solved is how to adopt such system in a distributed way and specifically how to select the cluster with higher cardinality in a distributed way, since each node would assume a final opinion without knowing the opinions of the other clusters. As a matter of fact, the network of distributed agents split into several clusters of opinions: however, being the agents interconnected by means of a network topology a max-consensus may be setup in order to spread the value of the cluster with greater cardinality.

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REFERENCES

Blondel V., Hendrickx J., and Tsitsiklis J., 2009. On Krause's multi-agent consensus model with state-dependent connectivity, *Automatic Control, IEEE Transactions on*, vol. 54, no. 11, pp. 2586–2597.

Chatterjee S. and Seneta E., 1977, Toward consensus: some convergence theorems on repeated averaging. *J. Appl. Prob.*, vol. 14, pp. 89–97.

Constantin Morarescu I. and Girard A., 2010. Opinion Dynamics with Decaying Confidence: Application to Community Detection in Graphs, *Transaction on Automatic Control*.

Dempster A.P., 1967. Upper and lower probabilities induced by a multivalued mapping, *The Annals of Mathematical Statistics*, vol. 38, no. 2.

Dittmer J.C., 2001. Consensus formation under bounded confidence, *Nonlinear Analysis-Theory Methods and Applications*, vol. 47, no. 7, pp. 4615–4622.

French J. R. P., 1956. A formal theory of social power, *Psychological Review*, vol. 63, pp. 181–194.

Gasparri A. and Oliva G., 2012. Fuzzy opinion dynamics, *American Control Conference 2012 (ACC2012),*.

Groot M. H. D., 1974. *Reaching a consensus*. Wiley.

Hegselmann R. and Krause U., 2002. Opinion dynamics and bounded confidence models, analysis, and simulation, *Journal of Artifical Societies and Social Simulation (JASSS) vol*, vol. 5, no. 3.

Kurza S. and Rambaua J., 2010. On the hegselmann-krause conjecture in opinion dynamics, *Journal of Difference Equations and Applications*, vol. 17, no. 6, pp. 859–876.

Leherer K., 1975. *Social consensus and rational agnoiology*. Wiley.

Lorenz J., 2005. A stabilization theorem for continuous opinion dynamics, *Physica A*, vol. 355, no. 1, pp. 217–223.

Lorenz J., 2006. Consensus strikes back in the Hegselmann-Krause model of continuous opinion dynamics under bounded confidence, *Journal of Artificial Societies and Social Simulation*, vol. 9, no. 1, p. 8.

Mirtabatabaei A. and Bullo F., 2011. Opinion Dynamics in Heterogeneous Networks: Convergence Conjectures and Theorems, *Arxiv preprint arXiv:1103.2829*, no. March, pp. 1–22.

Moshtagh N. and Jadbabaie A., 2007. Distributed geodesic control laws for flocking of nonholonomic agents, *IEEE Transactions on Automtic Control*, vol. 54, no. 4, pp. 681 – 686.

Olfati-Saber R, Fax J. A. and Murray R. M., 2007. Consensus and cooperation in networked multi-agent systems, *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233,

Oliva G., 2012. Stability and Levelwise Representation of Discrete-time Fuzzy Systems, International Journal of Fuzzy Systems, vol. 14, n.2, pp. 185-192.

Oliva G., Panzieri S. and Setola R., 2012. Distributed synchronization under uncertainty: A fuzzy approach, Fuzzy Sets and Systems, vol. 206 pp. 103–120.